Compensation of emittance and bunch length variations induced by insertion devices using damping wigglers in fourth-generation light sources

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Abstract

Recent investigations suggest that in a fourth-generation light source, where both fundamental-frequency andharmonic RF cavities are commonly used for bunch lengthening, variations in radiation energy loss per turn (U0)could cause a significant change in bunch length. It is necessary to compensate for the U0 variations caused bygap changes of the insertion devices (IDs). In this paper, we investigate the approach of using two horizontal ortwo vertical damping wigglers to simultaneously compensate for horizontal emittance and U0 variations inducedby IDs. Theoretical analysis and a specific example of application are presented.

Full Text

Compensation of Emittance and Bunch Length Variations Induced by Insertion Devices Using Damping Wigglers in Fourth-Generation Light Sources

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Recent investigations suggest that in a fourth-generation light source, where both fundamental-frequency and harmonic RF cavities are commonly used for bunch lengthening, variations in radiation energy loss per turn (U_0) could cause

significant changes in bunch length. It is necessary to compensate for the U_0 variations caused by gap changes of the insertion devices (IDs). In this paper, we investigate the approach of using two horizontal or two vertical damping wigglers to simultaneously compensate for horizontal emittance and U_0 variations induced by IDs. Theoretical analysis and a specific application example are presented.

Keywords: Emittance, Radiation energy loss per turn, Vertical damping wiggler, Parameters compensation

Introduction

Currently, numerous laboratories worldwide have either constructed or are planning to construct fourth-generation light sources (4GLSs) [1–18]. A remarkable feature of 4GLSs is that the storage ring emittance is reduced to near or even reaches the X-ray diffraction limit through the use of multi-bend achromat (MBA) lattices [19–25]. Thanks to the reduction in beam emittance and the adoption of state-of-the-art insertion devices (IDs) [26–29], the brightness of 4GLSs is approximately two orders of magnitude higher than that of third-generation light sources. More significantly, the coherence of synchrotron radiation is improved, laying the foundation for achieving higher spatial resolution.

In 4GLSs, IDs cause radiation energy loss that matches or surpasses that in bending magnets. Adjustments of the gaps (and thus magnetic fields) of IDs according to experimental requirements will lead to evident variations in radiation energy loss per turn (U_0) and other beam parameters, such as emittance, energy spread, and bunch length. Among these parameters, variations in emittance would lead to fluctuations in photon brightness and distribution, which impact specific beamline experiments, such as those conducted in scanning transmission X-ray microscopy beamlines [30]. This issue can be resolved by adopting a horizontal damping wiggler (HDW) to compensate for horizontal emittance variations [31, 32], and two HDWs have been proposed to simultaneously compensate for variations of both emittance and energy spread [33].

Recent investigations suggest that in a 4GLS where both fundamental-frequency and harmonic RF cavities are commonly used for bunch lengthening, variations in U_0 could cause evident changes in bunch length. Maintaining bunch length stability is of significant importance for mitigating the intra-beam scattering (IBS) effect, enhancing beam stability, and prolonging beam lifetime.

To compensate for bunch length variations, one approach is to integrate an additional feedback system into the RF cavities to adjust the relevant RF parameters accordingly, so as to keep the bunch length unchanged. In this paper, we investigate another approach, i.e., using two HDWs or vertical damping wigglers (VDWs) to simultaneously achieve compensation for horizontal emittance and U_0 without the need for an additional RF feedback system. Compared to the dual HDWs scheme, the dual VDWs scheme can directly generate vertical emittance without requiring additional methods and can compensate for the vertical emittance variations induced by non-planar IDs.

This paper is organized as follows. In Section II, we analyze the characteristics of HDW and VDW and theoretically explain how two HDWs or VDWs achieve compensation for horizontal emittance and U_0 variations. In Section III, we provide a specific example to facilitate a more intuitive understanding. Section IV presents the conclusion.

II. Theoretical Analysis

In this section, we elucidate the reason for persistence of U_0 variations following HDW' s compensation for horizontal emittance. Additionally, we clarify how two HDWs or VDWs address this issue.

