Fractal Dimension, Cohaesal and Imaginary Dimension

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Abstract

Numerous irregular and fragmented complex forms in nature can be effectively generated and described by fractal geometry through self-similar iteration. However, fractal self-similarity in nature is limited to a specific scale range, with a clear scale boundary. An imaginary dimensional cohaesal iteration was proposed to overcome this problem. Through imaginary dimensional cohaesal iteration, it generates complete systems, where fractals can be serve as generating subsystems. Through variations in iteration, it achieves pattern evolution, breaks through scale limitations, and better describes complex object systems in nature (such as from branches to leaves). While cohaesal has expanded the specific scale range of fractals, it still has certain size constraints and overlooks details. Studies have demonstrated that under imaginary iteration, patterns derived from different original patterns exhibit mutual similarity. It can simulate various natural and physical processes (e.g., leaf veins developing mesophyll), facilitate the modeling of processes such as egg hatching, and realize the iterative generation of images from branches to leaves.

Full Text

Preamble

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Abstract

Objective: Many irregular and fragmented complex forms in nature can be effectively generated and described by fractal geometry through self-similar iteration. However, fractal self-similarity in nature is limited to a specific scale range with clear scale boundaries. To overcome this limitation, we propose imaginary dimensional cohaesal iteration.

Methods: Through imaginary dimensional cohaesal iteration, we generate complete systems where fractals serve as generating subsystems.

Results: This approach achieves pattern evolution through iterative variations, breaks through scale limitations, and better describes complex object systems in nature (such as from branches to leaves).

Limitations: While cohaesal has expanded the specific scale range of fractals, it still has certain size constraints and overlooks details.

Conclusions: Studies have demonstrated that under imaginary iteration, patterns derived from different original patterns exhibit mutual similarity. This method can simulate various natural and physical processes (e.g., leaf veins developing mesophyll), facilitate modeling of processes such as egg hatching, and realize iterative generation of images from tree branches to leaves.

Keywords: Fractal; Imaginary Dimension; Cohaesal; Mutual Similarity; Graphic Evolution

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Introduction

The major scientific achievements of the 20th century include relativity, quantum mechanics, DNA double helix structure, and fractals-chaos. Among these, fractal geometry [1-7] broke through the traditional framework of Euclidean geometry, while chaos theory [8-12] explained the core characteristics of "sensitivity to initial conditions" and "topological transitivity" through mathematical definitions. Since the 21st century, fractal and chaos theories have been deeply integrated with complex systems science [13-22] and have become a common research framework across multiple disciplines such as physics and biology [23-29].

Fractals originate from the geometric abstraction of "irregular and complex systems" exemplified by natural forms such as coastlines, tree branches, and lightning. Although such systems have irregular shapes, they possess the core characteristic of "infinite nested self-similarity." Based on this theory, we can calculate that the fractal dimension of a coastline section may lie between 1 and 2 [30]. Fractals were the first to expand geometric dimensions from integer to non-integer dimensions.

Before fractal theory emerged, traditional Euclidean geometry [31] described the world using regular shapes such as straight lines, circles, and cones. It struggled to effectively handle the irregular, fragmented, and complex forms ubiquitous in nature. Fractals provided a mathematical language for describing complex natural forms and introduced the powerful tool of "fractal dimension," which can quantify the "roughness" or "complexity" of these forms.

Fractals offer a brand-new "grammar" and "vocabulary" to interpret, quantify, and simulate the natural universe, which is inherently irregular, complex, and rough. Fractals have proven that simple iterative rules lie behind "complexity," marking a major leap in human cognition.

Fractals are a cross-disciplinary universal mathematical tool, providing researchers in different fields with a common language and perspective. In medicine, fractals can analyze vascular networks, tumor vascular distribution, lung bronchial tree structures, neuron morphologies, and electrocardiograms [32-37], offering new quantitative indicators for disease diagnosis. In materials science, fractals aid in studying fracture surfaces and porous media structures (such as rocks and soil) while establishing connections between these structures and physical properties (such as strength and permeability) [38-40]. In geology, fractals can be applied to analyze rock structures [41-42]. This universality has facilitated interdisciplinary collaboration and innovation.

Currently, fractal theory applications still have limitations [43], particularly regarding "multi-physical field coupling" [44-46]. Although most fractals can be constructed through iterative functions, real-world fractals (e.g., mountain ranges, cloud formations) are often shaped by combined multi-scale physical processes that mathematical models struggle to fully capture. In engineering systems (e.g., aero-engines, chips), "mechanical-thermal-electromagnetic" multi-physical field coupling [47] is widespread. However, existing fractal and chaos models are mostly limited to single-physical field research (e.g., analyzing only vibration fractal characteristics) [48-49] and fail to fully consider coupling effects between different physical fields.

