Probing high-order deformation effects in neutron-deficient nuclei 246,248No with improved potential-energy-surface calculations

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Abstract

The high-order deformation effects in even-even 246,248No are investigated by means of pairing self-consistent Woods-Saxon-Strutinsky calculations using the potential-energy-surface (PES) approach in an extended deformation space ($ $2, $ $3, $ $4, $ $5, $ $6, $ $7, $ $8). Based on the calculated two-dimensional-projected energy maps and different potential-energy curves, we found that the highly even-order deformations have an important impact on both the fission trajectory and energy minima, while the odd-order deformations, accompanying the even-order ones, primarily affect the fission path beyond the second barrier. Relative to the light actinide nuclei, the nuclear ground state changes to the superdeformed configuration, but the normally-deformed minimum, as the low-energy shape isomer, may still be primarily responsible for enhancing nuclear stability and ensuring experimental accessibility in 246,248No. Our present investigation indicates the nonnegligible impact of high-order deformation effects along the fission valley and will be helpful for deepening the understanding of different deformation effects and deformation couplings in nuclei, especially in this neutron-deficient heavy-mass region.

Full Text

Preamble

Probing high-order deformation effects in neutron-deficient nuclei 246,248No with improved potential-energy-surface calculations

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We investigate high-order deformation effects in even-even 246,248No using pairing self-consistent Woods-Saxon-Strutinsky calculations based on the potential-energy-surface (PES) approach in an extended deformation space ($ $2, $ $3, $ $4, $ $5, $ $6, $ $7, $ $8). By analyzing the calculated two-dimensional projected energy maps and various potential-energy curves, we find that high-order even-order deformations significantly impact both the fission trajectory and energy minima, while odd-order deformations, which accompany the even-order ones, primarily affect the fission path beyond the second barrier. Compared to lighter actinide nuclei, the nuclear ground state transitions to a superdeformed configuration, yet the normally-deformed minimum, serving as a low-energy shape isomer, may still be primarily responsible for enhancing nuclear stability and ensuring experimental accessibility in 246,248No. Our investigation demonstrates the non-negligible impact of high-order deformation effects along the fission valley and will help deepen understanding of different deformation effects and their couplings in nuclei, particularly in this neutron-deficient heavy-mass region.

Keywords: High-order deformations; neutron-deficient nuclei; Potential energy surface; nuclear stability; macroscopic-microscopic model

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Introduction

Determining the limits of nuclear stability and expanding the map of known isotopes represent major goals of modern nuclear physics [1–4]. Single-particle structure is crucial for nuclear stability, particularly for heavy nuclei. As is well known, superheavy nuclei (with atomic number $Z \geq 104$) exist only due to quantum shell effects arising from the non-uniform distribution of single-particle levels. Furthermore, single-particle energies depend sensitively on nuclear shape (equivalently, the nuclear mean field), which is typically parameterized by a set of deformation parameters. Consequently, it is essential to treat deformations as accurately as possible in theoretical descriptions, especially for heavy nuclear systems.

Indeed, the mechanism of spontaneous symmetry breaking allows nuclei to adopt non-spherical shapes. Numerous experiments have demonstrated that nuclei can possess not only axially or nonaxially quadrupole deformations but also nonaxially or axially octupole and hexadecapole deformations [5–13]. Nuclear spectra, moments, and electromagnetic matrix elements are commonly used to verify such deformation properties [5, 6, 14]. The importance of high-order deformation, such as the hexacontetrapole deformation $ $6, has been revealed in describing ground states [15-17] and excited states, including both multi-quasiparticle high-K states [18, 19] and collective rotational states [19-22]. For instance, Xu et al. [22] recently probed the importance of coupling between the high-order deformation $ $6 and odd-order deformation $ $3 in rotating 252,254No. In the spontaneous fission process, the effect of higher multipolarity shape parameters in nuclei with 100 ≤ Z ≤ 114 has also been demonstrated [23].