A. Horizontal Emittance and U_0 Variations Induced by IDs

In an electron storage ring, the emittance (without linear coupling) and U_0 can be expressed in terms of synchrotron radiation integrals:

$= C\_q \gamma^2 (I_5 / (I_2 - I_4))$
$U_0 = C\_\gamma E^4 I_2$

where $C\_q = 3.84 \times 10^{-13}$ m, γ is the Lorentz relativistic factor, $C\_\gamma = 8.85 \times 10^{-5}$ m · GeV^{-3}, and E is the electron energy. The synchrotron radiation integrals are expressed by:

$I_2 = \ (1/\ ^2)$ ds
$I_4 = \ (/ \ ^3 + 2k)$ ds, where $k = (1/B\ )(\ B\_y/\ x)$
$I_5 = \ (H/|\ |^3)$ ds, where $H = \gamma\ ^2 + 2\alpha\ \ ' + \beta\ '\ ^2$

where is the curvature of the reference particle, is the dispersion function, B is the magnetic rigidity, and γ, α, β are the Twiss parameters.

Considering IDs (we first consider only planar IDs, as was done in previous research on horizontal emittance compensation [32, 33]), the horizontal emittance and U_0 are:

$= C\_q \gamma^2 (I_5b + \Sigma_{\ 1}\ I_5IDi) / ((I_2b + \Sigma_{\ 1}\ I_2IDi) - (I_4b + \Sigma_{\ 1}\ I_4IDi))$
$U_0 = C\_\gamma E^4 (I_2b + \Sigma_{\ 1}\ I_2IDi)$

where the subscripts b and ID represent synchrotron radiation integrals contributed by the bare lattice and IDs, and N is the number of IDs. During machine operation, users adjust ID magnetic fields due to experimental requirements. This results in alterations to the synchrotron radiation integrals contributed by IDs, leading to variations in horizontal emittance and U_0.

B. Horizontal Emittance and U_0 Compensation by Two HDWs

Evident variations in U_0 may still persist after compensating for horizontal emittance using an HDW. To simultaneously compensate for both horizontal emittance and U_0, we introduce two HDWs. For all theoretical considerations presented hereafter, the storage ring is assumed to contain perfectly achromatic straight sections.

For horizontal emittance compensation by an HDW, the compensation equation is:

= C_q γ^2 (I_5b + Σ_1 I_5IDi + I_5HDW) / ((I_2b + Σ_1 I_2IDi + I_2HDW) - (I_4b + Σ_1 I_4IDi + I_4HDW))

where:

I_2HDW = (B_H^2 L_H) / (2(B)^2)
I_5HDW = (λ_H^2 β B_H^5 L_H) / (15π^3(B)^5)

Here, B_H is the HDW magnetic field strength, L_H is the HDW length, λH *is the HDW period length, and* β *is the average horizontal beta function in the HDW. Accordingly,* B{IDi} is the ID_i magnetic field strength, L_{IDi} is the ID_i length, λ_{IDi} is the ID_i period length, and β _i is the average horizontal beta function in ID_i.

Here, Σ_1 I_4IDi and I_4HDW are ignored because their values are significantly smaller compared to Σ_1 I_2IDi and I_2HDW. When ID magnetic fields change, we can adjust the magnetic field strength of the HDW in real time to maintain the stability of horizontal emittance.

The equilibrium emittance of particle beams in a storage ring arises from a dynamic balance between two fundamental physical processes [34]. First, the quantum excitation caused by the quantized emission of photons contributes to an increase in emittance. Second, a damping effect, resulting from the loss of some transverse momentum of the particles during photon emission, contributes to a decrease in emittance. The quantum excitation effects caused by IDs and HDW are characterized by Σ_1 I_5IDi and I_5HDW. Following the compensation of horizontal emittance using an HDW, U_0 is automatically compensated only when Σ_1 I_5IDi and I_5HDW are sufficiently small to be considered negligible. In this scenario, the compensation equation can be expressed as follows:

= C_q γ^2 (I_5b / (I_2b + I_2HDW - I_4b)) = ^c

When the storage ring' s IDs consist primarily of low-field undulators, I_5b Σ_1 I_5IDi, indicating that after employing an HDW to compensate for horizontal emittance, the magnitude of U_0 variations is contingent upon I_5HDW. When the compensation amount of horizontal emittance variations is substantial, the magnetic field strength required for the HDW also increases. Thus, the quantum excitation induced by HDW will produce a significant effect, especially in low emittance rings (see Appendix A). The final result is that after compensating for horizontal emittance, U_0 still exhibits evident variations.

