Effects of Radius Measurement Uncertainty on RMF Model Parameters and Neutron Star Matter Properties

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Date: 2025-10-26T14:54:46+00:00

Abstract

We investigate the impact of neutron star radius measurement precision on the constraints of relativistic mean-field (RMF) model parameters and the equation of state (EOS) of dense matter within a Bayesian framework. Using the canonical neutron star radius $R_{1.4} = 11.9$ km with uncertainties $\sigma_R = 1.0$, 0.5 and 0.2 km, we analyze six high-density density-dependent coupling parameters. It is found that reducing the observational uncertainty from 1.0 km to 0.5 km yields only minor improvements, whereas further reduction to 0.2 km significantly tightens constraints, particularly on the isoscalar couplings $\alpha''_S(2n_0)$ and $\alpha''_V(2n_0)$, with uncertainty reductions up to $\sim 90\%$. High-precision data favor a softer EOS, lower central pressures, and narrower credible intervals for the EOSs of symmetric nuclear matter, while the symmetry energy response depends strongly on the adopted couplings at subsaturation densities. Our results highlight the decisive role of future radius measurements with $\sigma_R \lesssim 0.2$ km, as expected from next-generation X-ray and gravitational-wave observatories, in refining the high-density behavior of the EOS and disentangling the density dependence of the symmetry energy from that of symmetric nuclear matter.

Full Text

Preamble

We conduct a Bayesian inference analysis to systematically evaluate the impact of neutron star radius measurement precision on the constraints of relativistic mean-field (RMF) model parameters and the equation of state (EOS) of dense matter. Using the canonical radius constraint $R_{1.4} = 11.9$ km with observational uncertainties $\sigma = 1.0$, 0.5, and 0.2 km, we find that high-precision data ($\sigma = 0.2$ km) significantly tighten the constraints on the isoscalar couplings (α_S and α_V), favoring a softer symmetric nuclear matter (SNM) EOS with lower pressures across all density functionals (DD-ME2, TW99, PKDD). In contrast, the constraints on the symmetry-energy-related coupling α_{TV} exhibit strong model dependence: uncertainties broaden for DD-ME2 and TW99 due to compensatory softening effects but narrow for PKDD owing to its stiffer symmetry energy prior.

This divergence propagates to the proton fraction and sound speed, where uncertainties increase for softer functionals but decrease for PKDD under high precision. Our results underscore that future radius measurements with $\sigma \leq 0.2$ km will be transformative for resolving high-density EOS behavior and disentangling the density dependence of nuclear matter properties, while also highlighting the critical role of low-density functional characteristics in Bayesian inference outcomes.

Introduction

Understanding the equation of state (EOS) of dense strongly interacting matter remains a central goal in both nuclear physics and astrophysics [?]. Nevertheless, the properties of strongly interacting matter at large densities ($n_V \gtrsim 2n_0$) are still unclear [?] despite that nuclear matter being well understood at densities around the saturation density $n_0 = 0.16$ fm^{-3} [?]. This is mainly attributed to the non-perturbative nature of Quantum Chromodynamics (QCD), while perturbative calculations become reliable only at extremely large densities $n_V \gtrsim 40n_0$ [?, ?].

Fortunately, astrophysical observations of neutron stars (NSs) have achieved unprecedented precision. As natural laboratories that host matter under extreme densities and isospin asymmetries, NSs provide critical observational data to probe the EOS far beyond terrestrial conditions. For example, to reproduce the masses of pulsars PSR J1614-2230 ($1.928 \pm 0.017 M_\odot$) [?, ?] and PSR J0348+0432 ($2.01 \pm 0.04 M_\odot$) [?], the EOS for neutron star matter needs to be stiff. According to the measurements of the tidal deformability ($70 \leq \Lambda_{1.4} \leq 580$) and radii ($R = 11.9 \pm 1.4$ km) for $1.4 M_\odot$ neutron stars based on the binary neutron star merger event GRB 170817A-GW170817-AT 2017gfo [?], the EOSs of the corresponding neutron star matter need to be soft. Carrying out pulse-profile modelings using NICER and XMM-Newton data, combined mass-radius measurements for PSR J0030+0451, PSR J0740+6620, and PSR J0437-4715 have become feasible [?], which corroborates the constraints on the EOSs of neutron star matter from other observations—that the EOS is soft at low densities and becomes stiff at large densities. This indicates a unique signature for the speed of sound v_s of neutron star matter, i.e., there may exist a peak of v_s as a function of density which may correspond to a possible deconfinement phase transition [?].

