Postprint: Multiscale Homogenization-Based Simulation Study on the Eccentric Compression Performance of Reinforced Concrete Columns

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Abstract

Based on the equivalent homogenization theory, multi-scale simulations were conducted to investigate the influence of meso-scale factors of concrete aggregates on the mechanical behavior of reinforced concrete columns under eccentric compression by establishing meso-scale numerical models and meso-scale homogenized numerical models. The results demonstrate that the simulation results agree well with reference experimental results, thereby validating the effectiveness of the meso-scale homogenized numerical model at the component level; both the meso-scale numerical model and the meso-scale homogenized numerical model can effectively reflect the influence of random aggregate distribution on the mechanical properties and damage distribution of reinforced concrete under eccentric compression; compared with the meso-scale numerical model, the meso-scale homogenized numerical model exhibits lower sensitivity to different aggregate shapes; the meso-scale homogenized numerical model can save computational space and improve computational efficiency.

Full Text

Preamble

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Multi-scale Homogenization-Based Simulation Study on the Eccentric Compression Performance of Reinforced Concrete Columns

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Abstract: Based on equivalent homogenization theory, this study investigates the influence of meso-scale factors of concrete aggregates on the mechanical behavior of reinforced concrete columns under eccentric compression through multi-scale simulation using meso-scale numerical models and meso-homogenization numerical models. The results demonstrate that: (1) simulation outcomes agree well with reference experimental results, validating the effectiveness of the meso-homogenization numerical model at the component level; (2) both meso-scale and meso-homogenization numerical models can effectively capture the influence of random aggregate distribution on the mechanical properties and damage distribution of reinforced concrete under eccentric compression; (3) compared with the meso-scale numerical model, the meso-homogenization numerical model exhibits lower sensitivity to different aggregate shapes; and (4) the meso-homogenization numerical model can significantly reduce computational storage requirements and improve computational efficiency.

Keywords: eccentrically compressed reinforced concrete column; meso-numerical model; meso-homogenization numerical model; meso-scale factors of aggregate

Classification: TU375.3

Concrete is a multi-phase composite material composed of aggregates, mortar, microcracks, air voids, and other constituents. The random distribution and varied shapes of aggregates are the primary sources of concrete’ s randomness and heterogeneity, and their meso-scale composition influences the mechanical behavior of reinforced concrete members. Therefore, investigating the influence of concrete aggregate meso-scale factors on the mechanical behavior of reinforced concrete members holds significant importance.

In recent years, domestic and international scholars have established numerous meso-scale numerical models to study concrete member behavior from a meso-level perspective, including lattice models [1], random particle models [2], random mechanical property models [3], and random aggregate models [4-6]. Gao et al. [5] proposed a placement algorithm for two-dimensional polygonal and convex polyhedral random aggregates in concrete. Liu et al. [4] simulated the entire process from damage to fracture failure in single-edge notched tension specimens using random aggregate models. Du et al. [7] developed aggregate placement algorithms considering actual engineering aggregate gradation and volume fraction, and conducted compressive failure studies on three-dimensional concrete random aggregate numerical specimens.

To deeply investigate the influence of concrete meso-scale components on mechanical behavior at both material and component levels, many scholars have employed meso-scale numerical simulations to study macro-scale mechanical properties of concrete materials under uniaxial tension, compression, shear, and flexural tension [8-11], as well as reinforced concrete columns under axial and eccentric compression [12-15]. Additionally, Chen et al. [16-17] performed numerical simulations on non-contact spliced reinforced concrete bridge columns based on meso-scale models.

Meso-scale numerical models for concrete require fine meshing of interface layers between different materials to satisfy deformation compatibility requirements, leading to high computational cost and low efficiency [3-9]. To represent concrete heterogeneity while improving computational efficiency, Ma et al. [10] proposed a parallel multi-scale modeling method for bridge structures based on homogenization concepts. Jin [18] simulated the axial compression behavior of concrete rectangular and reinforced concrete columns using a concrete meso-scale equivalent model. Wang et al. [19] investigated the influence of random aggregate distribution and different shapes on the macro-scale tensile mechanical properties of concrete materials using a meso-homogenization model. However, the aforementioned concrete meso-homogenization mechanical models were based on elastic damage constitutive relationships and did not consider the influence of the concrete elastoplastic stage.