To address the issue of U_0 variations, we split the HDW into two parts (HDW1 and HDW2) without increasing the total length. The compensation equations are as follows:

= C_q γ^2 (I_5b + Σ $_1$ I_5IDi + I_5HDW1 + I_5HDW2) / ((I_2b + Σ $_1$ I_2IDi + I_2HDW1 + I_2HDW2) - (I_4b + Σ $_1$ I_4IDi + I_4HDW1 + I_4HDW2))

U_0 = C_γ E^4 (I_2b + Σ $_1$ I_2IDi + I_2HDW1 + I_2HDW2)

Compensation of horizontal emittance and U_0 can be achieved with varying ID magnetic fields by adjusting the magnetic field strengths (B_{H1} and B_{H2}) of the two HDWs. For given lengths of HDW (L_{H1} and L_{H2}), appropriate selection of emittance and U_0 compensation values enables numerical determination of B_{H1} and B_{H2} through computational procedures. The compensation equations can be further simplified:

Σ $_1$ I_2IDi + I_2HDW1 + I_2HDW2 = I_2^c
I_5HDW1 + I_5HDW2 = I_5^c

C. Horizontal Emittance and U_0 Compensation by Two VDWs

In this section, we propose an alternative method for simultaneously compensating for horizontal emittance and U_0. Two VDWs are introduced. VDW was originally proposed to generate round beams instead of traditional linear coupling methods [35, 36]. VDW is equivalent to rotating HDW by 90° around the longitudinal axis, thereby transforming the magnetic field from the vertical direction to the horizontal direction. Consequently, the quantum excitation effect is transferred to the vertical direction.

After adding a VDW, the emittances become:

= C_q γ^2 (I_5b + Σ $_1$ I_5IDi) / ((I_2b + Σ $_1$ I_2IDi + I_2VDW) - I_4b)
_y = C_q γ^2 (I_5VDW) / (I_2b + Σ $_1$ I_2IDi + I_2VDW)

where:

I_2VDW = (B_V^2 L_V) / (2(B)^2)
I_5VDW = (λ_V^2 β_y B_V^5 L_V) / (15π^3(B)^5)

Here, B_V is the VDW magnetic field strength, L_V is the VDW length, λ_V is the VDW period length, and β_y is the average vertical beta function in the VDW. In this case, vertical emittance is generated as a result of vertical dispersion induced by the horizontal field of the VDW.

In the scenario where I_5b Σ $_1$ I_5IDi, the contribution of Σ $_1$ I_5IDi to I_5 can be ignored. If a VDW is used to compensate for horizontal emittance variations, then U_0 is automatically compensated:

= C_q γ^2 (I_5b / (I_2b + Σ $_1$ I_2IDi + I_2VDW - I_4b)) = ^c
U_0 = C_γ E^4 (I_2b + Σ $_1$ I_2IDi + I_2VDW) = U_0^c

Since VDWs contribute radiation damping in both transverse planes but induce quantum excitation only in the vertical plane, the horizontal emittance benefits from increased damping without added excitation. This enables simultaneous stabilization of both horizontal emittance and U_0, while generating a small, controlled vertical emittance. We propose dividing the VDW into two units, designated VDW1 and VDW2, to maintain the stability of both horizontal and vertical emittance:

= C_q γ^2 (I_5b + Σ $_1$ I_5IDi) / ((I_2b + Σ $_1$ I_2IDi + I_2VDW1 + I_2VDW2) - I_4b) = ^c
_y = C_q γ^2 (I_5VDW1 + I_5VDW2) / (I_2b + Σ $_1$ I_2IDi + I_2VDW1 + I_2VDW2) = _y^c

Stability of horizontal and vertical emittance can be achieved with varying ID magnetic fields by adjusting the magnetic field strengths (B_{V1} and B_{V2}) of the two VDWs. Simultaneously, U_0 is automatically compensated. One can generate a round beam by making _y^c ^c. The compensation equations can be further simplified:

Σ $_1$ I_2IDi + I_2VDW1 + I_2VDW2 = I_2^c
I_5VDW1 + I_5VDW2 = I_5^c

III. Application to SAPS Lattice

To intuitively understand the compensation of horizontal emittance and U_0, we take the Southern Advanced Photon Source (SAPS) [4] as an example.