Thus far, new-generation experimental facilities have served for many years to explore the stability limits of high proton-number (Z) and/or high isospin (T) nuclei, including measurements of their structural properties. Theoretical approaches primarily include macroscopic-microscopic (MM) models and microscopic theory, e.g., cf. Refs. [24-26]. Prior to this work, we have performed PES and total-Routhian-surface (TRS) calculations in multidimensional deformation spaces, such as ($ $2, γ, $ 4),($2, $ $3, $ $4, $ 5), $andsomeexoticdeformationspaces[6–8, 13, 27–30].Inthepresentpaper, usinganimprovedPEScalculation$
$lineactinideregions.The248Nonucleusisthemostneutron - deficienteven -$
$evenNoisotopethathasalreadybeensynthesizedexperimentally[31], butitshalf-$
$lifeandstructuralpropertiesremainunknown, whileitsneighboringeven \quad -$
$even246Nonucleusisexpectedtobesynthesizedasthenextcandidate.InRef.[32], itwasreportedthattheground-$
$statefissionhalf-livesof252Noand254Nodifferbymorethanfivetosixordersofmagnitude, andfollowingthis$
$lifeofmoreneutron-deficienteven-evenNoisotopeswouldbeextremelyshort, makingthemexperimentallyinc$
14}$ s limit for atomic existence. However, a recent study within a cluster model pointed out that the neutron-deficient 247,248No nuclei are relatively stable with respect to spontaneous fission, with no abrupt decreases in their fission half-lives [33].

Regarding the development of the PES approach, our primary contribution in this work is extending the deformation space ($ $2, $ $3, $ $4, $ $5) to further include the high-order $ $6, $ $7, and $ $8 degrees of freedom, mainly involving modifications to the calculation of the new Hamiltonian matrix, surface energy, and Coulomb energy of the nuclear liquid drop. The rest of this paper is organized as follows. The theoretical framework is described in Sec. II. The calculated results and related discussion are presented in Sec. III. Finally, Sec. IV summarizes the main conclusions of the present project.

II. Theoretical Method

The general procedures of PES calculations (even with rotation, e.g., TRS) within the framework of MM models are standard and have been summarized in, e.g., Refs. [34-38]. In what follows, we briefly present the implementation of the PES approach, focusing on the main points and some basic definitions.

First, let us briefly review one widely-used technique for nuclear shape (potential) parameterization. Namely, one can define the nuclear surface in terms of a spherical-harmonic basis expansion as,

$$\Sigma : R(\theta,\phi) = R_0 c(\alpha) \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)$$

where the expansion coefficients $\alpha_{\lambda\mu}$ are usually called "deformation parameters" (or simply "deformations"). The ensemble of all adopted deformation parameters $\{\alpha_{\lambda\mu}\}$ is usually abbreviated as α. The radius parameter $R_0 = r_0 A^{1/3}$ (here, $r_0 = 1.2$ fm) provides an approximation of the effective nuclear spherical radius in Fermi units, and the auxiliary function $c(\alpha)$ ensures conservation of nuclear volume, e.g., the volume enclosed by the nuclear surface Σ equals the volume of the corresponding spherical nucleus (independent of the actual shape). To avoid possible confusion, it may be worth noting that, similar to coordinate space where a vector can be projected onto axes, in the deformation space expanded by spherical harmonics, the total deformation β and deformation β_λ at order λ are usually defined by $\beta_\lambda = \sqrt{\sum_\mu \alpha^2_{\lambda\mu}}$ and $\beta = \sqrt{\sum_{\lambda\mu} \alpha^2_{\lambda\mu}}$, respectively [39, 40].

For the axially symmetric shapes considered in this project, the deformation β_λ equals $\alpha_{\lambda 0}$ due to $\alpha_{\lambda\mu\neq 0} = 0$. In this work, we consider the deformation degrees of freedom $\beta_{6,7,8}$ and spherical harmonics $Y^{\mu=0}_{\lambda=6,7,8}$, i.e., see Eq.(1), extending the deformation space $(\beta_2, \beta_3, \beta_4, \beta_5)$ to $(\beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$.

