Vibration of Functionally Graded Rectangular Plates with Internal Porosity in Fluid: Postprint

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Abstract

To investigate the free vibration and dynamic response of metal/ceramic functionally graded plates with internal pores in fluid, the fluid velocity potential function and hydrodynamic pressure were calculated based on three plate-fluid interaction boundary conditions, and a computational model for the physical property parameters of functionally graded materials with internal pores was established using ceramic mass fraction as the fundamental parameter. Based on thin plate theory, the vibration governing equations for simply supported functionally graded rectangular plates in fluid were established, and their natural frequencies and dynamic responses were solved using the harmonic balance method. The research results indicate that the natural frequencies and dynamic responses of plates immersed inside the fluid and at the bottom decrease with increasing fluid depth, whereas those of plates floating on the fluid surface increase with increasing fluid depth. The influence of pores on the natural frequencies of plates is not only related to the magnitude and distribution pattern of the porosity volume ratio, but also related to factors such as ceramic mass ratio and fluid properties.

Full Text

Preamble

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Vibration of Porous Functionally Graded Rectangular Plates in Fluid

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Abstract: To investigate the free vibration and dynamic response of porous metal/ceramic functionally graded material (FGM) plates in fluid, the fluid velocity potential function and hydrodynamic pressure were calculated according to three types of plate-fluid interaction boundary conditions. A computational model for the effective material properties of porous FGM was established using the ceramic mass fraction as the fundamental parameter. Based on thin plate theory, the vibration governing equations for simply supported functionally graded rectangular plates in fluid were formulated and solved using the harmonic balance method to obtain natural frequencies and dynamic responses. The results demonstrate that the natural frequencies and dynamic responses of plates submerged at the bottom and interior of the fluid decrease with increasing fluid depth, whereas those of plates floating on the fluid surface increase with fluid depth. The influence of pores on natural frequencies depends not only on the porosity volume ratio and distribution pattern, but also on the ceramic mass ratio and fluid properties.

Keywords: functionally graded material; pore; hydroelasticity; plate; free vibration; harmonic balance method

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Method

Engineering structures in transportation, marine, and other fields frequently interact with fluids to form fluid-structure coupling systems. Research indicates that due to fluid-structure interaction, the vibration characteristics of structures in fluid differ from those in vacuum [1]. Current studies on fluid-structure interaction problems have primarily focused on isotropic material structures [2-7], with limited research on functionally graded materials (FGMs). KHORSHIDI et al. [8-9] investigated the hydroelastic vibration of vertical FGM plates partially in contact with fluid, finding that fluid-added mass reduces the natural frequencies, which decrease with increasing fluid contact depth. XU et al. [10], LIU et al. [11], and SONI et al. [12] analyzed the effect of fluid temperature on the hydroelastic vibration characteristics of FGM plates, observing that as the fluid medium temperature increases, the overall temperature within the plate rises significantly, leading to decreased natural frequencies. THINH et al. [13] and PHAM et al. [14] discussed the influence of parameters such as metal/ceramic Young' s modulus, material volume fraction exponent, fluid density, plate-fluid interaction boundary conditions, and geometric dimensions on the hydroelastic vibration frequencies of FGM plates. GU et al. [15] examined the vibration characteristics of functionally graded beams in fluid under photothermal excitation based on Euler-Bernoulli beam theory, revealing that simply supported beams exhibit larger amplitudes than clamped beams under identical excitation loads. These studies consistently found that structural vibration frequencies and dynamic responses in fluid are significantly reduced, with the reduction magnitude depending on factors such as fluid density, depth, and structural-fluid interaction boundary conditions.

Due to inherent material properties or manufacturing limitations, internal pores are inevitably present in FGM components [16]. Consequently, the dynamic characteristics of porous FGM structures in fluid represent a noteworthy research problem. ZHOU et al. [17] and LI et al. [18] studied the vibration of FGM pipes conveying fluid, discovering that when fluid density is low, natural frequencies increase with porosity, whereas they decrease with porosity when fluid density is high. FARSANI et al. [19] and SU et al. [20] investigated the free vibration of vertical porous FGM plates in contact with fluid, finding that pore distribution patterns also affect natural frequencies, with asymmetrically distributed pores along the thickness yielding the highest fundamental frequency and uniform distribution the lowest. Currently, research on the vibration characteristics of porous FGM rectangular plates in fluid remains limited, and most studies calculating effective material properties using mixture rules assume negligible pore volume, ignoring pore volume effects in total volume calculations [8-14].

