

Mechanisms of mirror energy difference for states exhibiting Thomas-Ehrman shift: Gamow shell model case studies of $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$

Authors: Li, Dr. Kunhao, Wang, Dr. Peiyan, Jianguo Li, Michel, Dr. nicolas, Dr. Mengran Xie, Ma, Dr. Chun-Wang (Nuclear physics), Zuo, Dr. Wei, Dr. Jianguo Li

Date: 2025-10-21T15:41:53+00:00

Abstract

The mirror energy difference (MED) of the mirror state, especially for states bearing the Thomas-Ehrman shift, serves as a sensitive probe of mirror symmetry breaking. We employ the Gamow shell model, which includes the inter-nucleon correlation and continuum coupling, to investigate the MED for *sd*-shell nuclei by taking the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ as examples. Our GSM provides good descriptions for the excitation energies and MEDs for the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$. Moreover, our calculations also reveal that the large MED of the mirror states is caused by the significant occupation of the weakly bound or unbound $s_{1/2}$ waves, giving the radial density distribution of the state in the proton-rich nucleus more extended than that of mirror states in deeply-bound neutron-rich nuclei. Moreover, our GSM calculation shows that the contribution of Coulomb is different for the low-lying states in proton-rich nuclei, which significantly contributes to MEDs of mirror states, which is well-recognized. Furthermore, our GSM calculation points out that the contributions of the nucleon-nucleon interaction are different for the mirror state, especially for the state of proton-rich nuclei bearing the Thomas-Ehrman shift, which also contributes to the significant mirror symmetry breaking with large MED.

Full Text

Preamble

Mechanisms of mirror energy difference for states exhibiting Thomas-Ehrman shift: Gamow shell model case studies of $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$

Kun-Hao Li,^{1, 2} Pei-Yan Wang,^{2, 3} Jian-Guo Li,^{2, 3, 4, †} Nicolas Michel,^{2, 3, ‡}
Meng-Ran Xie,^{2, 3} Chun-Wang Ma,^{1, 5, §} and Wei Zuo^{2, 3, 4}

¹College of Physics, Center for Theoretical Physics, Henan Normal University,
Xinxiang 453007, China

²CAS Key Laboratory of High Precision Nuclear Spectroscopy, Institute of Mod-
ern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

³School of Nuclear Science and Technology, University of Chinese Academy of
Sciences, Beijing 100049, China

⁴Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern
Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province,
China

⁵Institute of Nuclear Science and Technology, Henan Academy of Sciences,
Zhengzhou 450015, China

The mirror energy difference (MED) of mirror states, particularly for states bearing the Thomas-Ehrman shift, serves as a sensitive probe of mirror symmetry breaking. We employ the Gamow shell model, which includes inter-nucleon correlation and continuum coupling, to investigate the MED for sd-shell nuclei using $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ as examples. Our GSM provides good descriptions for the excitation energies and MEDs of $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$. Moreover, our calculations reveal that the large MED of mirror states is caused by the significant occupation of weakly bound or unbound $s_{1/2}$ waves, giving the radial density distribution of the state in the proton-rich nucleus a more extended character than that of mirror states in deeply bound neutron-rich nuclei. Our GSM calculation also shows that the contribution of the Coulomb interaction differs for low-lying states in proton-rich nuclei, which significantly contributes to MEDs of mirror states, as is well recognized. Furthermore, our GSM calculation points out that the contributions of the nucleon-nucleon interaction are different for mirror states, particularly for states in proton-rich nuclei bearing the Thomas-Ehrman shift, which also contributes to significant mirror symmetry breaking with large MED.

Keywords: mirror symmetry breaking, Thomas-Ehrman shift, mirror energy difference, continuum coupling, Gamow shell model

INTRODUCTION

Exotic nuclear structures in drip-line nuclei have become a subject of great interest in recent years, as they are characterized by unique properties that exhibit significant differences compared to those of stable nuclei [?]. One of the most significant phenomena observed in these systems is the Thomas-Ehrman shift (TES) [?, ?]. This effect is most pronounced in nuclei close to the proton drip lines, where the balance between the strong force and the Coulomb force is most delicate. States exhibiting TES effects are often weakly bound or unbound, characteristic of open quantum systems, while their neutron-rich mirror counterparts remain deeply bound, resulting in a large mirror energy difference

(MED) in their isobaric states [?]. These large MEDs are attributed to their proximity to near-threshold effects, in which continuum effects need to be properly treated. Consequently, a thorough comprehension of the Thomas-Ehrman shift is pivotal for elucidating the dynamics of weakly bound and unbound nuclear systems and understanding the mechanisms underlying mirror symmetry breaking in mirror nuclei.

