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Abstract

According to Planck's law, the energy density for photons with radiation frequency f is based on the assumption that the photons' frequency has a continuous distribution, with an infinitesimal frequency interval df approaching zero. By integrating over the continuous frequencies, the energy density of photons with frequencies between f and $f+df$ can be derived. In this paper, the energy density of discrete frequencies is rigorously derived from Planck's law, when the frequency interval df defined by Planck's law is a non-zero constant. Then, according to the cosmic resonant cavity model, there exists a fundamental radiation frequency (minimal frequency interval) f_0 in cosmic space. Therefore, the equation for energy density of any single discrete frequency (any multiple of f_0) is derived, which is verified by exhaustive calculations for any temperature T. The derivation of equations for discrete radiation frequencies facilitates more precise quantum analysis of blackbody radiation in the future.

Full Text

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Abstract— According to Planck's law, the energy density for photons with radiation frequency f is based on the assumption that the photons' frequency has a continuous distribution, with an infinitesimal frequency interval df approaching zero. By integrating over the continuous frequencies, the energy density of photons with frequencies between f and f + df can be derived. In this paper, we derive the energy density of discrete frequencies according to Planck's law via critical derivation, when the frequency interval df defined by Planck's law is a non-zero constant. Then, according to the cosmic resonant cavity model, there is a fundamental radiation frequency (minimal frequency interval) f_0 in cosmic space. Therefore, the equation for energy density of any single discrete frequency (any multiple of f_0) is derived consequently, which is verified by detailed traversal calculation results for any temperature T. The derivation of equations for discrete radiation frequencies is helpful for more precise quantum analysis of blackbody radiation in the future.

Keywords— resonant cavity, Planck's law, blackbody radiation, energy density of discrete radiation frequency, infinitesimal frequency interval df

I. Background and Related Works

Planck's law [**?**] describes the distribution of radiation from a blackbody. It states that the energy of radiation is not continuously distributed, but rather consists of discrete energy states, introducing the concept of quantized photon energy for the first time. Planck's law reveals the quantized nature of electromagnetic radiation. The discreteness of radiation energy can be explained by grouping energy by frequency, which can be seen as a series of "energy gradients," where the height and width of each gradient depend on frequency. This spectrum is often referred to as the "energy spectrum" or "radiant energy density per unit frequency." Due to the quantum nature of radiant frequency, the amount of energy contained in each frequency band is determined by the number of photons [**?**].

Research related to Planck's Law mainly includes both computational and experimental aspects. In recent years, quantized energy distribution and spectral distribution have become key tools for studying thermodynamics and equations of state, in research fields including quantum coherence and quantum processes [**?**][**?**][**?**]. The resonant cavity model of electromagnetic waves refers to a model where electromagnetic waves are confined within a closed space, forming a resonant mode. In short, it is a model where electromagnetic waves are confined between two reflective mirrors and form a pattern. In this model, electromagnetic waves can propagate and reflect repeatedly within the space, forming resonant modes that result in standing wave patterns at specific frequencies. These standing wave patterns have discrete energy values and frequencies, and only electromagnetic waves (photons) with specific frequencies can remain in the cavity in a standing wave state. Therefore, only photons whose frequencies match the frequencies of the modes confined within the resonant cavity can be retained within the cavity.

The main theories and research work regarding the frequency distribution and evolution of photons within the resonant cavity are as follows. Optical cavity quantum electrodynamics is a theoretical method based on Maxwell's optics and quantum electrodynamics, used to calculate and describe quantum optics and its nonlinear response within a closed resonant cavity. This approach provides a quantum mechanical explanation of high-order nonlinear phenomena within the resonant cavity and predicts effects such as single-photon coupling, cavity quantum electrodynamics (CQED) energy levels, and photon coherence [?][?]. Photon coherent states are the optimal state as they minimize the product of photon number and variance. Photon statistical distribution is a probability distribution function that describes the distribution of photons within the resonant cavity, such as the Bose-Einstein distribution [?]. These distributions describe the probability distribution functions of photon distribution within the resonant cavity under different physical conditions. The study of photon statistical distributions mainly involves quantum optics and statistical physics.

This paper uses the resonant cavity model to prove that the minimum frequency interval f_0 is a finite value, rather than approaching zero. Based on this, the energy density of any single discrete frequency f is derived, and the correctness of the equations for discrete frequency energy density is verified via derivation and traversal calculation for any radiation temperature T, according to Planck's Law.

II. The Frequency of Photons in a Resonant Cavity

The radiation frequency interval is called the spectral line resolution in classical physics, which is limited by the radiation wavelength and the harmonics of the spectrum. From the perspective of quantum mechanics, the frequency interval lines are limited by the quantization of photon spectral energy. For the frequency interval of spectral lines, the energy difference between photons in adjacent spectral lines can only be an integer multiple of Planck constant h multiplied by the frequency difference Δf, that is, $\Delta E = N\ h\Delta f$, where N is an integer [?][?]. Therefore, when the energy of the photon is small enough, it cannot be further divided into smaller energy units, and the frequency difference at this point approaches the fundamental frequency. It can be assumed that when the resonant cavity radius R is large enough, the frequency interval of spectral lines Δf approaches the fundamental frequency f_g of light [?][?]. This can be derived by combining wave optics and quantum mechanics.