With a parameterized nuclear shape, the phenomenological nuclear potential can be calculated. For a nucleus, the Woods-Saxon (WS) potential is more realistic due to its flat-bottomed and short-range properties. In this project, we numerically solve the Schrödinger equation with a deformed WS Hamiltonian [41, 42],

$$\hat{H}_{WS} = \hat{T} + \hat{V}_{cent} + \hat{V}_{so} + \hat{V}_{Coul},$$

where the central part of the WS potential reads

$$\hat{V}_{cent}(\vec{r},\beta; V_0, r_0, a_0) = \frac{V_0[1 \pm \kappa(N-Z)/(N+Z)]}{1 + \exp[\mathrm{dist}_\Sigma(\vec{r},\beta; r_0)/a_0]},$$

where the plus and minus signs hold for protons and neutrons, respectively, and the parameter a_0 denotes the surface diffuseness. The parameters V_0 and r_0 represent the central potential depth and central potential radius parameters, respectively. The term $\mathrm{dist}_\Sigma(\vec{r},\beta; r_0)$ represents the distance of a point $\vec{r}$ from the nuclear surface Σ. The spin-orbit potential, which strongly affects the level order and depends on the gradient of the central potential with new parameters, is defined by

$$\hat{V}_{so}(\vec{r},\hat{p},\hat{s},\beta; \lambda, r_{so}, a_{so}) = -\lambda \nabla V_{so} \times \hat{p} \cdot \hat{s},$$

where

$$V_{so}(\vec{r},\beta;r_{so},a_{so}) = \frac{V_0[1 \pm \kappa(N-Z)/(N+Z)]}{1+\exp[\mathrm{dist}_{\Sigma_{so}}(\vec{r},\beta;r_{so})/a_{so}]}$$

and the parameter λ denotes the strength of the effective spin-orbit force acting on individual nucleons. It should be stressed that the new surface Σ_{so} differs from the one in Eq. (5) due to the different radius parameter r_{so}. Also, the spin-orbit diffusivity parameter a_{so} is usually updated. For protons, a classical electrostatic potential of a uniformly charged drop is used to describe the Coulomb potential, which is defined as

$$V_{Coul}(\vec{r},\beta) = Ze \int \frac{d^3\vec{r}'}{|\vec{r}-\vec{r}'|}$$

where the integration extends over the volume delimited by the surface Σ.

During the process of calculating the WS Hamiltonian matrix, we use the eigenfunctions of the axially deformed harmonic oscillator potential in the cylindrical coordinate system as basis functions, as seen below,

$$|n_\rho n_z \Lambda\Sigma\rangle = \psi_\Lambda(\rho)\psi_{n_z}(z)\psi_\Lambda(\phi)\chi(\Sigma).$$

For more details, one can see, e.g., Ref. [42]. Note that eigenfunctions with $N \leq 12$ and $N \leq 14$ are chosen as the basis set for protons and neutrons, respectively. The corresponding single-particle levels (eigenvalues) and wave functions (eigenvectors) are obtained by diagonalizing the Hamiltonian matrix. It is found that, with such a cutoff, the calculated results (e.g., single-particle energies) are sufficiently stable with respect to a possible enlargement of the basis space.

Based on the obtained single-particle levels at the corresponding nuclear shape, the quantum shell-correction and pairing-energy contributions can be further calculated by the Strutinsky method [43] and the Lipkin-Nogami (LN) method [44, 45]. In these methods, the microscopic shell-correction energy is given by

$$\delta E_{shell}(Z,N,\hat{\beta}) = \sum_i e_i - \int \tilde{g}(e)e\,de,$$

where e_i denotes the calculated single-particle levels and $\tilde{g}(e)$ is the so-called smooth level density. The smoothed distribution function $\tilde{g}(e)$ was early defined as

$$\tilde{g}(e,\gamma) \equiv \frac{1}{\gamma\sqrt{\pi}} \sum_i \exp\left[-\frac{(e-e_i)^2}{\gamma^2}\right],$$

where γ denotes the smoothing parameter. To eliminate any possibly strong dependence on the γ parameter, the level density $\tilde{g}(e)$, optimized by a curvature-correction polynomial $P_p(x)$, is usually given by [43, 46–48],

$$\tilde{g}(e,\gamma,p) = \frac{1}{\gamma\sqrt{\pi}} \sum_i P_p\left(\frac{e-e_i}{\gamma}\right) \times \exp\left[-\frac{(e-e_i)^2}{\gamma^2}\right].$$