Additionally, natural fractals (such as cloud formations, coastlines, and mountain ranges) are not strictly self-similar; they only exhibit approximate self-similarity in a statistical sense. Their self-similarity is also limited to a specific scale range with clear minimum and maximum scale boundaries. Beyond this limited range, self-similarity ceases to exist [50]. For example, a tree's branching structure may exhibit 5 to 6 levels of self-similar characteristics from trunk to twigs, but when the scale is reduced to leaves or cells, self-similarity no longer exists.

Fractal dimension, while capable of converting form complexity into quantifiable values, compresses the rich information inherent in the shape itself. Key characteristics such as branching patterns, texture details, and local morphological differences are weakened or even overlooked. It is a single statistical measure, and two objects that are strikingly different in shape and structure may have identical or highly similar fractal dimensions.

A counterintuitive phenomenon occurs when using fractal analysis methods: two structures with different initial forms and fractal dimensions may, after multiple iterations, converge to a macroscopic structural shape that is visually identical or highly similar. This characteristic stems from the inherent laws of nonlinear dynamical systems. The final fractal form presentation depends on the initial shape, iteration rules, and key parameters. Differences in initial dimensions may be "weakened" or even "neutralized" by parameter synergies during repeated iterations, ultimately leading to similar structural characteristics [51]. For example, in certain iterated function systems or chaotic systems with identical strange attractors, regardless of initial point differences, system evolution eventually converges to the same fractal structure. In "bifurcating tree fractals" used for natural form simulation, if two sets of initial parameters are set—one being binary trees where each branch level length is 0.63 times the previous level with a 30° bifurcation angle; the other being ternary trees where each branch level length is 0.4 times the previous level with a 45° bifurcation angle—after 10 iterations, both simulation results present vein-like patterns with "main veins extending and lateral veins distributed in a crisscross pattern." Visually, no significant differences exist between them; differences only appear at the microscopic level.

The aforementioned bifurcating trees only consider length-wise branch proportions and fail to account for lateral branch dimensions. Consequently, binary and ternary tree models can describe the longitudinal iteration of tree branches, leaf veins, and blood vessels but cannot describe the iteration of leaf mesophyll or animal tissue.

By introducing a new dimensional extension—from real dimensions to imaginary dimensions—and performing iteration within imaginary dimensions (with variations occurring during iteration), this paper breaks through the scale limitations of real-dimensional fractals. This enables leaf veins to grow mesophyll, blood vessels to grow tissue, and achieves iterative image generation from tree branches to leaves, thereby better describing complex natural objects. We also found that through imaginary dimension iterative generation, different iteration parameters can lead to similarity between different patterns; that is, mutual similarity is achieved through different iterations.

Fractal iteration determines contour shape and high-frequency details. Through imaginary iteration, the high-frequency information of fractal iteration is covered, resulting in higher mutual similarity. The fractal of leaf veins can be obtained by removing the imaginary part of leaves, and the bifurcation tree can be obtained by removing the imaginary part of the crown. Thus, cohaesal is the inverse process of fractals.

Real fractal dimension is obtained through iteration in a certain direction. If positioning is performed by establishing coordinates in real dimensions, some positions with imaginary dimensions deviate from real-dimensional coordinates.

These positions depend on, attach to, and are parasitic on real-dimensional coordinates. They are neither independent nor free real dimensions and cannot be separated from real-dimensional coordinates.

Imaginary coordinates are established based on imaginary dimensions. In addition to uniform expansion in the imaginary direction, they can also follow exponential, trigonometric functions, or even probability distributions. In particular, probability distributions may provide a new coordinate system for quantum mechanics: wave function propagation can be transformed into a description using imaginary dimensions, which may greatly simplify quantum mechanical calculations. For example, electronic motion has a probability distribution in the direction perpendicular to motion, which is attached to the direction of real dimension motion.