Two possible reasons exist for states with large MED, of either external or internal character. If extended single-particle wave functions of weakly bound or unbound s- or p-waves are significantly occupied in the considered states, the large MED is of external nature, as in the TES states [?, ?, ?]. The second possibility, related to configuration mixing (see Refs. [?, ?]), is of internal nature. In this case, the extended wave function arises via strong configuration mixing in which a few nodal states of s or p waves are included in the calculations. These two external and internal effects are different but can be intertwined in a complex manner. For instance, the inversion of ground states in the ^{16}F and ^{16}N mirror nuclei is primarily due to the unbound proton $1s_{1/2}$ orbital, which can also be well described in GSM calculations within the configuration mixing framework [?, ?].

The sd-shell nuclei, situated at the boundary between light and heavy nuclei, exhibit a wide range of nuclear structure phenomena that remain somewhat mysterious [?]. In recent years, these nuclei have been extensively studied using a variety of experimental techniques [?]. A wealth of information on the Thomas-Ehrman shift has been gleaned from sd-shell proton drip-line nuclei, where numerous states exhibiting significant TES effects have been identified [?, ?]. For instance, the mirror pairs $^{18}\text{Ne}/^{18}\text{O}$ [?, ?] and $^{19}\text{Na}/^{19}\text{O}$ [?] serve as notable examples. For sd-shell nuclei, TES is mainly driven by s-waves. Indeed, the proton $1s_{1/2}$ orbital is weakly bound or unbound in proton drip-line nuclei, whereas the neutron $1s_{1/2}$ is well-bound in their mirror neutron-rich nuclei.

Several theoretical models have been developed to probe mirror asymmetry for mirror nuclei, such as the standard shell model (SM) [?], mean-field calculations [?, ?], and ab initio approaches [?, ?]. Within standard SM calculations, weakly bound and unbound wave functions on eigenenergies are indirectly considered via phenomenologically adjusting the matrix element related to the $1s_{1/2}$ orbit [?, ?]. Mean-field calculations, such as Skyrme-Hartree-Fock, have also been extensively employed in MED studies [?, ?]. However, these models involve parameters that are constrained by data [?, ?, ?]. In recent years, ab initio approaches such as the ab initio valence-space in-medium similarity renormalization group have also been applied to study MEDs of sd-shell nuclei [?, ?, ?, ?, ?], in which the extended many-body wavefunctions are partially described by using a large number of HO spaces.

Moreover, current theoretical calculations have pointed out that TES is caused by the repulsive Coulomb interaction and the occupations of weakly bound or unbound s- or p-waves for valence protons. However, detailed studies on the mechanism of TES are also lacking. In recent shell model calculations [?], TES

has been investigated using calculated spectroscopic factors. Moreover, in our previous work [?], we compared MED results calculated using the shell model with spectroscopic factors and the ab initio valence-space in-medium similarity renormalization group approach, finding that the two models give similar results.

One of the major challenges in studying drip-line nuclei is accounting for the interplay between configuration mixing and continuum effects. The Gamow shell model (GSM) [?, ?, ?, ?, ?] has emerged as a powerful tool in this regard, as it provides a unified framework to describe the structure of nuclei close to the particle emission threshold and allows for an accurate understanding of exotic properties in drip-line nuclei. Based on this situation, we employ GSM to investigate significant mirror symmetry breaking with large MED values and the underlying mechanism for sd-shell nuclei, taking the 18Ne/18O and 19Na/19O mirror partners as examples.

II. METHOD

GSM is a multiconfiguration shell model framework that works in the picture of a core plus valence nucleons [?, ?, ?]. At the heart of GSM lies the utilization of the one-body Berggren basis [?], which possesses bound, resonance, and scattering states generated by a finite-range potential, typically of Woods-Saxon (WS) type (see details in Refs. [?, ?, ?]). The GSM Hamiltonian matrix is characterized as complex symmetric [?, ?]. The overlap method along with the Jacobi-Davidson method extended to complex-symmetric matrices is adopted to diagonalize and identify many-body resonance eigenstates [?, ?, ?]. Consequently, GSM calculations include both inter-nucleon correlations and continuum coupling [?, ?, ?].