Among various observational probes, the radius of neutron stars, particularly for canonical stars with mass $1.4 M_\odot$, has emerged as a key quantity for constraining the pressure and composition of dense matter [?, ?]. Recent advances in X-ray observations, especially from NICER, along with gravitational-wave measurements from LIGO and Virgo, have significantly improved the precision of such radius estimates [?]. High-precision NS radius measurements have been identified by the astrophysics community as a primary scientific objective for next-generation X-ray pulse-profile observatories, such as the enhanced X-ray timing and polarimetry mission (eXTP) [?] and the spectroscopic time-resolving observatory for broadband energy X-rays (STROBE-X) [?]. Concurrently, these measurements are a major science driver for third-generation gravitational-wave detectors, including the Einstein Telescope [?] and Cosmic Explorer [?].

As Bayesian statistical inference has proven to be a powerful tool for quantifying uncertainties in model parameters and systematically incorporating observational data into theoretical models [?], numerous recent studies have successfully applied it to constrain relativistic mean-field (RMF) models and the neutron star equation of state using contemporary multi-messenger data [?]. While these investigations robustly delineate the current state of knowledge, a distinct but critical question remains: What level of future observational precision is required to achieve decisive constraints on the high-density EOS? The present work addresses this prospective question by conducting a controlled numerical experiment. Instead of using fixed observational uncertainties, we employ a Bayesian framework to investigate the impact of progressively higher-precision measurements (with uncertainties $\sigma = 1.0$, 0.5, and 0.2 km) of the canonical neutron star radius $R_{1.4} = 11.9$ km. To isolate the effect of data precision on high-density physics, we introduce a novel RMF parametrization with a piecewise-defined coupling scheme that specifically targets the behavior at suprasaturation densities. This approach allows us to quantitatively forecast how future radius measurements with $\sigma \leq 0.2$ km, as anticipated from next-generation observatories, will refine our understanding.

Our results reveal that such high-precision data provide a transformative improvement, strongly constraining the isoscalar sector (scalar and vector couplings) and leading to significantly tighter bounds on the internal pressure and sound speed. A key finding is that the constraints on the symmetry energy and isovector parameters depend critically on the adopted low-density functional, highlighting the interplay between crust and core physics. Therefore, the main innovation of our work lies not only in providing a targeted forecast for future missions but also in introducing a framework to quantitatively assess the payoff of increased observational precision and to disentangle the density dependence of different components of the neutron star matter EOS.

The paper is organized as follows. The theoretical frameworks of the RMF model and Bayesian inference approach are presented in Sec. II. The obtained constraints on the coupling constants and EOSs are illustrated in Sec. III. We draw our conclusion in Sec. IV.

Theoretical Framework

A. Relativistic Mean-Field Framework with Density-Dependent Couplings

The Lagrangian density for the relativistic mean-field (RMF) model [**?**], applicable to finite nuclei and uniform nuclear matter, is expressed as

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\partial_\mu - \gamma^\mu(g_\omega\omega^\mu + g_\rho\rho^\mu\tau_3) - M - g_\sigma\sigma]\psi + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu$$

where ψ denotes the nucleon field, M the bare nucleon mass, τ_3 the third isospin component, and $(\sigma, \omega^\mu, \rho^\mu)$ the meson fields. The field tensors are $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu$. For uniform matter, meson fields reduce to time components due to time-reversal symmetry, and only the third component of isospin for the ρ meson is considered to maintain charge conservation.

In uniform nuclear matter, derivative terms of mean fields vanish as the corresponding source currents do not vary with time or space coordinate, and Eq. (1) can be simplified into a point-coupling form, i.e.,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi - \alpha_S(\bar{\psi}\psi)^2 - \alpha_V(\bar{\psi}\gamma^\mu\psi)^2 - \alpha_{TV}(\bar{\psi}\gamma^\mu\tau_3\psi)^2,$$

where $\alpha_S = g_\sigma^2/m_\sigma^2$, $\alpha_V = g_\omega^2/m_\omega^2$, and $\alpha_{TV} = g_\rho^2/m_\rho^2$ correspond to the couplings in isoscalar-scalar, isoscalar-vector, and isovector-vector channels, respectively. To incorporate in-medium effects, we adopt density-dependent couplings $\alpha_S(n_V)$, $\alpha_V(n_V)$, and $\alpha_{TV}(n_V)$ instead of nonlinear self-interactions $U(\sigma, \omega)$. The Dirac equation derived from Eq. (2) is then

$$[-i\alpha\cdot\nabla + \beta(M - \alpha_S n_S)]\psi = (\epsilon - \Sigma_R - \alpha_V n_V - \tau_3\alpha_{TV}n_{TV})\,\psi,$$

where $n_S = \langle\bar{\psi}\psi\rangle$, $n_V = \langle\bar{\psi}\gamma^0\psi\rangle$, and $n_{TV} = \langle\bar{\psi}\gamma^0\tau_3\psi\rangle$ are scalar, vector, and isovector densities. The rearrangement term Σ_R arises from density-dependent couplings, i.e., $\Sigma_R = -\frac{1}{2}\left[n_S^2\frac{d\alpha_S}{dn_V} + n_V^2\frac{d\alpha_V}{dn_V} + n_{TV}^2\frac{d\alpha_{TV}}{dn_V}\right]$.