Assuming that there is a monochromatic light beam with a frequency of f, a wavelength of λ, and a corresponding energy of E, when it passes through a medium, its velocity changes, resulting in a change in frequency and wavelength. According to Fermat's principle [?], the propagation path is determined based on the principle of minimum optical path length, so the path of the incident ray and the reflected ray must be equal in the medium, that is: $2nd = N\ \lambda \rightarrow \lambda =$ MATH_0 $\rightarrow f =$ MATH_1 where n is the refractive index of the medium, d is the thickness of the medium, and N is an integer.

Quantum mechanics analysis reveals that only allowed energy states in the medium can be occupied by photons. These allowed energy states correspond strictly to the wavelength and frequency of light. Therefore, when a photon passes through a medium, it can only occupy those energy states that are integer multiples of the fundamental frequency of the incident light. This phenomenon can be explained as follows: only energy states that are integer multiples of the fundamental frequency can be occupied by photons. When photons occupy non-integer multiples of energy states, their propagation paths in the medium do not satisfy the principle of minimum optical path length, thus they will be scattered or absorbed [?][?][?].

This article proposes a resonant cavity model for cosmic space. Based on the Big Bang model [?], cosmic space originated from a finite-sized closed space. Since the beginning of the Big Bang, the age of the universe is a finite time, estimated to be about 13.8 billion years until now [?][?]. According to different models of cosmic expansion, the current cosmic radius R is on the order of tens of billions of light-years [?][?][?], which is a finite value rather than tending towards ∞. Therefore, cosmic space is a finite closed space, which conforms to the characteristics of a resonant cavity.

According to equation (1), d represents the thickness of the medium. In the cosmic resonant cavity model, the thickness of the medium d can be replaced by the cosmic radius R, yielding: MATH_2 $\rightarrow$ f = MATH_3 . Let integer N = 1 for fundamental frequency f_0. The refractive index n = 1 in vacuum. Therefore, $\Delta f = f_0 =$ MATH_4 . Since the cosmic radius R is a finite value, according to equation (3), the minimum frequency interval $\Delta f = f_0$ is a non-zero finite value. Furthermore, based on Planck's Law, the following sections derive the energy density equation for any single discrete frequency N f_0, where N is an integer. Then the correctness of the derived equations is verified through simulations for any radiation temperature T.

III. Radiation Energy Density of Discrete Frequency According to Planck's Law

A. The Distribution of Discrete Radiation Frequencies

The following energy density equations (4) and (5) according to Planck's law are based on Bose-Einstein statistics, in which the number of photons per unit volume is extremely large, which could be equivalent to the number of infinite in statistics. When performing statistical analysis on a large number of photons' frequency distribution, the minimum frequency interval df is assumed to be infinitesimally small, resulting in the assumption that frequencies have a continuous distribution. This simplifies the mathematical expressions and facilitates obtaining analytical solutions. However, the assumption of continuous frequency distribution is merely an idealized approximation and does not represent the real-world scenario where photon frequencies are not entirely continuous. In the original Bose-Einstein statistical analysis, each photon has a discrete frequency, implying the presence of discrete intervals between photon frequencies. The minimum interval between frequencies df is a non-zero constant. If more accurate computational results are required, the discreteness of photon frequencies still needs to be considered.

B. Energy Density of Discrete Frequencies

According to Planck's law, the equation for energy density at frequency f is as follows [?]:

U(f, T) = MATH_5

The energy density U(f,T) is the energy of photons within frequencies between f and f + df per unit volume V, where df represents an infinitesimal change in frequency according to Planck's law [?]. Therefore, the total energy density of all photons within frequency f where $0 \leq f < \infty$ is given as follows:

U(T) = MATH_6 = MATH_7

Since df is constant for $0 \leq f < \infty$, therefore f = N df, where N are consecutive integers 0,1,2,3…∞, as shown in Figure 1. Since each frequency f equals the discrete frequency N df, hence it is the differential distribution of discrete frequencies, with df being the differential naturalization factor. Let C_{df} be the naturalization factor coefficient for df. Then, equation (4) can be rewritten as follows:

U(f, T) = U(N df, T) = C_{df} MATH_8

U(T) = MATH_9

Let x = MATH_{10} , then:

U(T) = C_{df} 8πh(df)^3 (kT/h)^4 MATH_{11}

Therefore:

U(T) = C_{df} 8πh(df)^3 (kT/h)^4 MATH_{12}

Since U(T) = C_{df} MATH_{13} according to Planck's law, hence:

C_{df} = df for any df > 0

Equation (8) proves that the naturalization factor coefficient C_{df} equals the naturalization factor df. Hence, for any naturalization factor (frequency interval) df, then:

U(f, T) = U(N df, T) = MATH_{14}

IV. The Energy Density of a Single Discrete Frequency N f_0

U(f, T) = U(N df, T) = MATH_{15}

U(T) = MATH_{16}

For instance, let df = 1 Hz, then:

U(T) = MATH_{17}

U(N df, T) = U(N, T) = MATH_{18}

The traversal calculation for any radiation temperature T also verifies that MATH_{19} , which is consistent with equation (11), as described in the following simulation results in Appendix.