2.1 Bifurcating Trees with Fractal Iteration

A binary tree fractal iteration was designed with the following parameters: The angle between a branch and its parent branch is fixed at 30° to ensure the fractal structure exhibits regular and symmetrical divergent morphology. The recursive depth is set to 10 levels; that is, the fractal starts from the initial main branch and generates 10 levels of branches layer by layer, ultimately forming a tree-like structure with rich hierarchical levels. The initial main branch length is 100 pixels, and all branch widths are 1 pixel to ensure clarity and visual consistency of branch lines. The branch length decay coefficient is set as r=0.63, meaning each new branch length is 0.63 times its parent branch length. Controlling branch decay rhythm through this proportion ensures the overall fractal is neither too sparse nor crowded, ultimately generating a binary tree fractal graph with coordinated morphology and distinct details, as shown in Fig. 1 [Figure 1: see original paper].

Figure 1 shows that the binary tree fractal pattern perfectly exhibits expected structural characteristics under these parameter settings: starting from the initial main branch at the visual center, branches on both left and right sides symmetrically diverge at a strict 30° angle, forming a highly regular symmetric morphology. This symmetry persists across all iterative levels, maintaining consistent left-right mirror distribution throughout.

Fig. 1 intuitively demonstrates the core fractal characteristic—self-similarity. This characteristic, defined as "structural similarity between local parts and the whole," endows the pattern with both mathematical model rigor and the implicit growth rhythm of natural trees, resembling a simplified abstract form of a natural tree.

A ternary tree fractal iteration was further designed, as shown in Fig. 2 [Figure 2: see original paper], with parameters and design details as follows:

The angle between a branch and its parent branch is set to 45°. Three branches diverge uniformly from each node with a 45° angle between adjacent branches, preventing spatial overlap in subsequent iterations. The recursive depth is set to 10 levels, iterating according to the rule of "one upper-level branch generating three lower-level branches" to balance detail density and visualization range. The initial main branch length is 100 pixels, and all branch widths are 1 pixel to ensure clarity and visual consistency. The branch length ratio is 0.4; since the ternary tree has more branches, this small ratio helps balance quantity and space, preventing stacking or sparsity.

2.2 Bifurcating Trees and Cohaesal Iteration

For the aforementioned fractal trees, branch width is set to 1 pixel. However, actual tree branches exhibit varying widths when viewed from the side: the width iteration ratio from trunk to branches at all levels generally differs from the length ratio. Considering that width-lateral expansion in nature presents multiple forms—including uniform, exponential, trigonometric, and probabilistic expansion, corresponding to different density distributions—this expansion is entirely distinct from longitudinal fractal fractional dimension. Specifically, a real fractional dimension exists in the longitudinal direction, while lateral expansion iteration corresponds to the imaginary dimension.

The fractal dimension is defined as follows: If a graph is composed of N similar graphs, each obtained by reducing the original graph by a factor of r, then the fractal dimension is. [FORMULA MISSING]

An imaginary-real iteration experiment was conducted on the binary tree with these parameters: the angle between a branch and its parent branch is fixed at 30°, recursive depth is 10 levels, branch length decay coefficient is 0.63, and branch width decay coefficient is 0.7.

The generated graph is shown in Fig. 3 [Figure 3: see original paper]. Figure 3 reveals that the binary tree cohaesal more closely resembles natural trees, making the graph more rigorous as a mathematical model while expressing the growth rhythm of natural trees.

Imaginary-real iteration was performed on the ternary tree with these parameters: the angle between a branch and its parent branch is fixed at 45°, recursive depth is 10 levels, branch length ratio coefficient is 0.4, and branch width ratio coefficient is 0.55. The generated Ternary Tree 1 graph is shown in Fig. 4 [Figure 4: see original paper]. Fig. 5 [Figure 5: see original paper] presents Ternary Tree 2, with branch length ratio coefficient of 0.5 and branch width ratio coefficient of 0.5. As observed in Figures 4 and 5, these ternary tree cohaesal graphs more closely resemble natural trees—they enhance mathematical model rigor while conveying natural tree growth rhythm.

2.3 Bifurcation Trees and Cohaesal Variational-Iteration

The cohaesal variant iteration of bifurcating trees was analyzed. From a graphical perspective, this variation iteration results in graph evolution from branchlike to leaf-like forms.

Binary tree iteration was performed: for the first 5 iterations, the branch width ratio coefficient remained constant at 0.7 (displayed in black). Variation occurred after the 6th iteration, where subsequent branch width ratio coefficients were 0.9, 1.1, 1.3, 1.5, 1.7, and 1.9; variant parts are displayed in green, as shown in Fig. 6 [Figure 6: see original paper]. The boxed local area shows a magnified view of the variation region.

This process resembles natural leaf transition—from spring sprouting to summer full development.