The nucleon chemical potential μ_i is fixed by

$$\mu_i = \sqrt{\nu_i^2 + M^{*2}} + \Sigma_R + \alpha_V n_V + \tau_{3,i}\alpha_{TV}n_{TV},$$

where Fermi momentum ν_i is related to density $n_i = \nu_i^3/(3\pi^2)$, and $M^* = M - \alpha_S n_S$ is the nucleon effective mass.

The energy density $\mathcal{E}_{NM}$ and pressure P_{NM} of nuclear matter are:

$$\mathcal{E}_{NM} = \mathcal{E}_k + \alpha_S n_S^2 + \alpha_V n_V^2 + \alpha_{TV}n_{TV}^2,$$

$$P_{NM} = \mu_n n_n + \mu_p n_p - \mathcal{E}_{NM},$$

where $\mathcal{E}_k$ is the kinetic energy. For neutron stars, leptons ($l = e, \mu$) contribute additional energy and pressure:

$$\mathcal{E} = \mathcal{E}_{NM} + \sum_l \mathcal{E}_l, \quad P = P_{NM} + \sum_l P_l,$$

under charge neutrality ($n_p = n_e + n_\mu$) and β-equilibrium ($\mu_n = \mu_p + \mu_l$) conditions. The energy per nucleon in symmetric nuclear matter (SNM) is defined through the energy density $\mathcal{E}_{NM}$ indicated in Eq. (6) and the nucleon rest mass M as

$$\epsilon_0(n_V) = \frac{\mathcal{E}_{NM}|_{n_p=n_n}}{n_V} - M.$$

The symmetry energy is expressed as

$$\epsilon_{\mathrm{sym}}(n_V) = \alpha_{TV} n_V,$$

where $n_p = n_n = n_V/2 = \nu^3/3\pi^2$ and $\mathcal{E}^* = \nu^2 + M^{*2}$.

We partition the density domain into three regions, that is, $n_{\mathrm{on}} \le n_V \le n_0$, $n_0 \le n_V \le 2n_0$, and $n_V > 2n_0$ with $n_{\mathrm{on}} = 0.1$ fm^{-3} and n_0 being the saturation density. In each region, couplings follow an exponential form:

$$\alpha_\xi(n_V) = \alpha''_\xi(n_I) \left[\frac{\alpha'_\xi(n_I)}{\alpha''_\xi(n_I)}\right]^2 \left\{1 - \exp\left[-\frac{\alpha''_\xi(n_I)}{\alpha'_\xi(n_I)}(n_V - n_I)\right]\right\} + \alpha'_\xi(n_I)(n_V - n_I) + \alpha_\xi(n_I),$$

where $\xi = S, V, TV$ and $n_I \in \{n_{\mathrm{on}}, n_0, 2n_0\}$. Continuity of α_ξ and its first derivative α'_ξ is enforced at intersection densities n_I, while second derivatives $\alpha''_\xi(n_I)$ are free parameters to be determined by observational data.

Note that several density-dependent point-coupling parametrizations already exist, e.g., DD-PC1 [?]. In principle, adopting the exponential form (11) for the coupling constants and employing different coefficients in different density regions enables us to reproduce essentially all existing RMF models including interactions in isoscalar-scalar, isoscalar-vector, and isovector-vector channels, where the density-dependent couplings can then be inferred within the Bayesian statistical framework according to astrophysical observations.