According to equation (3), the minimum frequency interval $\Delta f = f_0$ in the cosmic resonant cavity is a non-zero finite value, where f_0 is the fundamental frequency in a resonant cavity. Therefore, all radiation frequencies in the cosmic resonant cavity are discrete frequencies N f_0, where N are consecutive integers 0,1,2,3…∞. Hence, the minimum frequency interval between these discrete frequencies is f_0. If f_0 = df, then the energy density U(N f_0,T) of any single discrete frequency N f_0 is as follows, according to equation (6), where f_0 = df = C_{df}.

U(N f_0, T) = U(N df, T) = MATH_{20}

U(N f_0, T) = MATH_{21}

Figure 1 shows the energy densities of every discrete frequency f = N df = N f_0 = N Hz according to equation (11), where f_0 = df = 1, and M = 1. It shows that the energy density distribution of frequency spectrum is discrete, rather than a continuously smooth curve. Let f_0 = 1/M Hz, where M is a real number where M > 0. Figure 2 shows an example for energy densities of every discrete frequency N f_0 according to equation (12), where f_0 = 1/M, and M = 4. Therefore, there are 4 discrete frequencies per Hz. Note that M = 4 is an example in figure 2. The exact value of f_0 = 1/M Hz is still unknown, which needs to be measured in future experiments.

The following traversal calculation for any temperature T and any f_0 = L Hz in the Appendix, where L is a real number and L > 0, also verifies that MATH_{22} , which is consistent with equations (10) and (12).

V. Conclusion

This paper derives the energy density of discrete frequencies according to Planck's law via critical derivation, when the frequency interval df defined by Planck's law is a non-zero constant. Then, according to the cosmic resonant cavity model, there is a fundamental radiation frequency (minimal frequency interval) f_0 in cosmic space. Therefore, the equation for energy density of any single discrete frequency (any multiple of f_0) is derived consequently, which is verified by detailed traversal calculation results for any temperature T. The derivation of equations for discrete radiation frequencies is helpful for more precise quantum analysis of blackbody radiation in the future.

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Appendix

Simulation Results

Traverse calculations for any temperature T are performed according to equations (10), (11) and (12). The calculation results are shown in Table I and Table II.

As shown in Table I, when radiation temperature $T = 7 \cdot 10^{-11}$ K, and the minimum frequency interval $f_0 = df = 1$ Hz, therefore every discrete frequency f = N f_0 = N, where N are consecutive integers 0,1,2,3…∞. The calculation result shows that MATH_{23} , where MATH_{24} is the sum energy density of each discrete frequency N, where N are consecutive integers from 0 to ∞. U(T) = MATH_{25} is the total energy density of all frequencies according to Planck's law [?].

Table II also shows that, when L = 1, $f_0 = df = 1$ Hz, then for temperature $T \geq 10^{-10}$ K, MATH_{26} , which is consistent with the derivation of equations (8). Therefore, it verifies that df = 1 for any temperature $T \geq 10^{-10}$ K, where df is the frequency interval defined by Planck's law [?]. Table II shows that, when L = 1, and $f_0 = df = 1$ Hz, if temperature $T < 7 \cdot 10^{-11}$ K, then, MATH_{27} .

This is because the minimum frequency interval f_0 = df is too sparse for such low temperature T, causing the distribution of all discrete frequencies to not be equivalent to the distribution of frequencies from 0 to ∞ according to the derivation of MATH_{28} in equation (7). Hence, when L = 1, and f_0 = df = 1 Hz, if temperature $T < 7 \cdot 10^{-11}$ K, then, MATH_{29} .

Similarly, as shown in Table II, for any L > 0 and f_0 = df = L Hz, if $T < 7 \cdot L \cdot 10^{-11}$ K, then, MATH_{30} . However, as shown in Table II, for any L > 0 and f_0 = df = L Hz, if $T \geq L \cdot 10^{-10}$ K, then, MATH_{31} , which is consistent with equation (13). Therefore, the energy density U(N f_0,T) of discrete frequency (N f_0) according to equations (10) and (12) are verified.

[TABLE:I]

[TABLE:II]

End of Appendix

Note: Figure translations are in progress. See original paper for figures.

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