The many-body Schrödinger equation of the GSM Hamiltonian can be solved within the so-called cluster orbital shell model (COSM) formalism [?] (see Refs. [?, ?, ?]). The GSM Hamiltonian in COSM coordinates reads [?, ?, ?]:

$$\text{Aval}(\text{cid:88}) \hat{H}_{\text{GSM}} = (\text{cid:18}) p^2 + \hat{U}(c) (\text{cid:19}) \text{Aval}(\text{cid:88}) (\text{cid:18}) \hat{V}_{ij} + (\text{cid:19}) \pi_i \cdot \pi_j$$

where Aval is the number of valence nucleons, π_i is the effective mass of the nucleon, $\hat{U}(c)$ is represented by a one-body WS potential mimicking the inert core, \hat{V}_{ij} is the residual inter-nucleon interaction, which is modeled by a pionless effective field theory (EFT) interaction [?, ?], in which only two-body contact terms up to next-to-next-leading order are considered. The regularization approach adopted in Refs. [?, ?] is used. The last term embodies the recoil effects induced by the finite mass of the core Mc. The EFT interaction is optimized to reproduce the low-lying states of selected nuclei.

In the present work, the 18Ne/18O and 19Na/19O mirror partners are taken as examples. The doubly magic nucleus 16O is chosen as the inert core, and the $s_{1/2}$, $p_{1/2,3/2}$ and $d_{3/2,5/2}$ partial waves are represented by the Berggren basis, in which 40 discretization points are used in total for continuum states in each

partial wave, and the $f_{5/2,7/2}$ partial waves are treated using the HO basis, in which 6 HO states are adopted for each partial wave. To estimate the effects of higher orbitals, we compare the results after adding the $g_{7/2,9/2}$ partial waves under the HO basis; the difference in binding and excitation energy in the states studied in the present work is less than 5 keV. As such, higher partial waves are neglected. Only the Coulomb force is considered for the isospin non-conserving part of the GSM Hamiltonian. The contribution of the isospin-dependent part of nuclear interaction to TES is small and is neglected in the present GSM calculations. The Hamiltonian used in Ref. [?] is adopted in the present work. The calculated excitation energies of $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror partners are presented in Tables 1 and 2, which show good agreement with experimental data [?]. In the following section, the mechanics of the mirror energy difference for the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror partners are investigated in detail.

III. RESULTS

Our GSM calculations accurately describe the excitation energies of low-lying states for the mirror partners $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$. To delve deeper into the significant mirror symmetry breaking observed in these mirror nuclei, we define the MED for a given state $J\pi$ as $\text{MED}(J\pi) = \text{Ex}(T<z, J\pi) - \text{Ex}(T>z, J\pi)$, where T_z denote the negative and positive isospin projection $T_z = (N - Z)/2$, respectively, for a mirror pair. We have calculated MED values for mirror states in $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$, as presented in Tables 1 and 2, along with experimental data. It is observed that in most states, the GSM-calculated MED values for low-lying states in the mirror partners $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ align with experimental data. Although the numerical differences in excitation energies between calculated and experimental values for some states are relatively large, they still exhibit the expected qualitative trend of MED. Meanwhile, SM results perform poorly, which demonstrates that coupling to continuum states plays an essential role in the study of mirror states bearing significant mirror symmetry breaking. However, an exception is noted for the $0+2$ state in the $^{18}\text{Ne}/^{18}\text{O}$ mirror nuclei, where our GSM calculations yield larger values than experimental data. Both our GSM calculations and experimental data highlight significant mirror symmetry breaking in the $3+1$ state of the $^{18}\text{Ne}/^{18}\text{O}$ mirror nuclei and the $1/2+1$ state of the $^{19}\text{Na}/^{19}\text{O}$ mirror nuclei, evidenced by their large MED values.

Table 1 . The calculated excitation energies of $^{18}\text{Ne}/^{18}\text{O}$ with GSM and SM calculations in harmonic oscillator single sd-shell model space without including higher orbitals, along with experimental data [?]. The unit of excitation energy is given in MeV, and the units of particle decay width and MED are given in keV.

Fig. 1 [Figure 1: see original paper]. The average occupation numbers for the $s_{1/2}$ and $d_{5/2}$ partial waves in the low-lying states of the $^{18}\text{Ne}/^{18}\text{O}$ mirror pair, calculated using the GSM above the ^{16}O core.

Table 2 . Similar to Table 1, but for $^{19}\text{Na}/^{19}\text{O}$.

Fig. 2 [Figure 2: see original paper]. Similar to Fig. 1, but for low-lying states in $^{19}\text{Na}/^{19}\text{O}$.