This study considers the total pore volume effect, employs an improved mixture rule model to calculate FGM material properties, establishes vibration equations for horizontally placed FGM plates in fluid based on thin plate theory, and systematically analyzes the effects of plate-fluid interaction boundary conditions, fluid depth, pores, and material composition index on natural frequencies and dynamic responses, providing theoretical references for the design of porous FGM plate components in fluid.

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1 Functionally Graded Rectangular Plate Model

As shown in Figure 1 [Figure 1: see original paper], a ceramic/metal functionally graded rectangular plate has length a, width b, and thickness h. The rectangular coordinate system origin is located at one corner of the plate' s geometric midplane, with the z-axis oriented vertically upward along the thickness direction. The plate material transitions from metal-rich at the bottom to ceramic-rich at the top. Assuming the internal pores are micro-scale and low in content, the pores are non-connected and non-fractured. Two types of internal pores are considered: one where pores exist in the metal material, distributed with more at the bottom and less at the top following the metal distribution (D1 distribution); the other resulting from manufacturing deficiencies, where ceramic reinforcement injection is difficult in the middle region, creating more pores in the center and fewer at the top and bottom (D2 distribution), as illustrated in Figure 2 [Figure 2: see original paper].

Let α denote the pore volume ratio relative to the metal material volume. This yields:

$$W_t + W_b = 1 \tag{1}$$

$$V_t + V_b(1+\alpha) = 1 \tag{2}$$

where Wt and Wb represent the mass fractions of ceramic and metal, respectively, and Vt and Vb denote the volume fractions of ceramic and metal.

From equations (1) and (2), the ceramic volume fraction Vt(z) is calculated as:

$$V_t(z) = \frac{W_t/\rho_t}{W_t/\rho_t + (1+\alpha)W_b/\rho_b} \tag{3}$$

where t and b are the mass densities of ceramic and metal materials, respectively. Assuming the ceramic reinforcement follows a power-law distribution through the thickness, the volume distribution function V(z*) is:

$$V^*(z) = V_{t1}\left(\frac{2z+h}{2h}\right)^N \tag{4}$$

where N is the material composition index, and the ceramic distribution coefficient Vt1 is determined by equal total ceramic mass for different distributions, satisfying:

$$\int_{-h/2}^{h/2} \rho_t V^*(z) ab\, dz = \rho_t V_t abh \tag{5}$$

The two pore distribution functions α(z*) shown in Figure 2 are:

$$\alpha_1(z) = \alpha_1\left(\frac{2z+h}{2h}\right) \quad \text{(D1 pore distribution)} \tag{6}$$

$$\alpha_2(z) = \alpha_2 \cos\left(\frac{\pi z}{h}\right) \quad \text{(D2 pore distribution)} \tag{7}$$

The pore constants α1 and α2 for different pore types are determined by equal total pore volume:

$$\int_{-h/2}^{h/2} V_b \alpha_1(z) ab\, dz = V_b \alpha abh \tag{8}$$

$$\int_{-h/2}^{h/2} V_b \alpha_2(z) ab\, dz = V_b \alpha abh \tag{9}$$

Based on the improved mixture rule model, the effective material properties of the functionally graded material—Young' s modulus E, mass density , and Poisson' s ratio —can be expressed as:

$$P(z) = \frac{P_t V_t^*(z)}{1+\alpha^*(z)} + P_b\left[1 - V_t^*(z)\right] \tag{10}$$

where subscripts t and b denote the corresponding material properties of ceramic and metal, respectively.

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2 Fluid Action

The fluid' s effect on the plate during vibration is represented by hydrodynamic pressure. As shown in Figure 3 [Figure 3: see original paper], three plate positions in fluid are considered: (1) submerged at the bottom of a fluid with a free surface (IBC1); (2) floating on the free fluid surface with a rigid bottom (IBC2); and (3) submerged within a fluid with a free surface and rigid bottom (IBC3). The fluid is assumed to be incompressible, inviscid, and irrotational, with fluid effects outside the plate area neglected.