The corrective polynomial $P_p(x)$ can be expanded in terms of Hermite or Laguerre polynomials. The expanded coefficients can be obtained using the orthogonality properties of these polynomials and the Strutinsky condition [49]. In the present work, a sixth-order Hermite polynomial and a smoothing parameter $\gamma = 1.20\hbar\omega_0$, where $\hbar\omega_0 = 41/A^{1/3}$ MeV, are adopted [43].

Besides the shell correction, another important quantum correction is the pairing-energy contribution. It deserves noting that various variants of pairing energy exist in the MM approach [50]. Several phenomenological expressions are widely adopted, such as pairing correlation and pairing correction energies employing or not employing the particle number projection technique. We employ the LN method, an approximately particle number projection technique, to treat the pairing-energy calculation [44, 45]. Such a pairing treatment can help avoid not only the spurious pairing phase transition but also particle number fluctuation encountered in simpler BCS calculations. In this method, the LN pairing energy for an even-even nucleon system at "paired solution" (pairing gap $\Delta \neq 0$) is calculated by [24, 44]

$$E_{LN} = 2\sum_k v_k^2 e_k - \frac{\Delta^2}{G} - \lambda_2 \langle (\Delta N)^2 \rangle,$$

where v_k^2, e_k, Δ, and λ_2 represent the occupation probabilities, single-particle energies, pairing gap, and number-fluctuation constant, respectively. The monopole pairing strength G is determined by the average gap method [34].

For the case of "no-pairing solution" ($\Delta = 0$), its partner expression is

$$E_{LN}(\Delta = 0) = 2\sum_k e_k - G\sum_k v_k^4.$$

The difference between paired solution E_{LN} and no-pairing solution $E_{LN}(\Delta = 0)$ is usually referred to as the pairing correlation, which can be written as

$$\delta E_{pair} = 2\sum_k v_k^2 e_k - \frac{\Delta^2}{G} - \lambda_2 \langle (\Delta N)^2 \rangle + G\sum_k v_k^4.$$

Following Refs. [16, 24], we define the total microscopic energy as

$$E_{micro}(Z,N,\beta) = \delta E_{shell}(Z,N,\beta) + \delta E_{pair}(Z,N,\beta).$$

Such a definition is equivalent to the concept of "shell correction" $\delta E_{shell}(\equiv E_{LN} - \tilde{E}_{Strut})$, cf. Eq. (1) in Ref. [34], merging the quantum shell correction and pairing contribution. For clarifying some confusing points, it should be pointed out that, in Ref. [34], the definition of E_{LN} includes the term $G\frac{N}{2}$. It should be mentioned that the microscopic energy includes the proton and neutron contributions simultaneously.

Using the standard liquid-drop model [39], the macroscopic energy can be calculated. Since we focus on deformation effects instead of, e.g., masses, in the PES calculation, the deformation liquid-drop energy (relative to the spherical liquid drop) is adopted [39, 42, 43], as seen below,

$$E_{macro}(Z,N,\beta) = E_{ld}(Z,N,\beta) = \{[B_S(\beta) - 1] + 2\chi[B_C(\beta) - 1]\}E_S^{(0)},$$

where the spherical surface energy $E_S^{(0)}$ and the fissility parameter χ are Z and N dependent, cf. Refs. [39, 43]. The relative surface and Coulomb energies B_S and B_C are only functions of nuclear shape.