For $n_V \le n_0$, parameters are fixed to replicate the relativistic density functionals DD-ME2 [?], TW99 [?], and PKDD [?] at $n_V = n_{\mathrm{on}}$, where the values of the coupling constants, their first and second derivatives at $n_V = n_{\text{on}}$ are listed in Table II. The predictions of those functionals for nuclear saturation properties are summarized in Table I, where K and J_0 respectively denote the incompressibility and skewness parameters for symmetric nuclear matter. L and K_{sym} stand for the slope and curvature coefficients of the symmetry energy, respectively. In such cases, the variations of saturation properties of nuclear matter can be examined, where the relativistic density functional DD-ME2 predicts a stiff EOS for symmetric nuclear matter with large K and J_0, PKDD predicts a stiff EOS for asymmetric nuclear matter with large L, and TW99 predicts EOSs with moderate stiffness. Finally, six high-density parameters remain unconstrained: $\alpha''_S(n_0)$, $\alpha''_V(n_0)$, $\alpha''_{TV}(n_0)$, $\alpha''_S(2n_0)$, $\alpha''_V(2n_0)$, and $\alpha''_{TV}(2n_0)$. According to our previous investigation [?], these parameters modulate the EOS at supranuclear densities ($n_V > n_0$) and exhibit linear correlations with coupling strengths constrained by various pulsar observations, showing reduced sensitivity at higher densities. For $n_V \leq n_{\text{on}}$, neutron star matter becomes nonuniform and we adopt the unified EOSs fixed with single-nucleus approximation employing the relativistic density functionals DD-ME2, TW99, and PKDD [?].

B. Bayesian Inference Approach

The foundational principle of Bayesian analysis is mathematically formalized through Bayes' theorem, which governs probability updating upon data acquisition:

$$P(M|D) = \frac{P(D|M)P(M)}{\int P(D|M)P(M)dM}.$$

In this formulation, $P(M|D)$ denotes the posterior probability of model M conditional on dataset D, representing updated belief after data assimilation. $P(D|M)$ represents the likelihood function, quantifying the probability of observing data D under model M. $P(M)$ corresponds to the prior probability, encapsulating pre-existing knowledge about M before data consideration. The denominator constitutes a normalization constant ensuring posterior probabilities integrate to unity across the model space, enabling rigorous model comparison.

The six density-dependent coupling parameters $p_{i=1,\ldots,6}$ ($\alpha''_S(n_0)$, $\alpha''_V(n_0)$, $\alpha''_{TV}(n_0)$, $\alpha''_S(2n_0)$, $\alpha''_V(2n_0)$, and $\alpha''_{TV}(2n_0)$) are sampled uniformly within prior bounds (Table III). Note that the initial parameter ranges (in units of 10^{-16} MeV^{-8}) are set to be $[-10, 10]$ for $\alpha''_S(n_0)$, $\alpha''_V(n_0)$, $\alpha''_{TV}(n_0)$, and $[-10, 50]$ for $\alpha''_S(2n_0)$, $\alpha''_V(2n_0)$, $\alpha''_{TV}(2n_0)$, with subsequent refinement via preliminary low-precision posterior analysis.

Sampled parameters are transformed into density-dependent couplings $\alpha_S(n_V)$, $\alpha_V(n_V)$, and $\alpha_{TV}(n_V)$ via equation (11). These couplings parameterize the RMF model to generate NS EOS under β-equilibrium. Neutron-star structure is determined by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [?, ?]:

$$\frac{dP}{dr} = -\frac{G\mathcal{E}M}{r^2}\left(1+\frac{P}{\mathcal{E}}\right)\left(1+\frac{4\pi r^3 P}{M}\right)\left(1-\frac{2GM}{r}\right)^{-1},$$

$$\frac{dM}{dr} = 4\pi r^2 \mathcal{E},$$

where $G = 6.707 \times 10^{-45}$ MeV^{-2} denotes the gravitational constant. Theoretical radii $R_{\rm th}$ derived from TOV solutions are confronted with observational constraints $R_{\rm obs} \pm \sigma_{{\rm obs},j}$ via the radius likelihood function

$$P_R[D|M(p_i)] = \frac{1}{\sqrt{2\pi}\sigma_{{\rm obs},j}} \exp\left[-\frac{(R_{\rm th}-R_{\rm obs})^2}{2\sigma^2_{{\rm obs},j}}\right].$$

In this work we adopt the observational data on the $1.4M_\odot$ NS from the LIGO/Virgo collaboration but with three uncertainties [?], i.e., $R_{\rm obs} = R_{1.4} = 11.9$ km, and $\sigma_{{\rm obs},j=1,2,3} = 1.0$, 0.5, and 0.2 km. The sampling process incorporates fundamental physical constraints by rejecting any EOS that is thermodynamically unstable ($dp/d\mathcal{E} < 0$), acausal ($c_s \geq c$), or cannot support a maximum neutron star mass of at least $1.97M_\odot$ ($M_{\rm max} < 1.97M_\odot$). Computationally, these are implemented as binary filters within the likelihood function:

$$P[D|M(\{p_i\})] = P_{\rm filter} \times P_{\rm mass,max} \times P_R,$$

where $P_{\rm filter}$ and $P_{\rm mass,max}$ assign a value of 1 if the EOS satisfies the aforementioned constraints and 0 otherwise, effectively setting the posterior probability to zero for invalid models. Posterior distributions are sampled via Markov Chain Monte Carlo (MCMC) using the Metropolis-Hastings algorithm. Parameter probability density functions (PDFs) are derived through marginalization:

$$P(p_i|D) \propto \int P(D|M) \prod_{k\neq i} dp_k.$$

Initial 40,000 samples (burn-in phase) are discarded to mitigate non-equilibrium sampling effects [?, ?]. Posterior statistics are computed from 10^6 subsequent samples, consistent with established convergence diagnostics.