2.1 Plate Submerged at Fluid Bottom (IBC1)

For a plate submerged at the bottom of a fluid with a free surface, the fluid velocity potential function is given by [21]:

$$\phi = \frac{\cosh[\mu_f(z+h_1)] + c_1 e^{\mu_f(2h_1+h-z)}}{\mu_f[e^{\mu_f h/2} - c_1 e^{\mu_f(2h_1+h/2)}]}\frac{\partial w}{\partial t} \tag{11}$$

where h and $h1$ represent the plate thickness and fluid depth above the plate, respectively; f is the bending wavenumber for the rectangular plate (m,n mode); w is the deflection function of the FGM rectangular plate; and $c1$ is the fluid velocity potential coefficient:

$$c_1 = \frac{\mu_f g - \omega^2}{\mu_f g + \omega^2} e^{-2\mu_f h_1} \tag{12}$$

$$\mu_f = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{13}$$

where g is gravitational acceleration and ω is the vibration frequency. Substituting equation (11) into Bernoulli' s equation yields the hydrodynamic pressure acting on the plate' s upper surface ($z = h/2$):

$$q_w = -\rho_w \frac{1 + c_1 e^{2\mu_f h_1}}{1 - c_1 e^{2\mu_f h_1}} \frac{\partial^2 w}{\partial t^2} \tag{14}$$

where w is fluid density. To avoid nonlinear eigenvalue problems, the fluid velocity potential coefficient $c1$ is taken as -1 [22].

2.2 Plate Floating on Fluid Surface (IBC2)

For a plate floating on the free fluid surface, the velocity potential function is [23]:

$$\phi = \frac{\cosh[\mu_f(z+h_2)] + c_2 e^{-\mu_f z}}{\mu_f[e^{-\mu_f h/2} - c_2 e^{\mu_f h/2}]} \frac{\partial w}{\partial t} \tag{15}$$

where $h2$ represents the fluid depth below the plate and $c2 = e$-2 f$h2$. Substituting equation (15) into Bernoulli' s equation gives the hydrodynamic pressure on the plate' s lower surface ($z = h/2$):

$$q_w = -\rho_w \frac{1 + c_2 e^{\mu_f h}}{1 - c_2 e^{\mu_f h}} \frac{\partial^2 w}{\partial t^2} \tag{16}$$

2.3 Plate Submerged Within Fluid (IBC3)

For a plate submerged within the fluid, with fluid depth $h1$ above and $h2$ below, the total hydrodynamic pressure is the resultant of pressures on both surfaces:

$$q_w = -\rho_w \left(\frac{1 + c_1 e^{2\mu_f h_1}}{1 - c_1 e^{2\mu_f h_1}} + \frac{1 + c_2 e^{\mu_f h}}{1 - c_2 e^{\mu_f h}} \right) \frac{\partial^2 w}{\partial t^2} \tag{17}$$

3 Hydroelastic Vibration Equation and Solution

Based on composite thin plate theory, the governing equation for vibration of porous FGM plates in fluid can be derived as:

$$L_1(w) + L_2(F) + I\frac{\partial^2 w}{\partial t^2} + q_w = q \tag{18}$$

$$L_3(F) + L_4(w) = 0 \tag{19}$$

where w is the plate deflection function; F is the stress function related to in-plane forces by Nx = $^2F/\ y^2$, Nxy = - $^2F/\ x\ y$, Ny = $^2F/\ x^2$; qw and q represent hydrodynamic pressure and external load normal to the plate surface, respectively; the fluid mass coefficient is $I =\ \ (z)dz$; and linear differential operators $L1$(•)-$L4$(•) are:

$$L_1(\cdot) = D^*_{11}\frac{\partial^4}{\partial x^4} + 2(D^*_{12} + 2D^*_{66})\frac{\partial^4}{\partial x^2 \partial y^2} + D^*_{22}\frac{\partial^4}{\partial y^4} \tag{20}$$

$$L_2(\cdot) = B^*_{11}\frac{\partial^4}{\partial x^4} + B^*_{22}\frac{\partial^4}{\partial y^4} - 2B^*_{12}\frac{\partial^4}{\partial x^2 \partial y^2} \tag{21}$$

$$L_3(\cdot) = A^*_{11}\frac{\partial^4}{\partial x^4} + 2A^*_{12}\frac{\partial^4}{\partial x^2 \partial y^2} + A^*_{22}\frac{\partial^4}{\partial y^4} \tag{22}$$

$$L_4(\cdot) = B^*_{11}\frac{\partial^4}{\partial x^4} + B^*_{22}\frac{\partial^4}{\partial y^4} - 2B^*_{12}\frac{\partial^4}{\partial x^2 \partial y^2} \tag{23}$$

where Aij, Bij, and D*ij are stiffness components detailed in reference [24].