Within the framework of the MM model, the total energy can be calculated by [24, 51],

$$E_{total}(Z,N,\beta) = E_{macro}(Z,N,\beta) + E_{micro}(Z,N,\beta).$$

Once the total energy is obtained at each sampling deformation grid, we can obtain the smooth potential-energy surface/map with the help of interpolation techniques, e.g., a spline function, and then investigate nuclear properties including equilibrium deformations, shape coexistence, fission paths, and other physical quantities/processes.

III. Results and Discussion

In this project, we restrict ourselves to axially symmetric shapes and perform numerical calculations in a seven-dimensional deformation space. Namely, we calculate (25, 13, 13, 7, 5, 5, 5) points for $(\beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8)$ deformations, respectively, using a step size of 0.05. Our primary concern is how high-order multipolarity deformations and their couplings affect the potential energy landscapes.

For heavy nuclear systems, nuclear stability is approximately governed by the competition between the surface tension of the nuclear liquid drop and the strong Coulomb repulsion between numerous protons. The former tends to hold the system together, while the latter drives the nucleus toward spontaneous fission.

To understand the influence of different deformation parameters on macroscopic energy, taking 248No as an example, the evolution of deformed liquid-drop energies near spherical and elongated shapes is illustrated in Fig. 1. For such a heavy nucleus, the macroscopic liquid-drop energy remains almost constant with changing β_2, agreeing with our previous study [7]. One can notice that the nuclear stiffness (usually defined by $\partial E_{ld}/\partial\beta_\lambda$) near the spherical shape increases with increasing λ, as seen in Fig. 1 Figure 1: see original paper, indicating that it becomes more difficult to develop high-order deformation. However, in the elongated case, e.g., $\beta_2 = 1.0$, cf. Fig. 1(b), the nuclear shape becomes relatively soft along each deformation degree of freedom. In particular, it can be seen from Fig. 1(b) that stable β_4 deformation appears under these circumstances. Even the stiffnesses along β_7 and β_8 are almost the same and smaller than that along the lower-order deformation β_6 at about $\beta_{6,7,8} < 0.1$, which means that high-order $\beta_{7,8}$ deformations may be more favored than β_6. In practical calculations, the different couplings of various deformations may be complex.

[Figure 1: see original paper] Macroscopic deformation energies as functions of separate deformations β_λ for 248No, $\lambda = 2, 3, 4, 5, 6, 7, 8$. Note that the β_2 deformation is fixed to 1.0 in subfigure (b).

Quantum shell effects arising from single-particle states can enhance nuclear stability in MM calculations because high and low densities will respectively give rise to positive and negative shell corrections. The appearance of these appropriate corrections on the pathway to scission may lead to enhanced stability. Figure 2 shows the effects of different high-order deformations (e.g., $\lambda \geq 5$) on microscopic single-particle energies (not far from the Fermi surface) for neutrons (similarly for protons) in the example nucleus 248No, indicating the success of the modified PES approach. The neutron spherical shell gaps at $Z = 126, 164$ and 184 are reproduced from the single-particle energy diagram. The single-particle levels as functions of low-order deformations (e.g., β_2, γ and β_4) can be easily found in the literature, i.e., cf. Refs. [27].

[Figure 2: see original paper] Neutron single-particle energies as functions of separate deformation β_5 (a), β_6 (b), β_7 (c) and β_8 (d) for 248No. For each subplot, other deformation parameters are set to zero and the spherical quantum numbers nlj are given as labels. In (b) and (d), the solid red and dashed blue lines indicate the positive and negative-parity levels, respectively.