Results and Discussion

Figures 1, 2, and 3 present the one-dimensional (1D) PDFs for six RMF model parameters and the associated two-dimensional (2D) PDFs illustrating their correlations. The NS observational constraints employed in the calculations are the radius of the canonical star $R_{1.4} = 11.9$ km with 1σ uncertainties of 1.0 km, 0.5 km, and 0.2 km. For the density region below saturation density, three different nuclear energy density functionals were utilized: DD-ME2 (Fig. 1 [Figure 1: see original paper]), TW99 (Fig. 2 [Figure 2: see original paper]), and PKDD (Fig. 3 [Figure 3: see original paper]). The 68% and 90% credible intervals for these six parameters are presented in Table IV. We observe that the results for $\sigma = 0.5$ km and $\sigma = 1.0$ km exhibit only minor differences, evident in both the 1D PDFs and the 2D PDFs. In contrast, significant changes emerge when comparing results for $\sigma = 0.2$ km to those for $\sigma = 1.0$ km for the three parameters defined at saturation density (listed in Table IV):

(1) Parameter $\alpha''_S(n_0)$: The peak value of the posterior distribution shifts from predominantly negative to predominantly positive. Specifically, the peak (mode) of the posterior distribution shifts from -0.56×10^{-16} MeV^{-8} for $\sigma = 1.0$ km to 0.12×10^{-16} MeV^{-8} for $\sigma = 0.2$ km when using the DD-ME2 functional. Similar shifts in the posterior peak are observed for the other functionals: from -0.56 to 0.44 (TW99) and from -0.68 to 0.20 (PKDD), all in units of 10^{-16} MeV^{-8}. The constraints on $\alpha''_S(n_0)$ (reflected in the width of the credible intervals) show no substantial improvement for DD-ME2 and TW99. However, for PKDD, the constraint tightens by approximately 40% at the 68% credible level.

(2) Parameter $\alpha''_V(n_0)$: Consistent with $\alpha''_S(n_0)$, its peak value also shifts from negative to positive: from -0.36×10^{-16} MeV^{-8} to 0.04×10^{-16} MeV^{-8} (DD-ME2), from -0.24×10^{-16} MeV^{-8} to 0.24×10^{-16} MeV^{-8} (TW99), and from -0.32×10^{-16} MeV^{-8} to 0.12×10^{-16} MeV^{-8} (PKDD). Unlike $\alpha''_S(n_0)$, the constraints on $\alpha''_V(n_0)$ improve markedly for all scenarios adopting different functionals at subsaturation densities, with reductions in uncertainty of 54.5% (DD-ME2), 46.2% (TW99), and 75% (PKDD).

(3) Parameter $\alpha''_{TV}(n_0)$: For DD-ME2 and TW99 adopted at subsaturation densities, the value of $\alpha''_{TV}(n_0)$ decreases, while a substantial decrease is observed for PKDD. The constraints improve by approximately 50% for all three functionals. These results are influenced not only by the observational data and its precision but also by the intrinsic properties of the density functionals at saturation density, as detailed in Table I.

Utilizing the higher-precision observational data $\sigma = 0.2$ km, we find that the PDFs for the parameters $\alpha''_S(2n_0)$ and $\alpha''_V(2n_0)$ narrow significantly, with uncertainty reductions of the order of 90%. In contrast, the changes for the symmetry energy parameter $\alpha''_{TV}(2n_0)$ are less pronounced. Notably, relative to their prior ranges (see Table III), the PDFs for $\alpha''_{TV}(2n_0)$ show minimal change under the DD-ME2 and TW99 functionals adopted at $n_V < n_0$. Meanwhile, a substantial improvement in the constraint is observed when adopting the PKDD functional at $n_V < n_0$. For the 2D PDFs, the sign of the correlations between parameters remains consistent with our previous calculations [?]. Crucially, the extent of the joint credible regions is markedly reduced for $\sigma = 0.2$ km. This demonstrates that high-precision measurement of $R_{1.4}$ not only imposes strong constraints on the 1D PDFs of the parameters but also significantly constrains their 2D PDFs.