This study proposes a concrete homogenization mechanical model based on the CDP (Concrete Damaged Plasticity) model and composite material meso-mechanics. Considering the random distribution and different shapes of aggregates, meso-scale models and corresponding meso-homogenization models of reinforced concrete columns containing randomly distributed circular, elliptical, and polygonal aggregates were established to efficiently simulate the mechanical behavior and damage distribution of reinforced concrete columns under eccentric compression. The simulation results were compared with existing experimental data [20] to verify the feasibility and effectiveness of the meso-homogenization model, providing an important tool and simulation foundation for multi-scale modeling and efficient mechanical behavior analysis of reinforced concrete columns.

1 Homogenization Theory and Constitutive Relations

1.1 Aggregate, Mortar, and Steel

Aggregate is typically described using a linear elastic constitutive model, with its stress-strain relationship represented by an inclined line segment. The slope represents the elastic modulus E_{ag}, with endpoints $A(\varepsilon_{c.ag.max}, \delta_{c.ag.max})$ and $A'(\varepsilon_{t.ag.max}, \delta_{t.ag.max})$ corresponding to the compressive and tensile peak points of aggregate, respectively, as shown in [Figure 1: see original paper].

Due to the presence of microcracks, air voids, and other defects, mortar exhibits softening behavior during loading. Therefore, a plastic damage constitutive model is adopted to describe its mechanical properties. E_{mo} represents the mortar elastic modulus, x_0 is the compressive limit elastic strain, and the compressive and tensile peak points are $A(\varepsilon_{c.mo.max}, \delta_{c.mo.max})$ and $A'(\varepsilon_{t.mo.max}, \delta_{t.mo.max})$, respectively, as shown in [Figure 2: see original paper].

Plastic damage factors are calculated through Sidoroff' s energy equivalence principle [21]. Mortar plastic damage is characterized by compressive and tensile plastic damage factors d_c and d_t, which represent the damage degree of mortar under compression and tension, respectively, typically ranging from 0 to 1. When d_c and d_t equal 0, mortar is undamaged; when they equal 1, mortar has failed due to damage.

The compressive plastic damage factor expression is as follows:

When $x \leq 1$:

$$d_c = 1 - \frac{E_0 \varepsilon_c}{[\alpha_a + (3 - 2\alpha_a)x + (\alpha_a - 2)x^2}$$

When $x > 1$:

$$d_c = 1 - \frac{E_0 \varepsilon_c}{\alpha_d (x-1)^2 +}$$

The tensile plastic damage factor expression is as follows:

When $x \leq 1$:

$$d_t = 0$$

When $x > 1$:

$$d_t = 1 - \frac{1}{1.7}$$

where E_0 is the initial elastic modulus of mortar; f_c^* is the mortar compressive strength; α_a and α_d are parameters for the ascending and descending branches of the mortar compressive plastic damage constitutive model, respectively; and α_t is the parameter for the descending branch of the mortar tensile plastic damage constitutive model.

Referencing existing experimental data [20], material parameters for aggregate and mortar were obtained through meso-scale simulation back-analysis, as listed in .

Steel is described using an ideal plastic constitutive model, a classical model for steel mechanical behavior. The curve initially appears as an inclined straight line with slope equal to the steel elastic modulus E_s. After reaching the steel compressive and tensile yield points, it becomes a horizontal line, as shown in [Figure 3: see original paper]. Steel material parameters are listed in .

1.2 Connection Elements Between Steel and Concrete

Nonlinear Spring2 spring elements connect steel and concrete to simulate bond-slip behavior between them. The bond-slip constitutive relationship references the "Standard for Design of Concrete Structures" (GB/T 50010—2010) [22]. [Figure 4: see original paper] shows the bond stress-slip constitutive relationship curve between steel and concrete. This curve is primarily determined by characteristic points, including the cracking point $A_{cr}(\tau_{cr}, s_{cr})$, peak point $A_u(\tau_u, s_u)$, and residual point $A_r(\tau_r, s_r)$, with characteristic values listed in .