Fractals can only perform branch length iteration. However, through imaginary-dimensional cohaesal iteration, graphical evolution can be achieved via variation, such as leaf iteration, flower iteration, and fruit iteration. If iterated over time, temporal change characteristics can be analyzed.

Before variation, each branch' s real dimension along the longitudinal direction is three-dimensional; there are two perpendicular directions, one extended to imaginary dimension i and the other to imaginary dimension j. After iterative variation into leaves, the leaf is quasi-two-dimensional from a real dimension perspective (the thickness direction can be neglected graphically).

From an imaginary dimension perspective, one imaginary dimension disappears (i.e., the thickness direction) while the other is enhanced—this is also a variation result. Such variation increases contact area between the tree and external environment, enhancing biological efficiency. The three-dimensional branch changes to a two-dimensional leaf. The tree's contact area with air and sunlight increases after variation from branch to leaf, facilitating photosynthesis and gas exchange. Similarly, biological processes such as development from blood vessels to capillaries to tissue, from bronchi to lung, and from neurons to brain—all expand contact scope, thereby maximizing biological efficiency.

3. Cohaesal Similarity Analysis

Figures 3 to 8 show that some graphs look similar before and after imaginary iteration. To analyze the relationship between imaginary iteration and similarity, cross-correlation analysis was adopted, covering two scenarios: cross-correlation of the same bifurcating trees before and after imaginary iteration, and cross-correlation of different bifurcating trees.

The cross-correlation function of two images f(x, y) and g(x, y) is: [FORMULA MISSING]

Similarity degree between two graphs can be determined by calculating the cross-correlation value. When the cross-correlation value is 1, it corresponds to autocorrelation, indicating the two graphs are identical. Higher cross-correlation values indicate greater similarity.

During calculation, all digital images consist of discrete pixels. Let the discrete functions of two two-dimensional images be denoted as f[m,n] (size M×N) and g[m,n] (size P×Q). For graph f[m,n], where m [0,M−1] and n [0,N−1], m and n represent horizontal pixel index (column) and vertical pixel index (row), respectively, and M×N denotes graph 1 size. For graph g[m,n], where m [0,P−1] and n [0,Q−1], m and n represent horizontal pixel index (column) and vertical pixel index (row), respectively, and P×Q denotes graph 2 size.

The cross-correlation function of graph1 and graph2 is: [FORMULA MISSING]

Normalized cross correlation (NCC) was calculated to eliminate brightness difference influence: [FORMULA MISSING]

Where, [VARIABLE MISSING] is the local mean value of image f in the current window and [VARIABLE MISSING] is the mean value of g.

3.1 Cross Correlation Between Binary Tree and Ternary Tree

Cross-correlation computation was performed on a binary tree (angle 30°, r=0.63, D=1.500203) and Ternary Tree 1 (angle 45°, r=0.4, D=1.198978). The normalized cross-correlation peak value is 0.229239, indicating some similarity but low similarity degree (Fig. 7 [Figure 7: see original paper]).

The same binary tree was compared with Ternary Tree 2 (angle 45°, r=0.5, D=1.584963). The normalized cross-correlation peak value is 0.249564, slightly higher but still indicating low similarity (Fig. 8 [Figure 8: see original paper]).

3.2 Cross Correlation After Cohaesal Iteration

Cross-correlation after superimposing imaginary iteration was analyzed. For the binary tree (length decay r=0.63, width decay=0.7) and Ternary Tree 1 (length decay r=0.4, width decay=0.55), the cross-correlation peak is 0.338765 (Fig. 9 [Figure 9: see original paper]).

For the same binary tree and Ternary Tree 2 (length decay r=0.5, width decay=0.5), the cross-correlation peak is 0.382762 (Fig. 10 [Figure 10: see original paper]). Normalized cross-correlation peak values for trees with superimposed imaginary iteration are all higher than those without.

3.3 Cross Correlation Between Internal Bifurcating Trees with Variational-Iteration

Cross-correlation of bifurcation tree cohaesal variant iteration was analyzed. Results for binary tree, Ternary Tree 1, and Ternary Tree 2 show a consistent trend: greater deviation of the width coefficient (w) from a benchmark value yields larger imaginary dimension and smaller internal cross-correlation value, indicating lower similarity (Fig. 11 [Figure 11: see original paper]).

The relationship curve between internal cross-correlation and imaginary dimension for binary tree, Ternary Tree 1, and Ternary Tree 2 is shown in Fig. 12 [Figure 12: see original paper].