It is found that the most probable values of the six parameters with $\sigma = 0.2$ km do not lie within the regions obtained with $\sigma = 0.5$ and $\sigma = 1.0$ km as listed in Table IV. This phenomenon can be understood through the principles of statistical inference and the specific physics constrained by high-precision data. In Bayesian inference, the posterior distribution represents the updated belief about model parameters after incorporating data. When the observational uncertainty decreases (e.g., from $\sigma = 1.0$ km to $\sigma = 0.2$ km), the likelihood function becomes sharper, acting as a stronger filter on the parameter space. This can lead to two effects: (1) The range of plausible parameter values shrinks; (2) If the high-precision data favor a different region of the parameter space that was only weakly supported by low-precision data, the maximum a posteriori estimate can move significantly. In our case, the shift in the peaks of parameters like α_S'' and α_V'' reflects that the high-precision radius measurement strongly disfavors equations of state with stiff high-density behavior. The $\sigma = 0.2$ km data exclude large portions of the parameter space that were marginally consistent with $\sigma = 1.0$ km but yield pressures too high to match the precise radius constraint. This forces the posterior to concentrate around values that produce a softer EOS, as seen in Table IV. For example, α_S'' shifts from negative to positive values for some functionals, indicating a reduction in repulsive vector interactions at high densities.

The coupling constants of the RMF model are derived from the six parameters given in Eq. (11). Consequently, the constraints on these parameters presented in Figs. 1, 2, and 3 directly influence the density dependence of the three coupling constants α_S, α_V, and α_{TV}, as illustrated in Fig. 4 [Figure 4: see original paper]. The top, middle, and bottom panels of Fig. 4 display the results obtained using the DD-ME2, TW99, and PKDD functionals at $n_V < n_0$, respectively. Overall, all three coupling constants decrease with increasing nucleon density and exhibit asymptotic behavior at high densities, consistent with our earlier calculations [?]. Compared with the constraints obtained with $\sigma = 1.0$ km and $\sigma = 0.5$ km, the uncertainty ranges for α_S and α_V narrow significantly for $\sigma = 0.2$ km, with the constraint on α_V showing particularly remarkable improvement. Conversely, the range for α_{TV} exhibits a substantial broadening, especially in the region where the nucleon density $n_V > 2n_0$. This indicates that the current neutron star observational data provide weaker constraints on the symmetry-energy-related coupling constant. It is worth noting that the application of higher-precision data $\sigma = 0.2$ km leads to a divergent effect on α_{TV}: the constraint weakens (i.e., the uncertainty range broadens) if the DD-ME2 and TW99 functionals are adopted at $n_V < n_0$, whereas it strengthens (i.e., the range narrows) if the PKDD functional is adopted at $n_V < n_0$. This contrasting behavior in the constraint on α_{TV} can be directly attributed to the distinct symmetry energy properties of the functionals at saturation density, specifically the slope parameter L (see Table I). PKDD, with its large L value (90.2 MeV), imposes a stiffer symmetry energy prior. The high-precision data ($\sigma = 0.2$ km), in conjunction with this stiff prior, tightly constrains the possible evolution of the symmetry energy at suprasaturation densities, leading to a narrowed posterior for α_{TV}. Conversely, DD-ME2 and TW99 have smaller L values of 51.2 and 55.3 MeV, respectively, implying a softer and more flexible symmetry energy. When confronted with the same high-precision data demanding a softer overall EOS (lower pressure), the model can accommodate a wider range of α_{TV} values to compensate, resulting in a weakened constraint.

This interpretation is visually confirmed by the behavior of $\epsilon_{\rm sym}$ in Fig. 5 [Figure 5: see original paper], where the credible interval for PKDD narrows significantly with $\sigma = 0.2$ km, while those for DD-ME2 and TW99 broaden.

Figure 5 presents the 90% credible intervals for the energy per nucleon in symmetric nuclear matter (ϵ_0), the nuclear symmetry energy ($\epsilon_{\rm sym}$), and the pressure (p) of neutron star matter as functions of nucleon density. For comparison, the 90% credible bands for $\epsilon_{\rm sym}(n_V)$ and $p(n_V)$ obtained in the DDB models in Ref. [**?**] are included, and the constraining band for $p(n_V)$ from GW170817 in Ref. [**?**] is shown in the figure. The analysis of these results under different observational uncertainties ($\sigma = 1.0$, 0.5, and 0.2 km) reveals several key findings regarding the impact of radius measurement precision on the equation of state (EOS). The most pronounced effect is observed in the symmetric nuclear matter sector. As the observational uncertainty decreases to $\sigma = 0.2$ km, the credible intervals for ϵ_0 narrow substantially across the entire density range, with the central values shifting toward lower energies. This indicates that high-precision radius measurements strongly constrain the isoscalar sector of the nuclear interaction, particularly the density dependence of the scalar and vector coupling constants. The physical origin of this effect lies in the direct relationship between the neutron star radius and the pressure at supranuclear densities, which is predominantly governed by the symmetric matter contribution.