1.3 Constitutive Relations of Homogenized Elements

The constitutive relationship of meso-homogenization elements is based on homogenization theory and composite material parallel models, combining the semi-empirical method of material mechanics with plastic damage constitutive models. Using aggregate area fraction as the independent variable, the elastic modulus and strength of aggregates and mortar within homogenized grids are equivalently processed. This procedure consists of three steps: (1) equivalent elastic modulus based on composite material parallel models; (2) equivalent tensile and compressive strength based on the semi-empirical method of material mechanics; and (3) derivation of the corresponding plastic damage constitutive model based on equivalent elastic modulus and peak strength.

Step 1: Elastic Modulus Equivalence is based on the Voigt parallel model [18]. Assuming aggregates and mortar undergo the same displacement during loading, as shown in [Figure 5: see original paper], the equivalent elastic modulus E' is obtained through equations (5) and (6):

$$S_{ag} + S_{mo} = 1$$

$$E' = E_{ag} \cdot S_{ag} + E_{mo} \cdot S_{mo}$$

where E_{ag}, E_{mo}, S_{ag}, and S_{mo} are the elastic moduli and area fractions of aggregate and mortar in the equivalent grid element, respectively.

Step 2: Peak Strength Equivalence is based on the semi-empirical method of material mechanics, which considers two scenarios for the strength of concrete composite materials in the elastic stage. The first scenario occurs when the aggregate content is relatively high, where the ultimate strain of the concrete composite is similar to the aggregate failure strain $(\varepsilon_{t,c})_{ag.max}$. In this case, concrete strength is primarily controlled by aggregates, referred to as the **aggregate-controlled strength stage**. The expressions for concrete tensile and compressive peak stresses in this stage are given by equation (8). The second scenario occurs when aggregate content is low, where the ultimate strain of the concrete composite far exceeds the aggregate failure strain $(\varepsilon_{t,c})_{ag.max}$. It is assumed that when the concrete composite reaches its ultimate strain, the small amount of aggregate has already failed, and concrete strength is completely controlled by mortar, referred to as the **mortar-controlled strength stage**. The expressions for concrete tensile and compressive peak stresses in this stage are given by equation (9).

$$V_{ag} = \frac{c_{mo.max}}{1 - V_{c.mo.max}} \frac{1 - V_{ag.max}}{V_{ag}} + \delta_{t,c} = \delta_{t,c} = \delta_{t,c} = \delta_{t,c} = \delta_{t,c}$$

where V_{ag} is the proportion of aggregate within the equivalent grid; $\delta'_{t,c}$ is the tensile/compressive peak stress of the equivalent material grid in the elastic stage; and $(\delta_{t,c})_{ag.max}$ and $(\delta_{t,c})_{mo.max}$ are the tensile/compressive peak stresses of aggregate and mortar, respectively.

Using the equivalence of concrete tensile peak stress in [Figure 6: see original paper] as an example, where $(\delta_{t,c})_{mo.m}$ represents the tensile/compressive stress of mortar corresponding to the aggregate tensile/compressive peak strain. Figure 6: see original paper shows the stress-strain relationship curves for aggregate and mortar in tension. It can be observed that the aggregate tensile peak strain $(\varepsilon_t)_{ag.max}$ is significantly smaller than the mortar tensile peak strain $(\varepsilon_t)_{mo.max}$. Based on the above formulas, with aggregate proportion V_{ag} as the horizontal axis and overall concrete tensile peak stress as the vertical axis δ, these two scenarios can be plotted as two intersecting lines, as shown in Figure 6: see original paper. The concrete strength curve in the aggregate-controlled stage is a positively sloped ascending line, while the concrete strength curve in the mortar-controlled stage is a negatively sloped descending line. The red-highlighted portion represents effective values, showing that the peak stress of equivalent grids first decreases and then increases, essentially following a "V-shaped" development.

It is worth noting that the ascending branch of the concrete tensile stress-strain relationship curve is a straight line, which is fully applicable to the above equivalent tensile peak stress method for homogenized grids. The ascending branch of the concrete compressive stress-strain relationship curve consists of elastic and plastic stages. The above method can only determine the compressive peak stress in the elastic stage. The compressive peak stress of homogenized grids must be obtained by multiplying the elastic stage compressive peak stress by an amplification factor K_i:

$$2 + \alpha_a - i + \alpha_a - \alpha_a + 3 - 2\alpha + \alpha_{axi} + 3 - 2\alpha$$

$$\sigma_c = K_i \cdot \delta'$$

where α_a is the parameter for the ascending branch of the concrete compressive stress-strain relationship curve; x_i is the elastic-plastic boundary point; δ' is the compressive peak stress in the concrete elastic stage; and σ_c is the compressive peak stress of concrete.