SAPS is a mid-energy ultra-low emittance light source proposed to be built adjacent to the China Spallation Neutron Source (CSNS) [37]. The main parameters of SAPS are listed in Table 1 . The circumference of SAPS is 810 m and encompasses 32 periods. The component layout and beam optics of one period are illustrated in Fig. 1 [Figure 1: see original paper]. Nine IDs are scheduled for construction in the first phase of SAPS. The IDs of SAPS consist of three types: in-air undulator (IAU), in-vacuum undulator (IVU), and cryogenic permanent magnet undulator (CPMU), and the parameters are presented in Table 2 .

TABLE 1. Main parameters of SAPS.

Parameter	Value
Beam energy	2.2 GeV
Beam current	500 mA
Circumference	810 m
Natural emittance	26.3 pm • rad
Tune (H,V)	78.21, 44.16
Momentum compaction factor	3.61×10^{-5}
Radiation energy loss per turn without IDs	0.768 MeV

The effect of IDs on horizontal emittance and U_0 was evaluated using synchrotron radiation integral formulas. The results, presented in Fig. 2 [Figure 2: see original paper], were obtained by sequentially incorporating each ID into the calculations (we assume that all IDs are at minimum gap). The leftmost point represents the scenario where no IDs are included, while the rightmost point represents the scenario where all IDs are included. Horizontal emittance decreases by 14.76% (3.9 pm • rad) and U_0 increases by 28.34% (0.22 MeV) due to the influence of IDs.

In practice, horizontal emittance variations induced by IDs are less than 3.9 pm • rad, as it is improbable that all IDs will simultaneously operate at either the minimum or maximum gap. The SAPS has not yet been constructed; therefore, no operational data is available. We use a computer program to generate a different set of random numbers F_i between 0 and 1 for 9 IDs to simulate user behavior. Then the value of magnetic field strength for the i-th ID used in calculation is F_i B_ωmax_i. We use 200 sets of randomly generated numbers (each consisting of 9 digits) to simulate ID states operating at 200 different times. The results are presented in Fig. 3 [Figure 3: see original paper]. Horizontal emittance varies between 23.5 and 26.1 pm • rad, while U_0 varies between 0.78 and 0.92 MeV, with corresponding variations of 10.81% (2.6 pm • rad) and 17.57% (0.14 MeV), respectively.

First, we employ an HDW to compensate for horizontal emittance. The capacity of an HDW to compensate for horizontal emittance depends on its capacity to reduce horizontal emittance (see Appendix A). We choose an HDW with a length of 5 m and a periodic length of 42 mm. The horizontal emittance compensation capacity of the HDW is 2.7 pm • rad, slightly higher than horizontal emittance variations induced by IDs (2.6 pm • rad). The HDW can be used to compensate for horizontal emittance variations, maintaining it at 23.4 pm • rad. The details of parameter selection and relevant calculations about the HDW can be found in Appendix A.

For simplicity, the IBS effect [38] is neglected in this initial study. Future work may include a more complete treatment, and we assume that the influences of IDs and HDWs (VDWs) on beam optics can be effectively corrected [39, 40].

The simulation results of horizontal emittance compensation are shown in Fig. 4 [Figure 4: see original paper]. The maximum magnetic field strength required by the HDW is 2.48 T, corresponding to the moment when horizontal emittance reaches its maximum value of 26.1 pm • rad before compensation. The quantum excitation effect induced by the HDW is significant. As analyzed in Section II, even after compensating for horizontal emittance, U_0 still exhibits evident variations, as shown in Fig. 4c [Figure 4: see original paper]. Compared to the state before HDW compensation, although variations of U_0 have been reduced, it still exhibits variations of 10.12%.

The U_0 variations will lead to evident changes in bunch length. In SAPS, in order to reduce beam density, a harmonic cavity is used to stretch the bunch length. In steady state, the bunch length can be calculated through bunch density (z):

$(z) = {}_0 \exp(-\omega\_\{rf\}\ T_0\ E\ \alpha c\ \sigma\delta^2 / (2U_0)\quad \Phi(z)\ dz)$

The bunch length squared is: $\sigma\_z^2 = (\ \ z^2\ \ (z)\ dz - [\ \ z\ \ (z)\ dz]^2) / (\ \ \ (z)\ dz)$

The potential $\Phi(z)$ is: $\Phi(z) = U_0\ z + V_1(\cos(\omega\{rf\}\ z/c + \phi_1) - \cos \phi_1) + V_2(\cos(n\ \omega\{rf\}\ z/c + \phi_2) - \cos \phi_2)$

where $\omega\_\{rf\}$ is the RF cavity fundamental frequency, T_0 is the revolution time, α_c is the momentum compaction factor, and c is the speed of light. For a given U_0, RF voltage (V_i) and phase (ϕ_i) satisfy the optimal conditions for bunch lengthening:

$\sin \phi_1 = \surd(n^2 - 1)\ (U_0 / eV_1)$
$\tan \phi_2 = -\surd((n^2 - 1)^2 - (eV_1 / U_0)^2) / (n^2 - 1)$
$V_2 = V_1 \surd(n^2 - 1)$

For SAPS, after HDW compensation, U_0 fluctuates between 0.92 and 1.02 MeV, and $\sigma\delta$ *varies between 1.05‰ and 1.16‰. We take the average value of* U_0 (U_0\{ave\} = 0.97 MeV) to set the RF voltage and phase (n = 3, eV_1 = 1.4 U_0*\{ave\}). When the parameters of the RF cavity remain unchanged, the bunch length change induced by* $\sigma\delta$ variations can be negligible. However, the bunch length change resulting from U_0 variations is highly significant, as illustrated in Fig. 5 [Figure 5: see original paper]. Variations in U_0 will lead to the failure of bunch length stretching. It should be noted that our analysis has only focused on the simplified case of an idealized harmonic cavity under flat-potential conditions, whereas more complex scenarios—including beam loading in the cavities, variations in the filling pattern, distinctions between passive and active cavities, and the influence of machine impedance—warrant further investigation.

To address the issue of U_0 variations, we replace the 5 m HDW with two 2.5 m VDWs. The quantum excitation effect of VDW has been transferred to the vertical direction, as described by Equations (20) and (21). Thus, the capacity of VDW to compensate for horizontal emittance is directly proportional to the square of the magnetic field strength and is no longer constrained by its quantum excitation effect. However, if the magnetic field is excessively strong, it will lead to a rapid increase in vertical emittance. We select the compensation value for horizontal emittance as 19.6 pm·rad and for vertical emittance as 5 pm·rad; the results are displayed in Fig. 4 [Figure 4: see original paper]. The magnetic field strength required for the two VDWs is also presented. After two VDWs compensate for horizontal emittance, U_0 is almost automatically compensated, as shown in Fig. 4c [Figure 4: see original paper]. The U_0 exhibits variations of only 0.1% (negligible variations) resulting from a small change in the $\Sigma\ \ _1$ I_5IDi value.

We also simulated the compensation using two HDWs. The magnetic field required by HDW during the compensation process is identical to that of VDW (see Appendix B). Therefore, both achieve the same U_0 and energy spread (σ_δ) state after compensation. The difference is that ^{HDW} (25.2 pm · rad) is slightly greater than ^{VDW} + _y^{VDW} (24.6 pm · rad), as shown in Fig. 4a [Figure 4: see original paper] and Table 3 (see Appendix B for explanations).

Finally, we calculate the change in energy spread after adding VDWs (HDWs); the result is presented in Fig. 6 [Figure 6: see original paper] and Table 3 . Energy spread has increased (to approximately 0.125%) but remains acceptable, with variations decreasing from 5.61% before compensation to 4.33% after compensation. If necessary, more complex scenarios involving energy spread compensation can be further explored [33]. The energy spread growth may influence the bunch length and photon brightness. We calculated the fundamental brightness of the CPMU16 and the bunch length after harmonic cavity stretching under four different scenarios. In the calculations, the energy spread and U_0 values were set to the mean values of each case. For Case 1, the emittance was taken as the average value, while for Cases 2 and 3, a linear coupling coefficient of 0.25 was applied to the emittance (to maintain consistency with Case 3). Compared to the uncompensated case, the two VDWs (or two HDWs) compensation scheme results in an approximately 12% reduction in brightness (as shown in Table 3). The increase in energy spread has a negligible effect on the bunch length. We consider the trade-off of an acceptable brightness reduction for improved stability in photon distribution, brightness, and bunch length to be scientifically justified.

TABLE 3. Parameter variations and fundamental brightness after DW compensation.