For the symmetry energy $\epsilon_{\rm sym}$, the response to improved precision shows significant model dependence. When using the DD-ME2 and TW99 functionals at subsaturation densities, the uncertainty bands for $\epsilon_{\rm sym}$ broaden with $\sigma = 0.2$ km, while the PKDD functional exhibits a narrowing of the credible intervals. This divergent behavior can be attributed to the compensation effects between the isoscalar and isovector sectors: the strong constraint on the isoscalar couplings forces the isovector parameters to vary over a wider range to maintain agreement with the radius data, with the specific pattern depending on the low-density properties of each functional.

The combined effect on the total pressure p demonstrates a consistent trend toward a softer EOS with reduced uncertainties. The high-precision data ($\sigma = 0.2$ km) exclude EOSs with strong repulsive interactions, leading to lower central pressures and significantly tighter constraints. This effect is particularly evident when compared to existing multi-messenger constraints, such as those from GW170817 as shown by the yellow shadow in Fig. 5, where our high-precision forecast provides substantially more stringent limits on the pressure at high densities. These results underscore the critical importance of future high-precision radius measurements in advancing our understanding of dense matter physics. The significant improvement in constraints achieved with $\sigma = 0.2$ km highlights the potential of next-generation observatories to resolve long-standing uncertainties in the high-density behavior of nuclear matter.

Complementing the results in Fig. 5, Fig. 6 [Figure 6: see original paper] presents the PDFs of the internal pressure p and symmetry energy $\epsilon_{\rm sym}$ within neutron stars at $2n_0$ and $3n_0$. Our key findings are as follows: (1) For the DD-ME2 and TW99 functionals adopted at $n_V < n_0$, the PDFs of $\epsilon_{\rm sym}(2n_0)$ and $\epsilon_{\rm sym}(3n_0)$ obtained using neutron star observational data of three distinct precisions (σ = 1.0, 0.5, 0.2 km) are not significantly different. However, applying higher-precision radius data ($\sigma = 0.2$ km) weakens the constraints on $\epsilon_{\rm sym}$ at $2n_0$ and $3n_0$ (broadening the uncertainty range) under these functionals. Conversely, for the PKDD functional, the distributions narrow significantly. (2) The $\epsilon_{\rm sym}(2n_0)$ and $\epsilon_{\rm sym}(3n_0)$ predicted using the PKDD functional at $n_V < n_0$ is stiffer than that obtained with DD-ME2 or TW99. Notably, increasing the observational precision further stiffens them within the PKDD framework. (3) At both $2n_0$ and $3n_0$, the pressure p decreases and its PDF width narrows (indicating improved constraints) as observational precision improves. This pressure behavior is universal across the different density functionals. (4) The pressure at $2n_0$ exhibits strong sensitivity to the observational precision. In contrast, the PDF of the pressure at $3n_0$ remains nearly unchanged between $\sigma = 1.0$ km and $\sigma = 0.5$ km. (5) For comparison, we show results from Ref. [**?**], which utilize neutron star data from the NICER and LIGO/Virgo collaborations within piecewise-polytropic (PP) and constant speed of sound (CS) models, incorporating constraints from chiral effective field theory (χEFT) for densities $n_V \leq 1.5n_0$. Despite our results being derived solely under the constraint $R_{1.4} = 11.9$ km, we find remarkable agreement, particularly at $\sigma = 0.5$ km, with the findings of Ref. [**?**].

Figure 7 [Figure 7: see original paper] presents the 90% credible intervals for the squared sound speed (v_s^2) and proton fraction (y_p) of neutron star matter as functions of nucleon density. A particularly interesting observation emerges for the proton fraction y_p when using the DD-ME2 and TW99 functionals. The uncertainty bands for y_p broaden significantly when the observational precision improves to $\sigma = 0.2$ km, compared to the constraints obtained with $\sigma = 1.0$ km and 0.5 km. This counter-intuitive result represents an important finding of our Bayesian analysis, reflecting the complex interplay between observational constraints and model parameter spaces. The physical origin of this phenomenon can be traced to parameter compensation effects between the isoscalar and isovector sectors of the nuclear interaction.