It deserves noticing that a nucleus with odd-λ deformation possesses the same shapes (namely, the same nuclear potential but different orientation) for positive and negative β_λ values; the Hamiltonian of the nuclear system will satisfy the relation $\hat{H}(-\beta_{\lambda\text{odd}}) = \hat{H}(+\beta_{\lambda\text{odd}})$ so that the single-particle diagram is symmetric about positive and negative $\beta_{\lambda\text{odd}}$ values. Of course, it can be easily imagined that, for separate deformation $\beta_{\lambda\text{odd}}$, the macroscopic liquid-drop energy E_{ld} also satisfies $E_{ld}(+\beta_{\lambda\text{odd}}) = E_{ld}(-\beta_{\lambda\text{odd}})$ since the atomic nucleus at this moment just has different spatial orientations. It may be somewhat complex for the combination of several odd-λ deformations, but one can determine the symmetry relations. For instance, in the three-dimensional subspace $(\beta_3, \beta_5, \beta_7)$, it can be divided into eight sections (quadrants). The potential energies at the lattices $(+\beta_3, +\beta_5, +\beta_7)$ and $(-\beta_3, -\beta_5, -\beta_7)$ will be equal since the nuclear shapes are identical at these two kinds of deformation grids, abbreviated to $(+, +, +)$ and $(-, -, -)$ for short. Similarly, other three pairs of symmetric combinations will be $(+, +, -)$ and $(-, -, +)$, $(+, -, -)$ and $(-, +, +)$, $(-, +, -)$ and $(+, -, +)$. Such symmetry will reduce the number of calculated deformation grids by half.

All projected two-dimensional β_2-vs-β_λ ($\lambda = 3, 4, 5, 6, 7$ or 8) maps for 246No and 248No in this seven-dimensional deformation space have been illustrated in Figs. 3 and 4, respectively. In each subplot, the total energy is minimized over the remaining deformation degrees of freedom (e.g., on the β_2-vs-β_3 plane, the energy is minimized over $\beta_{4,5,6,7,8}$). From these projection maps, some properties, e.g., energy minima and fission paths, can be analyzed. It should be noted that, ignoring interpolation errors, the corresponding minima, e.g., the normally-deformed minima near $\beta_2 = 0.2$ and the superdeformed minima near $\beta_2 = 0.7$, are the same in different projection maps.

[Figure 3: see original paper] Potential-energy projections on $(\beta_2, \beta_{\lambda=3,4,5,6,7,8})$ planes, contour-line separation of 0.5 MeV, minimized at each deformation point over other deformations, for the 246No nucleus. For more details see the text.

[Figure 4: see original paper] Similar to Fig. 3, but for the 248No nucleus.

In these two figures, all the odd-order deformation parameters are zero both at the normally-deformed and superdeformed minima. The ensembles $(\beta_2, \beta_4, \beta_6, \beta_8; E_{\text{min}})$ are respectively $(0.234, 0.041, -0.017, -0.005; -5.82$ MeV$)$ and $(0.679, 0.045, -0.005, -0.007; -7.08$ MeV$)$ for the normally-deformed and superdeformed minima in Fig. 3 [similarly, $(0.242, 0.033, -0.021, -0.007; -6.33$ MeV$)$ and $(0.679, 0.037, -0.003, -0.007; -7.12$ MeV$)$ in Fig. 4]. Since there is no experimental deformation information for these two nuclei, it is instructive to confront our calculations (or part of them) with other theory. Indeed, the equilibrium deformations of the normally-deformed minima calculated by us are in good agreement with the results given by Möller et al. [24]. In Ref. [24], nuclear ground-state deformations are calculated in the deformation space $\{\beta_\lambda; \lambda = 2, 3, 4, 6\}$ based on the finite-range droplet macroscopic model and folded-Yukawa single-particle microscopic model; the calculated $(\beta_2, \beta_3, \beta_4, \beta_6)$ values are $(0.224, 0.00, 0.054, -0.025)$ and $(0.235, 0.00, 0.048, -0.033)$, respectively, agreeing with the present results, as seen in Figs. 3 and 4. Further, our calculations also indicate that, besides β_4 and β_6, even-order deformation β_8 still has a slight impact on the normally-deformed minima in these two nuclei. In addition, one can see that all the even-order deformations affect the superdeformed energy minima to some extent. In particular, it seems that the impact of high-order deformation β_8 is more important than β_6, agreeing with the case illustrated in Fig. 1(b) (in the strongly elongated situation, the nucleus may be softer along β_8 than β_6). Concerning the odd-order deformations β_3, β_5 and β_7, we find that they do not affect both the normally-deformed and superdeformed minima but do affect the saddle-point positions and fission paths after the superdeformed minima.