2 Establishment of Meso-scale and Meso-homogenization Models

2.1 Establishment of Meso-scale Models

2.1.1 Random Aggregate Model Generation Based on reference experiments [20], eccentric column specimens have dimensions of 960 mm × 150 mm, with a total concrete aggregate volume fraction of 0.7, of which coarse aggregate accounts for 0.37. Using the Fuller gradation curve [23] and Walraven formula [24] conversion, two-dimensional random aggregate model gradation parameters were obtained, as listed in .

Treating concrete as a two-phase composite material consisting of aggregates and mortar, the Monte-Carlo method [4] was employed to randomly place two-dimensional circular, elliptical, and polygonal aggregates through Matlab programming, generating five two-dimensional eccentric column geometric models, as shown in [Figure 7: see original paper].

2.1.2 Finite Element Setup of Meso-scale Models The concrete meso-scale model was created using the "partition method" to separately generate aggregate and mortar regions and assign material properties. Assuming perfect bonding between aggregates and mortar during loading, the interaction between aggregate and mortar regions was set as a tie constraint. Aggregate and mortar mesh shapes were set as triangular and quadrilateral (primarily), with element types CPS3 (three-node plane stress triangle) and CPS4R (four-node bilinear plane stress quadrilateral), respectively, to improve convergence and reduce iteration counts. The mesh size was controlled at 2 mm. To prevent stress concentration during simulated loading, rigid pads were placed at both ends of the eccentric column model and tied to the concrete. Reference points were established at the loading and constrained sides of the eccentric column model, with "point-to-surface" coupling constraints between reference points and pad surfaces.

2.2 Establishment of Meso-homogenization Models

Using a 960 mm × 150 mm concrete eccentric column with circular random aggregates as an example, and considering maximum aggregate size and specimen dimensions, the eccentric column random aggregate model was divided into 480 homogenized elements using 20 mm × 15 mm quadrilateral grids, as shown in [Figure 8: see original paper].

Through Matlab and Python programs, the constitutive relationships of aggregates and mortar within each homogenized element were equivalently processed and input as ABAQUS parameters to establish the reinforced concrete eccentric column meso-homogenization model. The distributions of elastic modulus and tensile/compressive strength for each homogenized element are shown in [Figure 9: see original paper]. The elastic modulus, compressive strength, and tensile strength of homogenized elements essentially follow normal distributions, reflecting the randomness of aggregate placement and the rationality of homogenized element sizes.

2.3 Steel Finite Element Setup

Steel mesh elements are T2D2 two-node two-dimensional truss elements with a mesh size of 2 mm. Steel material parameters are listed in . The eccentric column member dimensions and reinforcement layout are shown in Figure 8: see original paper, with stirrup spacing of 150 mm and densified ends to prevent local failure during loading. Spring2 spring elements connect steel and concrete nodes to simulate bond-slip behavior.

Similar to the meso-scale model, rigid pads were placed at the loading and constrained sides of the eccentric column to prevent stress concentration in concrete that could affect numerical simulation of mechanical behavior. Concrete and pads were tied, while steel and pads were coupled through "point-to-surface" constraints. Reference points were established at eccentric positions on the upper and lower pad surfaces, with "point-to-surface" coupling between reference points and pad surfaces. Vertical eccentric displacement loading was applied at the upper reference point with eccentricity $e_0 = 0.1 \times h_0$, where h_0 is the section depth of 150 mm. The lower reference point constrained X and Y displacements at the eccentric position while allowing rotation.