To investigate the significant mirror symmetry breaking and the associated large MEDs, we begin by calculating the average occupations of low-lying states through GSM. The focus is particularly on the $s_{1/2}$ and $d_{5/2}$ partial waves above the ^{16}O core for the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror nuclei, as illustrated in Figs. 1 and 2. Notably, other partial waves like $d_{3/2}$, $f_{5/2,7/2}$ exhibit negligible occupations and are therefore excluded from these figures. The calculated average occupations reveal almost identical patterns for mirror states within the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ pairs. Our GSM calculations further indicate that states exhibiting significant mirror symmetry breaking with large MED values also show significant occupancy in the $s_{1/2}$ partial waves—markedly higher than in their respective ground states. For instance, the occupations of the $s_{1/2}$ partial wave for the $3+1$ and $1/2+1$ states in the $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror pairs, respectively, are substantially greater than those of the ground states. Additionally, our calculations show that the $0+2$ states of $^{18}\text{Ne}/^{18}\text{O}$ demonstrate notable $s_{1/2}$ partial wave occupations compared to the ground states, resulting in a large MED. Contrastingly, experimental data give a smaller MED value, hinting at a complex structure of the $0+2$ states in $^{18}\text{Ne}/^{18}\text{O}$ that might not be fully captured by the ^{16}O plus valence particles picture.

Aligned with results from other theoretical frameworks, such as the standard shell model and the *ab initio* VS-IMSRG approach, our results indicate that the significant mirror symmetry breaking with large MEDs observed in mirror states stems primarily from the extensive occupation of the $s_{1/2}$ partial waves, which are weakly bound or unbound in the proton-rich nucleus but deeply bound in its mirror neutron-rich nucleus—this is the TES. However, a deeper understanding of mirror states bearing significant mirror symmetry breaking with large MED values is lacking. GSM is a very suitable model that properly treats both inter-nucleon correlations and continuum coupling to describe the properties of dripline nuclei, including a precise description of the many-body wave function in the asymptotic regions [?, ?, ?, ?].

To elucidate the underlying mechanism of large MEDs, we conduct a detailed analysis of the radial density distributions of mirror states in $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ pairs using GSM. The results allow us to systematically compare the radial distributions of valence protons in proton-rich nuclei and valence neutrons in their neutron-rich mirror counterparts, as presented in Figs. 3 and 4 for $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror partners, respectively. Our GSM results reveal that states characterized by minor mirror symmetry breaking with small MEDs exhibit almost identical radial density distributions, which decline sharply in the asymptotic regions, such as the ground states of both $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$. This phenomenon is largely attributed to the dominance of $d_{5/2}$ partial waves, which are constrained within the nuclear region by high centrifugal and Coulomb barriers, despite the state being unbound. Conversely,

GSM calculations depict the radial density distributions of the $3+1$ state of ^{18}Ne and the $1/2+1$ state of ^{19}Na as more extended in the asymptotic region than their neutron-rich counterparts, ^{18}O and ^{19}O , respectively. This disparity stems from the non-existent centrifugal barrier for the $s_{1/2}$ partial wave, leading to a more pronounced distribution in the proton-rich nucleus due to the weakly bound or unbound nature of the $s_{1/2}$ partial wave.

A similar mechanism underlies the formation of halo nuclei, where valence nucleons occupy weakly bound s - or p -partial waves, resulting in an extended density distribution due to the minimal or absent centrifugal barrier [?, ?, ?]. In previous works, these exotic phenomena resulting from the mechanism for the extension of the s -partial wave have been widely recognized. However, in reality, most theoretical models cannot directly provide the properties of this extension and its impact on physical phenomena when performing calculations. For example, the results of our SM calculations within a single sd -shell model space in Table 1 show that the MEDs of all states for $^{18}\text{Ne}/^{18}\text{O}$ are quite small.