The high-precision radius data ($\sigma = 0.2$ km) imposes stringent constraints on the isoscalar couplings (α_S and α_V), effectively requiring a softer overall equation of state. For functionals like DD-ME2 and TW99 that feature relatively soft symmetry energies (characterized by smaller slope parameters L), this strong isoscalar constraint necessitates greater flexibility in the isovector sector to maintain consistency with the radius measurement. Consequently, the uncertainty in the symmetry energy ϵ_{sym} broadens under high-precision conditions, as indicated in Fig. 5. Since the proton fraction y_p is sensitively determined by ϵ_{sym} through the β-equilibrium condition ($\mu_n - \mu_p = \mu_e \approx 4\epsilon_{\text{sym}}\delta$, where $\delta = 1-2y_p$), the increased uncertainty in the symmetry energy directly propagates to the proton fraction, explaining the broader credible intervals observed at $\sigma = 0.2$ km. In striking contrast, the PKDD functional exhibits the opposite behavior, with both y_p and ϵ_{sym} showing narrowed uncertainties under high-precision constraints. This divergent response stems from PKDD' s inherently stiffer symmetry energy, whose prior constraints are more compatible with the specific adjustments required to satisfy the high-precision radius measurement.

For the squared sound speed v_s^2, defined through $v_s = \sqrt{dp/d\mathcal{E}}$, the high-precision data generally produces lower values and reduced uncertainties, consistent with the overall softening of the EOS observed in Fig. 5. However, at high densities ($n_V > 4n_0$), a notable reversal occurs with a broadening of the v_s^2 uncertainty band, particularly for the DD-ME2 and TW99 functionals. This behavior reflects the growing influence of the poorly constrained symmetry energy sector at extreme densities. The significantly tightened constraints on both v_s^2 and y_p achieved with $\sigma = 0.2$ km for the PKDD functional demonstrate the critical importance of future high-precision radius measurements. These results highlight the complex, model-dependent nature of extracting symmetry energy information from astrophysical observations and underscore the need for complementary constraints from nuclear experiments and theories.

Conclusion

We have performed a Bayesian analysis of the density-dependent relativistic mean-field model using neutron star radius data for a $1.4M_\odot$ star with three levels of observational precision: $\sigma = 1.0$, 0.5, and 0.2 km. Our primary aim was to quantify how improvements in radius measurements affect the constraints on high-density coupling parameters, the corresponding EOS of neutron star matter, and derived stellar properties.

The results show that a modest improvement from $\sigma = 1.0$ km to 0.5 km yields only limited changes in the posterior distributions, whereas a further reduction to $\sigma = 0.2$ km leads to substantial tightening of constraints, particularly for the isoscalar couplings $\alpha''_S(2n_0)$ and $\alpha''_V(2n_0)$, whose uncertainties are reduced by up to $\sim 90\%$. The isovector coupling $\alpha''_{TV}(2n_0)$ remains weakly constrained adopting the functionals DD-ME2 and TW99 at $n_V < n_0$ but shows notable improvement for PKDD. While the high-precision data generally narrow the credible intervals of the isoscalar couplings and soften the pressure of neutron star matter, their effect on the symmetry-energy-related coupling α_{TV} is functional dependent, i.e., broadening if DD-ME2 and TW99 are adopted at $n_V < n_0$, but narrowing if PKDD is adopted.

At the EOS level, $\sigma = 0.2$ km data favor a softer neutron star matter EOS, with lower central pressures and reduced uncertainty bands across most densities. The EOS of symmetric nuclear matter is particularly sensitive to observational precision, while the symmetry energy exhibits opposite trends depending on the underlying functional at $n_V < n_0$. The pressure at $2n_0$ is highly responsive to improved constraints, whereas the pressure at $3n_0$ changes little between $\sigma = 1.0$ and 0.5 km. High-precision measurements also lead to better constraints on the proton fraction and sound speed, although the latter shows increased uncertainty above $\sim 4n_0$.

Our findings in the present work demonstrate that future high-precision radius measurements, at the level of $\sigma \lesssim 0.2$ km as anticipated from upcoming X-ray timing missions and third-generation gravitational-wave detectors, will significantly sharpen constraints on the high-density behavior of the EOS and the associated RMF couplings. Such measurements will be essential for disentangling the density dependence of the symmetry energy of nuclear matter, thereby improving our understanding of neutron-rich matter under extreme conditions.

Acknowledgments

C.J.X. is supported by the National Natural Science Foundation of China (Grant No. 12275234) and the National SKA Program of China (Grant No. 2020SKA0120300). W.J.X. is supported by the Open Project of Guangxi Key Laboratory of Nuclear Physics and Nuclear Technology under Grant No. NLK2023-03, and the Central Government Guidance Funds for Local Scientific and Technological Development, China, under Grant No. Guike ZY22096024.

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