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3 Numerical Analysis of Eccentric Columns

3.1 Validation of Meso-scale and Meso-homogenization Models

3.1.1 Stress Field Analysis [Figure 10: see original paper] shows the stress distributions of circular aggregate eccentric columns for both meso-scale and meso-homogenization models. From left to right, the images correspond to the elastoplastic stage, peak stage, descending stage, residual stage, and steel stress distribution at failure. During the elastoplastic stage, large compressive stress regions appear on the compression side of the column, with a few high-stress red zones. High-stress transmission paths primarily follow "aggregate-mortar-aggregate." At the peak stage, high-stress red zones increase on the compression side, with maximum stresses of approximately 144.7 MPa in aggregate regions and 62 MPa in mortar regions for the meso-scale model. During the descending stage, mortar elements lose capacity after damage, reducing high-stress red zones on the compression side. In the residual stage, high-stress red zones completely disappear as the column has failed, with stress transmission patterns completely altered and new transmission paths forming in previously undamaged regions. Steel stress distributions show both longitudinal bars in compression, with compression-side longitudinal bars yielding first and stirrups exhibiting outward bulging. Stress distribution locations and evolution processes are essentially identical between meso-scale and meso-homogenization models, demonstrating that the meso-homogenization model can accurately reflect stress distribution processes and validating the feasibility of the meso-homogenization model at the component level.

3.1.2 Failure Modes [Figure 11: see original paper] compares compressive damage distributions and experimental failure results for circular aggregate eccentric columns between meso-scale and corresponding homogenization models. Compressive damage primarily occurs in mortar regions above and below aggregates, with final failure manifested as local compressive failure on the compression side at mid-height. Simulated damage ranges and shapes are essentially consistent with reference experiments. Comparison between meso-scale and meso-homogenization simulation results shows that meso-scale model damage primarily develops around aggregates with larger distribution ranges, mainly located at the mid-height compression side. Meso-homogenization model damage distribution locations are essentially consistent with the meso-scale model, demonstrating good capability in reflecting eccentric column concrete damage distribution and failure modes.

3.1.3 Load-Mid-Height Deflection Curves [Figure 12: see original paper] compares the corresponding load-mid-height deflection curves. The meso-scale model peak load is $F_0 = 1,021.3$ kN with corresponding mid-height deflection $l_0 = 2.77$ mm. The meso-homogenization model peak load is $F_1 = 1,023.4$ kN with deflection $l_1 = 2.83$ mm. The reference experimental peak load is $F = 1,007.0$ kN with deflection $l = 2.71$ mm. Differences are within 5%, indicating good agreement between simulation and experimental results. Meso-scale and meso-homogenization simulation curves essentially coincide in the elastic stage. In the elastoplastic stage, the homogenization model shows slower elastic modulus degradation due to regular rectangular meshing and more uniform stress distribution, resulting in slightly higher bearing capacity than the meso-scale model. In the descending branch, the homogenization model exhibits faster capacity degradation than the meso-scale model because all homogenized grids are described by plastic damage constitutive models without elastic aggregate portions, leading to more concentrated damage.

3.2 Comparative Analysis of Meso-scale and Homogenization Models

[Figure 13: see original paper] compares compressive damage distributions among three meso-scale models and corresponding homogenization models with randomly distributed circular aggregates. Meso-scale model results show that plastic damage concentration locations are essentially identical, with final failure modes all being crushing failure on the compression side. However, damage locations shift due to different aggregate distributions: Model a shows a significantly higher damage location; Model b shows damage precisely at the mid-height compression side; Model c shows damage at both upper and lower positions. These variations result from differences in aggregate spacing—regions with smaller aggregate spacing more easily form "aggregate-mortar-aggregate" stress transmission paths. Thus, random aggregate distribution significantly influences compressive damage distribution locations in eccentric columns.

Since the homogenization method essentially "averages" the elastic modulus and strength of aggregates and mortar within equivalent units, high-stress "aggregate" regions decrease while low-stress "mortar" regions increase in meso-homogenization models. Consequently, some compressive damage present in meso-scale models does not appear in homogenization models. Furthermore, because meso-homogenization models lack elastic aggregates and all mechanical behavior is described by plastic damage constitutive models, damage concentration occurs more readily during loading, resulting in more localized damage distribution.