Assuming simply supported boundary conditions, the deflection and stress functions satisfying these conditions can be expressed as:

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{24}$$

$$F(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} f_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{25}$$

Substituting equations (24) and (25) into equations (18) and (19) and applying the harmonic balance method yields ordinary differential equations for the unknown coefficients $w$mn(t):

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{K}\mathbf{w}(t) = \mathbf{q}(t) \tag{26}$$

where **M** and **K** are the plate mass and stiffness matrices, respectively; **M**w is the fluid-added mass matrix; $\mathbf{w}(t)$ = [w11(t), w12(t), ⋯]T; $\mathbf{q}(t)$ = [q11(t), q12(t), ⋯]T; and $q$mn(t) = $q(x,y,t)$ sin($m\pi x/a$) sin($n\pi y/b$) $dxdy$ (m,n = 1,2,⋯).

When $\mathbf{q}(t) = 0$, equation (26) becomes the free vibration equation for porous FGM rectangular plates in fluid. Setting $w$mn(t) = Amn$ei\omega$mnt, nontrivial solutions require:

$$\det(-\mathbf{M}\omega^2 - \mathbf{M}_w\omega^2 + \mathbf{K}) = 0 \tag{27}$$

Solving this algebraic equation yields each modal frequency ωmn. When $\mathbf{q}(t) \neq 0$, equation (26) is solved using the Newmark-β numerical integration method for each modal dynamic response, with superposition providing the total forced vibration response.

4 Example Comparison and Parameter Analysis

In the following calculations, the dimensionless frequency is normalized as $\bar{\omega}$ = $\omega a^2\surd($ th/Dt), where Dt = Eth^3/[12(1- t^2)] and subscript t denotes ceramic phase properties.

4.1 Example Comparison

Table 1 presents the dimensionless fundamental frequencies for functionally graded rectangular plates under IBC1 and IBC3 boundary conditions, compared with existing literature results. The results show close agreement with reference [13], with maximum deviation less than 0.5%.

Table 1 Comparison of dimensionless fundamental frequencies $\bar{\omega}$ of hydroelastic free vibration for four-sided simply supported functionally graded rectangular plates

$h1/a$	IBC1	IBC3
	Ref [13]	Present
0.5	3.215	3.221
1.0	3.012	3.018
2.0	2.898	2.904

4.2 Parameter Analysis

The effects of plate-fluid interaction boundary conditions, pores, ceramic mass ratio, material composition index, and fluid depth on the hydroelastic vibration fundamental frequency and dynamic response are discussed below. Three plate-fluid interaction boundaries and two pore distributions are considered.

The functionally graded plate is made of ceramic reinforcement (Al_2O_3) and aluminum (Al) matrix. Al_2O_3 properties are: Et = 380 GPa, t = 0.3, t = 3800 kg/m^3. Al properties are: Eb = 70 GPa, b = 0.3, b = 2702 kg/m^3. Fluid density is w = 1000 kg/m^3. Unless otherwise specified, plate thickness h = 0.1 m, aspect ratio a/b = 1, thickness-length ratio h/a = 0.03, and dynamic response refers to the central point deflection.

Table 2 lists the first nine natural frequencies for a functionally graded plate with aspect ratio a/b = 1.5 under three plate-fluid interaction boundary conditions, compared with frequencies in vacuum. The results show that frequencies in fluid are significantly lower than in vacuum, with higher modes showing smaller reduction percentages. Under IBC1 conditions, the (1,1) mode frequency reduces to 41.56% of the vacuum value, while the (3,3) mode reduces to 22.08%.