Case	(pm · rad)	_y (pm · rad)	U_0 variations	σ_δ variations	Fundamental brightness of CPMU16 (photons/s/mm^2/$mrad^2$/0.1%B.W)
1. without DW	23.5-26.1	0.066	17.57%	5.61%	2.84×10^{22}
2. one HDW	23.4	0.066	10.12%	11.21%	2.92×10^{22}
3. two HDWs	25.2	0.066	0.10%	4.33%	2.47×10^{22}
4. two VDWs	19.6	5.0	0.10%	4.33%	2.49×10^{22}

IV. Conclusion

In this paper, we have demonstrated the approach of compensating for horizontal emittance and U_0 variations with two HDWs or two VDWs. Taking the SAPS as an example, we show that compensation can be achieved by adopting two HDWs or VDWs with a total length of 5 m. After compensation, the variations in emittance decreased from 10.81% to 0%, and the variations in U_0 decreased from 17.57% to 0.1%.

It is noted that this approach does not aim to compensate for energy spread variations. In the presented example, the energy spread variations remain at the same level with two HDWs or two VDWs. We consider that such a level of variation could be acceptable for most user beamlines. If necessary, more complex scenarios involving energy spread compensation can be further explored. In addition, we also show that if two VDWs, rather than two HDWs, are used, the variations in vertical emittance can be compensated. For light sources with non-planar IDs (such as the vertical or elliptical polarization modes of elliptically polarized undulators), this will be particularly beneficial. Another benefit of using VDW rather than HDW is that nonzero vertical emittance is required anyway for lifetime and intra-beam scattering reasons, and using the VDW to generate the vertical emittance allows a smaller horizontal emittance to be achieved. However, if VDWs are to be employed, this may potentially affect the horizontal aperture of the storage ring and consequently affect off-axis injection; adoption of this approach necessitates thorough investigation to assess its feasibility and implications.

Appendix A

In this section, we demonstrate that a lower natural emittance of the ring results in a more rapid increase in the quantum excitation effect generated by an HDW as magnetic field intensifies. This means that in rings with lower natural emittance, an HDW has a lower capacity to compensate for horizontal emittance variations because its compensation capacity is limited by the quantum excitation effect. It also means that in rings with lower emittance, after compensating for horizontal emittance by an HDW, the variations in U_0 become more pronounced.

The HDW can utilize its low magnetic field region (B_H < B_{Hcri}, see following text) to compensate for horizontal emittance, as well as its high magnetic field region (B_H > B_{Hcri}). The distinction is that the high magnetic field maintains horizontal emittance at a higher value and induces a more significant quantum excitation effect, thus exacerbating variations in U_0. Thus, this section focuses solely on the low magnetic field region of HDW.

In the low magnetic field region, the capacity of HDW to compensate for horizontal emittance variations induced by IDs depends on its capacity to reduce horizontal emittance. We assume that the capacity of HDW to reduce horizontal emittance is denoted as Δ ^{HDW}*{max} and the corresponding magnetic* *field strength of HDW is denoted as B*{Hopt}. When HDW is not included, the range of horizontal emittance variations induced by IDs is denoted as min to max (for SAPS, min = 23.5 pm · rad, max = 26.1 pm · rad). By employing an HDW, horizontal emittance can be consistently maintained at the value of max - Δ ^{HDW}*{max} (for SAPS, max - Δ ^{HDW}*{max} = 23.4 pm · rad).

The compensation equation is: = C_q γ^2 (I_5b + Σ $_1$ I_5IDi + I_5HDW) / ((I_2b + Σ $_1$ I_2IDi + I_2HDW) - I_4b) = max - Δ ^{HDW}_{max}

When ID gaps are configured to achieve an emittance of max, the magnetic field strength of HDW is set to B_{Hopt}, resulting in a horizontal emittance of max - Δ ^{HDW}*{max}. When ID gaps are set such that horizontal emittance is between min and max, the magnetic field strength of HDW should be reduced in real time to an appropriate value to maintain a stable horizontal emittance of max - Δ ^{HDW}*{max}. Therefore, the prerequisite for compensating horizontal emittance variations induced by IDs with an HDW is that Δ ^{HDW}_{max} ≥ max - min.

Strictly speaking, Δ ^{HDW}*{max} should be calculated under the state that horizontal emittance equals max (26.1 pm · rad). For simplicity, we calculate Δ ^{HDW}*{max} under the bare lattice state (without IDs), where horizontal emittance is b = 26.3 pm · rad. This has a negligible impact on the calculation results.