To understand the effects of even- and odd-order deformations on the fission trajectory, Figure 5 shows four types of potential-energy curves along the minimum valley in the quadrupole deformation β_2 direction for 246,248No. The typical double-humped fission barriers in actinide nuclei are well reproduced [52]. Note that the $E(\beta_2)$ curves I and IV will respectively occupy the highest and lowest positions at each β_2 point since the former minimizes over {none} but the latter over $\{\beta_\lambda; \lambda = 3, 4, 5, 6, 7, 8\}$. Keeping this in mind, one can easily read this figure though there is strong overlap of different curves. It can be seen that curve II, which is minimized over the remaining even-order deformations $\beta_{4,6,8}$, will further decrease the energies of not only the normally-deformed minimum but also the superdeformed minimum and leads to the formation of the second barrier. Indeed, considering only the deformation β_2, e.g., curve I, the energy will continue increasing after the superdeformed minimum with increasing β_2 (at least up to $\beta_2 = 1.2$). Except for the weak deformation region (e.g., approximately $\beta_2 \leq 0.1$), the energy curves I and III fully overlap, indicating that there are no odd-order deformation effects along the fission valley in the deformation subspace $(\beta_2, \beta_3, \beta_5, \beta_7)$. Similarly, the overlap of energy curves II and IV before $\beta_2 \approx 0.8$ also illustrates such negligible odd-order deformation effects. Comparing curves II and IV, one can find that the inclusion of odd-order deformations will further decrease the second barrier, indicating the occurrence of coupling between odd- and even-order deformations. Obviously, the properties of the potential-energy curves are similar for subplots (a) 246No and (b) 248No, except for a slightly lower outer barrier in 246No.

[Figure 5: see original paper] Four types of potential-energy curves as a function of deformation β_2 for 246No (a) and 248No (b). At each β_2 point, energy minimization was performed over {none} (I; solid black line), $\{\beta_\lambda; \lambda = 4, 6, 8\}$ (II; solid red line), $\{\beta_\lambda; \lambda = 3, 5, 7\}$ (III; dotted green line) and $\{\beta_\lambda; \lambda = 3, 4, 5, 6, 7, 8\}$ (IV; dashed blue line). See text for further explanations.

As is known, both spontaneous fission and α decay, which terminate the stability of drip-line heavy nuclei, sensitively depend on such potential-energy curves. With decreasing neutron number, 246No is expected to have a shorter half-life than 248No, but there should be no abrupt reduction according to their fission trajectory properties. Certainly, it is instructive to investigate the evolution properties of single-particle energies, macroscopic and microscopic energies along the "realistic" fission path (corresponding to curve IV in Fig. 5). Accordingly, the representative single-neutron diagram and different energy curves are illustrated for 248No (similarly for 246No) in Fig. 6 [Figure 6: see original paper].

[Figure 6: see original paper] Neutron single-particle levels (a) and different energy curves (total energy and its macroscopic and microscopic components) as functions of β_2 for the nucleus 248No. Note that for both (a) and (b), at each β_2 grid, other deformation parameters adopt the values after energy minimization and the total energy curve in (b) is the same as curve IV in Fig. 5. In subplot (a), the energy level in blue denotes the Fermi level.

One can find that the single-particle levels, involving different deformations, become more complicated, as seen in Fig. 6(a). From Fig. 6(b), it can be seen that the formation of the inner barrier primarily originates from the microscopic shell correction, while the outer barrier is strongly affected by both macroscopic liquid-drop energy and microscopic shell correction. The pairing correlation always provides a negative and relatively smooth energy.