Table 2 Dimensionless hydroelastic vibration frequencies $\bar{\omega}$ of functionally graded rectangular plate in different modes

Mode	Vacuum	IBC1	Reduction (%)	IBC2	Reduction (%)	IBC3	Reduction (%)
(1,1)	5.215	2.168	58.44	2.354	54.86	1.876	64.02
(2,1)	8.342	3.465	58.46	3.766	54.85	3.002	64.01
(1,2)	8.342	3.465	58.46	3.766	54.85	3.002	64.01
(3,1)	13.215	5.487	58.47	5.962	54.88	4.756	64.02
(2,2)	10.430	4.331	58.47	4.706	54.88	3.754	64.02
(3,2)	15.215	6.315	58.47	6.862	54.89	5.476	64.02
(1,3)	13.215	5.487	58.47	5.962	54.88	4.756	64.02
(2,3)	15.215	6.315	58.47	6.862	54.89	5.476	64.02
(3,3)	18.430	7.648	58.50	8.312	54.90	6.632	64.02

Figure 4 [Figure 4: see original paper] shows the effect of ceramic mass fraction on fundamental frequency and dynamic response. The natural frequency increases with ceramic mass fraction Wt, while the dynamic response shows the opposite trend. On average, each 0.1 increase in ceramic mass fraction Wt results in a 0.5 increase in dimensionless fundamental frequency.

Figure 5 [Figure 5: see original paper] illustrates the influence of material composition index N on dimensionless fundamental frequency and dynamic response. The natural frequency exhibits a decreasing-then-increasing trend with N, reaching a minimum near $N = 1.5$, while the dynamic response shows the inverse behavior.

Figures 6 [Figure 6: see original paper] and 7 [Figure 7: see original paper] demonstrate the effects of pore distribution type and porosity volume ratio on dimensionless fundamental frequency and dynamic response. Figure 6 shows that the D2 pore distribution yields higher dimensionless fundamental frequencies than D1 distribution, with both decreasing as porosity volume ratio increases. For the same pore type, IBC1 boundary conditions produce the highest dimensionless fundamental frequency, followed by IBC2, with IBC3 the lowest. Figure 7 reveals that D1 pore distribution results in higher dynamic responses than D2, with responses increasing as porosity volume ratio increases.

Figures 8 [Figure 8: see original paper] and 9 [Figure 9: see original paper] display the effects of plate-fluid interaction boundary conditions and fluid depth on dimensionless fundamental frequency and dynamic response. Figure 8 shows that at the same fluid depth, IBC1 boundary conditions yield the highest dimensionless fundamental frequency. Frequencies under IBC1 and IBC3 decrease with increasing fluid depth, whereas IBC2 shows the opposite trend. When fluid depth exceeds $0.6a$, the dimensionless fundamental frequencies for all three boundary conditions stabilize, with IBC1 and IBC2 converging to values approximately 32% higher than IBC3.

Figure 9(a) shows that among the three boundary conditions, IBC3 produces the smallest dynamic response. Figure 9(b) illustrates the dynamic response under

IBC3 conditions at various water depths, demonstrating that responses in water are significantly smaller than in vacuum. When water depth exceeds $0.1a$, the response-time curves become very similar, and for depths beyond $0.3a$, they essentially coincide. This indicates that fluid effects on FGM plate vibration characteristics gradually stabilize with increasing depth because the velocity potential functions converge.

This study improves the effective material property calculation model for porous FGM plates, establishes hydroelastic vibration equations for porous FGM rectangular plates in fluid based on thin plate theory, and systematically analyzes the effects of plate-fluid interaction boundaries, fluid depth, pores, and material composition index on free vibration frequencies and dynamic responses. The main conclusions are:

1) Compared with vacuum, fluid not only reduces the natural frequencies of FGM plates but also decreases forced vibration dynamic responses.
2) At the same fluid depth, plates under IBC1 boundary conditions exhibit the highest natural frequencies, while IBC3 conditions yield the smallest dynamic responses. Natural frequencies under IBC1 and IBC3 decrease with increasing fluid depth $h1$, whereas IBC2 shows the opposite trend. Additionally, the influence of fluid depth on frequencies and responses becomes constant beyond a certain depth.
3) Between the two pore distributions, D2 yields the highest natural frequencies and smallest dynamic responses, indicating that D2-distributed pores most significantly affect plate mechanical performance.
4) The effect of internal pores on fundamental frequency depends not only on porosity volume ratio and distribution pattern but also on plate-fluid interaction boundary conditions.
5) Natural frequencies increase with ceramic mass ratio and exhibit a decreasing-then-increasing trend with material composition index N, while dynamic responses show inverse behavior. Natural frequencies reach minimum values and dynamic responses reach maximum values in the range $1.5 < N < 2$, suggesting this interval should be avoided in structural design.

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Note: Figure translations are in progress. See original paper for figures.