The capacity of an HDW to reduce horizontal emittance can be characterized by the emittance ratio of the bare lattice with and without HDW. For a given length, the capacity of an HDW to reduce horizontal emittance depends on its magnetic field strength and period length. The length of a straight section of SAPS is 6 m. We consider using only one straight section to achieve compensation for horizontal emittance because the straight section is very valuable for a light source. We assume that the length of the HDW is 5 m. The emittance ratio varies under different values of B_H and λ*H, as illustrated in Fig. 7 [Figure 7: see original paper]. For a fixed* λH, there exists an optimal B_H (B*{Hopt}) that minimizes the horizontal emittance, corresponding to the maximum horizontal emittance reduction capacity of HDW (Δ ^{HDW}*{max}). The red dashed line represents the curve of B_{Hopt} for a given λ_H. From the figure, it is evident that the HDW with a short period and high field strength has a greater capacity to reduce horizontal emittance. Taking into account the manufacturing technology of IDs, a superconducting wiggler with a period length of 48 mm and a peak field strength of 4.2 T is selected, as it has been demonstrated to be constructible [42].

The capacity of the HDW to reduce horizontal emittance first increases and then decreases with the increase in magnetic field strength. It reaches a maximum of 2.7 pm · rad when B_H = B_{Hopt} and decreases to zero when B_H = B_{Hcri}.

The quantum excitation effect induced by HDW is proportional to the fifth power of its magnetic field strength, while the radiation damping effect is proportional to the square of its magnetic field strength. When 0 < B_H < B_{Hcri}, the radiation damping effect induced by HDW exceeds the quantum excitation effect, resulting in a decrease in horizontal emittance. When B_H = B_{Hcri}, the radiation damping effect cancels out the quantum excitation effect, and horizontal emittance remains unchanged. When B_H > B_{Hcri}, the quantum excitation effect exceeds the radiation damping effect, resulting in an increase in horizontal emittance.

B_{Hcri} can be determined by setting the emittance ratio equal to 1: ^{HDW} / b = 1

This yields: B_{Hcri} = (($15\pi^3$ m_0^3 v^3 b γ^2) / (C_q e^3 β λ_H^2))^(1/3)

where m_0, v, e represent the mass, velocity, and charge of the electron, respectively.

According to this equation, B_{Hcri} is proportional to the one-third power of the ring' s natural emittance (b); the lower the natural emittance, the smaller B_{Hcri}. This implies that the quantum excitation effect induced by HDW increases more rapidly for low emittance rings. The lower the natural emittance, the weaker the horizontal emittance compensation capacity of HDW. When horizontal emittance variations induced by IDs are significant, it is necessary to choose a sufficiently long HDW to ensure that its horizontal emittance compensation capacity is adequate when B_H = B_{Hopt}. The lower the natural emittance, the longer the required length of HDW under the same HDW parameters, and the more pronounced the quantum excitation effect produced by HDW. Consequently, the variations of U_0 after HDW horizontal emittance compensation remain evident due to the strong quantum excitation effect of HDW.

Appendix B

The compensation equations for HDW and VDW can be expressed in a unified form:

Σ $_1$ I_2IDi + I_2DW1 + I_2DW2 = I_2^c
I_5DW1 + I_5DW2 = I_5^c

where DW represents HDW or VDW. When β = β_y , HDW compensation equations have the same solutions as VDW compensation equations. When β $\neq$ β_y , we can make VDW compensation equations equivalent to HDW compensation equations by adjusting the lengths:

I_5VDW1 + I_5VDW2 = (β_y / β) (I_5HDW1 + I_5HDW2)

In this way, the HDW and VDW compensation equations have the same magnetic field solutions: B_{H1} = B_{V1}
B_{H2} = B_{V2}

The difference is that after compensation, the emittance has different values:

^{HDW} = C_q γ^2 (I_5b + Σ $_1$ I_5IDi + I_5HDW1 + I_5HDW2) / ((I_2b + Σ $_1$ I_2IDi + I_2HDW1 + I_2HDW2) - I_4b)

^{VDW} + _y^{VDW} = C_q γ^2 (I_5b + Σ $_1$ I_5IDi) / ((I_2b + Σ $_1$ I_2IDi + I_2VDW1 + I_2VDW2) - I_4b) + C_q γ^2 (I_5VDW1 + I_5VDW2) / (I_2b + Σ $_1$ I_2IDi + I_2VDW1 + I_2VDW2)

The relationship between ^{HDW} (emittance after HDW compensation) and ^{VDW} + _y^{VDW} (emittance after VDW compensation) depends on the values of β_y / β and I_4b (for SAPS, ^{HDW} > ^{VDW} + _y^{VDW}, see Section III).

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