Figure 12 shows that larger imaginary part of D results in lower internal cross-correlation for binary and ternary trees and lower similarity.

3.4 Cross-correlation Between Variational-Iterative Binary Trees and Ternary Trees

Cross-correlation analyses between different tree types (binary vs. ternary) with variational iteration were performed. Results show normalized cross-correlation peak values are generally low, indicating significant structural differences. However, as iterative width coefficient (w) increases, cross-correlation values can change. Notably, for some comparisons, cross-correlation gradually increases as w increases. When w exceeds a certain threshold (e.g., $w > 4.7$), cross-correlation values between different tree types can become higher than internal cross-correlation values of original trees. This indicates that with sufficiently large imaginary dimension (driven by w), it becomes difficult to distinguish the benchmark tree from which iterated graphs originated (Fig. 13 [Figure 13: see original paper]).

3.5 Cross-correlation Between Leaves of Binary Trees and Ternary Trees

Analysis focused on tree "leaf" portions generated in later variational iteration stages. Results reinforce previous findings. As branch width iteration coefficient (w) increases, imaginary dimension values increase and bifurcating tree leaf cross-correlation values become closer. Increasing w for both binary and ternary tree leaves significantly enhances cross-type similarity. When $w > 2.7$, cross-correlation between binary tree leaves and ternary tree leaves exceeds their respective internal cross-correlation values. This means the two patterns become highly similar and "cohaesal converge,"making origin distinction impossible (Fig. 14 [Figure 14: see original paper]).

4. Discussion

Imaginary iteration can be applied in numerous fields. In biological camouflage, for instance, eye patterns on butterfly wings can be generated through imaginary iteration of shapes combined with color. Additionally, shape evolution processes such as egg hatching, embryonic development, and fossil organism restoration can all be realized via imaginary iteration.

Beyond biology, imaginary iteration applies to visualization of processes in other disciplines—including pattern generation for chemical reactions and condensation processes in physics.

Through the inverse process of imaginary iteration, known as inverse imaginary iteration, veins can be obtained from leaves and branches from crowns. It also supports practical applications such as precise recognition and restoration of clear images from blurred ones.

For dynamic and continuous pattern transformations (e.g., human facial expression transition from "smiling" to "frowning"), such changes can be achieved using imaginary iteration. However, this transformation is achieved through discrete stepwise manifolds, making it more suitable for computer technology implementation.

Here we discuss a special case of imaginary iteration: self-iteration of patterns with constant capacity. Cross-correlation analyses were performed for characters "E" , " " , and "M" with varying stroke widths.

Figure 15 [Figure 15: see original paper] reveals several patterns:

For internal-symbol cross-correlation (i.e., cross-correlation of the same character, either "E" or " ") with varying stroke widths: as stroke width increases, cross-correlation values decrease, accompanied by similarity reduction.

For cross-correlation between "E" (stroke width = 1 pixel) and " " with varying stroke widths (1, 3, 5, 7, 9, 11, 13, 15 pixels): when both characters have 1-pixel stroke width, cross-correlation value is lower than their respective internal-correlation values but still exceeds 0.7, indicating certain similarity. As " "stroke width increases, cross-correlation value decreases.

For cross-correlation between " " (stroke width = 1 pixel) and "E" with varying stroke widths (1, 3, 5, 7, 9, 11, 13, 15 pixels): cross-correlation value is lower than respective internal-correlation values. However, when "E" stroke width is 1, 3, 5, 7, or 9 pixels, cross-correlation value remains above 0.7, demonstrating relatively high similarity. As "E" stroke width increases, cross-correlation value first increases then decreases.

For cross-correlation between "E" and " " with identical stroke widths (1, 3, 5, 7, 9, 11, 13, 15 pixels, with both characters sharing the same stroke width in each pair): as stroke width increases, cross-correlation value consistently rises. When stroke width exceeds 9 pixels, cross-correlation value even surpasses internal-correlation values of both "E" and " " . Specifically, at 15-pixel stroke width, cross-correlation value reaches 0.7849—far higher than the internal-correlation value of 0.6654 between "E1" (E with 1 pixel) and "E15" (E with 15 pixels), and the internal-correlation value of 0.6439 between " 1" (with 1 pixel) and " 15" (with 15 pixels).

Cross-correlation analyses were also conducted between "E" and "M," with key observations as follows:

For cross-correlation between "E" (stroke width = 1 pixel) and "M" with varying stroke widths (1, 3, 5, 7, 9, 11, 13, 15 pixels): cross-correlation values remained consistently low, indicating low similarity between the two characters.