[Figure 14: see original paper] shows load-mid-height deflection relationships for three groups of circular aggregate models with different distributions. The three meso-scale simulation curves exhibit small variations, with ascending branches matching well with experimental curves. Model a' s peak load is close to the experimental value, while Models b and c show slightly lower peak loads. Under eccentric compression, high-stress regions generally appear on the compression side, and regions with smaller aggregate spacing on the eccentric side tend to develop dense stress transmission paths. When stress increases sufficiently, these "dense" regions develop damage first. More extensive and concentrated damage leads to higher eccentric column bearing capacity.

3.3 Influence of Aggregate Shape on Eccentric Column Mechanical Properties

[Figure 15: see original paper] compares plastic damage distributions under eccentric compression among meso-scale and homogenization models of reinforced concrete columns with circular, elliptical, and polygonal aggregates generated from three groups of random aggregate models. Meso-scale model damage distributions show that overall damage distribution shapes are essentially similar across aggregate shapes, largely eliminating the influence of different aggregate distributions. However, differences in stress transmission patterns due to aggregate shape cause subtle variations in damage distribution. Compared with circular and elliptical aggregates, polygonal aggregates tend to develop stress concentration at corners during eccentric force transmission, with damage propagation paths often developing between polygonal aggregate sharp corners, resulting in essentially linear damage distribution that easily penetrates through. Compared with circular aggregates, elliptical aggregates exhibit two force transmission modes: (1) eccentric force acting on the long axis of elliptical aggregates, where the long side bears larger forces and the short side bears smaller forces, primarily showing compressive damage on both sides of the elliptical aggregate long axis; and (2) eccentric force facing the short side, where the short side bears larger forces and the long side bears smaller forces, primarily showing compressive damage on both sides of the elliptical aggregate short axis. Consequently, elliptical aggregate models exhibit more fragmented damage distribution that can form intersecting damage around aggregates.

From a macro-scale mechanical performance perspective, load-mid-height deflection curves for circular and polygonal aggregate models are similar, while elliptical aggregate models show slightly higher peak loads, confirming the above compressive damage analysis regarding how different aggregate shapes affect stress action and transmission patterns.

[Figure 16: see original paper] shows the corresponding load-mid-height deflection curves, where meso-homogenization model peak loads are slightly higher than meso-scale models. Purely from homogenization model compressive damage distributions, it is difficult to determine the influence of aggregate shape on eccentric compression mechanical properties. This limitation arises for two reasons: (1) since the homogenization method uses area percentages of aggregates and mortar in grids as variables, different aggregate shapes at the meso-scale have minimal impact on these area percentages, making homogenization models less effective for studying aggregate shape effects; and (2) homogenization model grids are regular squares or rectangles that cannot represent aggregate shapes even when containing elements with higher strength and elastic modulus, only allowing consideration of square or rectangular aggregate effects. These factors indicate that studying aggregate shape influence through meso-homogenization models has inherent limitations.

3.4 Computational Cost Comparison

compares computational costs between concrete column meso-scale and meso-homogenization models, with ratios of elements, nodes, and degrees of freedom approximately 1:110. After homogenization treatment, the eccentric column meso-scale model shows substantial reductions in element, node, and degree-of-freedom counts, demonstrating that meso-homogenization models significantly save computational storage and improve efficiency.

Conclusions

This study employed meso-scale numerical simulation and meso-homogenization numerical simulation to investigate reinforced concrete columns under eccentric compression, leading to the following main conclusions:

1. Both meso-scale and meso-homogenization models can effectively reflect the macro-scale mechanical properties and meso-scale damage distribution of eccentric columns.
2. Both models can adequately capture the influence of random aggregate distribution on eccentric column mechanical behavior. Random aggregate distribution affects macro-scale mechanical properties and meso-scale damage distribution by altering aggregate spacing.

3. Compared with meso-scale models, eccentric column meso-homogenization models exhibit lower sensitivity to aggregate shape due to larger grid sizes and exclusive use of elastoplastic constitutive models. Aggregate shape influences macro-scale mechanical properties and meso-scale damage distribution by altering stress transmission paths and patterns. Elliptical aggregate models show higher bearing capacity, while circular and polygonal aggregate models show lower capacity.
4. Two-dimensional meso-scale models have approximately 110 times more elements, nodes, and degrees of freedom than meso-homogenization models, demonstrating that meso-homogenization models substantially save computational storage and improve efficiency.

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