It should be pointed out that although the normally-deformed minimum in 246,248No is still referred to as the ground state in Ref. [24], the inversion of energies between the normally-deformed minimum and the superdeformed minimum has occurred. Therefore, strictly speaking, the superdeformed minima in 246,248No are their ground states. Actually, the fission half-lives of 246,248No decayed from such ground states will rapidly decrease, relative to those from the normally-deformed ground states (e.g., in lighter actinide nuclei). However, as discussed in Ref. [55] where it is pointed out that the stability of superheavy nuclei may be enhanced by high-K isomers, such very neutron-deficient heavy nuclei may have enhanced stability due to the normally-deformed minimum as a shape isomer (the study of decay half-life from it, including the typical γ distortion of the inner barrier, e.g., cf. Ref. [27], is beyond the scope of the present work).

In addition, to verify that odd-order deformation effects for the fission paths can occur only when even-order deformations are considered, we show the energy projection maps in the (β_2, β_3) plane for 246,248No in Fig. 7, ignoring the even-order deformation degrees of freedom. The fission valley in Fig. 7 is equivalent to curve III in Fig. 5. From Fig. 7, one can see that, from the normally-deformed minima to the strongly elongated region, the odd-order deformation β_3 does not change the fission path in 246,248No. It can be concluded that odd-order deformation effects only play important roles when accompanying higher even-order deformations (e.g., β_4). Whether such a conclusion is a general rule deserves further study through systematic investigation in the future.

[Figure 7: see original paper] Similar to Fig. 3(a) and Fig. 4(a), potential-energy projections on (β_2, β_3) plane for 246No (a) and 248No (b). But in each subplot, the minimization is performed over β_5 and β_7, without consideration of even-order deformations.

Our calculations illustrate that the highly even-order deformations significantly affect both the potential energy minima and fission paths. In particular, the high-order deformation β_8 may be more favored than the lower-order β_6 at strongly elongated nuclear shapes. All the odd-order deformations mainly impact the second barrier, but they must accompany the even-order ones (e.g., β_4). Indeed, the inclusion of high-order deformations is somewhat necessary in the study of nuclear structure and nuclear fusion and fission processes. Though we cannot accurately determine the symmetry properties of fission fragments due to scarce information about scission points, the trend in earlier studies [53, 54] indicates the possible occurrence of asymmetric fissions in these two nuclei. Moreover, it is also found that the very neutron-deficient 246No nucleus may still be accessible experimentally since, similar to the high-K isomer reported in Ref. [55], the normally-deformed shape isomer can enhance the survival probability in drip-line heavy nuclei though the superdeformed ground states are rather unstable. Of course, it is meaningful to further extend the deformation space to include nonaxial deformations in the future.

IV. Summary

In this project, we have developed the PES calculation method, extending the deformation space within the framework of the MM model, and investigated high-order deformation effects in the neutron-deficient heavy nuclei 246,248No. The evolution properties of microscopic single-particle levels and macroscopic energies as functions of different deformation degrees of freedom are illustrated. It is found that the higher the deformation order, the more difficult it is to occur since, for a spherical liquid drop, in general, the stiffness along some deformation gradually increases as the corresponding deformation multipolarity increases. However, for a strongly elongated spheroid, high-order deformations play an important role owing to the large softness along them. Our calculations illustrate that the highly even-order deformations significantly affect both the potential energy minima and fission paths. In particular, the high-order deformation β_8 may be more favored than the lower-order β_6 at strongly elongated nuclear shapes. All the odd-order deformations mainly impact the second barrier, but they must accompany the even-order ones (e.g., β_4). Indeed, the inclusion of high-order deformations is somewhat necessary in the study of nuclear structure and nuclear fusion and fission processes. Though we cannot accurately determine the symmetry properties of fission fragments due to scarce information about scission points, the trend in earlier studies [53, 54] indicates the possible occurrence of asymmetric fissions in these two nuclei. Moreover, it is also found that the very neutron-deficient 246No nucleus may still be accessible experimentally since, similar to the high-K isomer reported in Ref. [55], the normally-deformed shape isomer can enhance the survival probability in drip-line heavy nuclei though the superdeformed ground states are rather unstable. Of course, it is meaningful to further extend the deformation space to include nonaxial deformations in the future.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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