For cross-correlation between “E”and “M”with identical stroke widths (1, 3, 5, 7, 9, 11, 13, 15 pixels, with both characters sharing the same stroke width in each pair): cross-correlation values increased consistently as stroke width increased.

These results indicate “E”exhibits higher similarity to “ ”than to “M.”Specifically, mutual similarity and cohaesal between “E” and “ ” can be achieved through imaginary iteration.

5. Conclusion

This study introduces a novel conceptual framework, “cohaesal iteration,” by extending fractal geometry from real to imaginary dimensions. We have demonstrated that this approach successfully models the evolution of complex natural systems, breaking through scale limitations inherent in traditional fractal self-similarity. By introducing variations in the imaginary dimension during iteration, we can simulate development of complete systems from their generating subsystems, such as growth of leaf mesophyll from veins or formation of a full tree crown from branches.

This approach enables simulation of various natural and physical processes: it allows leaf veins to develop mesophyll, blood vessels to generate capillaries and eventually form muscle tissue, bronchi to develop alveoli and ultimately form lung organs, and neurons to develop into brains. It also supports modeling of processes such as egg hatching, embryonic development, fossil organism restoration, chemical reactions, and physical condensation. Additionally, this method realizes iterative image generation from tree branches to leaves, producing complete systems (fractals can be regarded as subsystem generators).

A key finding is the phenomenon of “cohaesal,” or mutual similarity, where patterns originating from structurally different initial forms converge toward high similarity after imaginary iteration. Our quantitative analysis using normalized cross-correlation confirms that as imaginary dimension component increases, similarity between disparate structures (e.g., binary and ternary trees) can exceed their internal similarity to their own less-developed variants. This suggests the imaginary dimension can mask underlying real fractal structure, leading to macroscopic form convergence.

The inverse process, termed inverse imaginary iteration, offers a powerful tool for feature extraction, allowing deconstruction of complex systems into fundamental fractal skeletons—for instance, extracting veins from leaves or restoring clear images from blurred ones.

Furthermore, the concept of an imaginary dimension based on probability distributions holds speculative but significant potential for simplifying quantum mechanical calculations, such as describing wave function propagation.

While cohaesal iteration provides a more holistic and dynamic descriptive language for complex, multi-scale natural systems, it still has certain size constraints and overlooks details. Future work will explore applications in biological modeling, materials science, and fundamental physics.

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Figure Legends

Figures

Fig. 1: Binary Tree with Angle of 30° Between Branches and the Parent Branch and a Length Decay Coefficient r=0.63

Fig. 2: Ternary Tree with Angle of 45° Between Branches and the Parent Branch and a Length Decay Coefficient r=0.4

Fig. 3: Binary Tree with an Angle of 30° Between Branches and the Parent Branch, a Branch Length Decay Coefficient of 0.63, and a Branch Width Decay Coefficient of 0.7

Fig. 4: Ternary Tree 1 with an Angle of 45° Between Branches and the Parent Branch, a Branch Length Decay Coefficient of r=0.4, and a Branch Width Decay Coefficient of 0.55

Fig. 5: Ternary Tree 2 with Branch Length Ratio Coefficient of 0.5 and Branch Width Ratio Coefficient of 0.5

Fig. 6: Binary Tree with Branch Width Iteration Coefficients of 0.7 for the First 5 Iteration Levels, and 0.9, 1.1, 1.3, 1.5, 1.7, and 1.9 After the 5th Iteration

Fig. 7: Cross correlation between Binary tree and Ternary Tree 1

Fig. 8: Cross correlation between Binary tree and Ternary Tree 2

Fig. 9: Cross correlation between Binary tree and Ternary Tree 1 after superimposing imaginary iteration

Fig. 10: Cross correlation between Binary tree and Ternary Tree 2 after superimposing imaginary iteration

Fig. 11: Relationship curve between internal cross correlation and w of Binary tree, Ternary Tree 1 and Ternary Tree 2

Fig. 12: The relationship curve between internal cross-correlation and imaginary dimension of Binary tree, Ternary Tree 1 and Ternary Tree 2

Fig. 13: The relationship curve between normalized cross-correlation and w

Fig. 14: Cross correlation peak curve of leaves

Fig. 15: Relationship between stroke width and cross correlation peak of different stroke widths of E, , M

Note: Figure translations are in progress. See original paper for figures.

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