However, mirror symmetry breaking can be calculated using traditional SM by introducing further approximations and modifications. For example, traditional SM calculations using the USDC interaction give the MED of the mirror $1/2+1$ state of $^{19}\text{Na}/^{19}\text{O}$ as -225 keV, which is significantly smaller than the experimental value of -727 keV. Moreover, the MED of the mirror $1/2+1$ state of $^{19}\text{Na}/^{19}\text{O}$ is mainly caused by the difference in optimized single-particle energy for valence protons and neutrons in the USDC interaction [?]. Nevertheless, since the calculated MED is significantly smaller than experimental data, they simulated the TES arising from continuum coupling through $\text{TES}_{\text{total}} = C_{2Ss_{1/2}} \text{TES}_{\text{sp}}$, where $C_{2Ss_{1/2}}$ is the spectroscopic factor of the proton $s_{1/2}$ orbital and TES_{sp} is the single-particle TES [?]. This approximate treatment of TES gives MED values of mirror states close to experimental data. Based on this, the relationship between TES and the extended $s_{1/2}$ partial wave in the mirror partner was indirectly determined; however, SM calculations cannot provide a direct description of TES. Moreover, phenomenological adjustments to the matrix elements related to the proton $s_{1/2}$ orbital have also been employed in SM to describe the indirect effects of weakly bound and unbound wave functions on eigenenergies. For $A \leq 20$ nuclei, the strength of effective SM interactions involving the loosely bound proton $s_{1/2}$ orbit is significantly reduced in comparison with their mirror nuclei to calculate significant mirror symmetry [?], where reduction factors of the two-body matrix elements related to the proton $s_{1/2}$ orbital are evaluated using the Woods-Saxon potential. The large MEDs in $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ are well reproduced.

Unlike SM calculations with the aforementioned corrections, our GSM calculations provide a more self-consistent direct calculation of the radial density distribution, yielding the expected MEDs without relying on other corrections and modifications. Moreover, our calculations directly unveil that mirror states demonstrating significant mirror symmetry breaking with large MEDs possess many-body wave functions in proton-rich nuclei that are more extended than

those in their neutron-rich mirror states, which further helps us understand the role of mirror symmetry breaking in shaping their properties.

Fig. 3 [Figure 3: see original paper]. The calculated radial density distribution of valence protons and valence neutrons for low-lying mirror states in $^{18}\text{Ne}/^{18}\text{O}$, respectively, above the ^{16}O inner core, using GSM.

Fig. 4 [Figure 4: see original paper]. Similar to Fig. 3, but for low-lying states in $^{19}\text{Na}/^{19}\text{O}$.

The GSM Hamiltonian, as shown in Eq. (1), can be divided into nuclear interaction (encompassing core-nucleons and nucleon-nucleon interaction) and Coulomb interaction (including one-body Coulomb (1BC) interaction between the inner core and valence protons, and two-body Coulomb (2BC) interaction between valence protons). We delve into further calculations to dissect the contributions from different parts of the Hamiltonian, aiming to shed light on the underlying mechanisms in mirror states exhibiting significant mirror symmetry breaking with large MED.

The computed energies for the low-lying mirror states in the pairs $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$, along with experimental data [?], are showcased in Figs. 5 and 6, respectively. To gain deeper insights, we also present the energy minus 2BC contribution (GSM-2BC) and energy minus 1BC and 2BC contribution (GSM-1BC-2BC) in proton-rich nuclei ^{18}Ne and ^{19}Na . Indeed, GSM-1BC-2BC corresponds to the contribution of nuclear interaction. Within the isospin symmetry picture, the difference in mirror state energies should solely stem from Coulomb interactions, implying that the GSM-1BC-2BC values for a state in a proton-rich nucleus would be the same as its mirror state in the neutron-rich nucleus.

Our GSM calculation gives that the GSM-1BC-2BC values for the ground states of ^{18}Ne and ^{19}Na closely align with the computed ground-state energies of their neutron-rich counterparts, ^{18}O and ^{19}O , respectively, indicating the preservation of mirror symmetry for these ground states. Conversely, for the excited $3+1$ state in $^{18}\text{Ne}/^{18}\text{O}$ and the $1/2+1$ state in $^{19}\text{Na}/^{19}\text{O}$, our GSM calculations showcase a deviation from this symmetry. To quantitatively examine this discrepancy, we introduce ΔE as the differential metric for significant mirror symmetry breaking. ΔE encapsulates the disparity between the GSM-1BC-2BC values in the state of the proton-rich nucleus and the energy calculated for the corresponding state in the neutron-rich mirror nucleus, which reads as $\Delta E = \langle \Psi_{\text{proton}} | H_{\text{NN}} | \Psi_{\text{proton}} \rangle - \langle \Psi_{\text{neutron}} | H_{\text{NN}} | \Psi_{\text{neutron}} \rangle$, where Ψ_{proton} and Ψ_{neutron} correspond to the many-body wave functions of proton-rich and neutron-rich nuclei, respectively. The ΔE corresponds to the difference in the contribution of nuclear interactions in the mirror state.

Our GSM calculations show that both ΔE and Coulomb interactions, including 1BC and 2BC, significantly influence the energy discrepancies observed in mirror states. Predominantly, the Coulomb interaction emerges as the dominant factor contributing to these differences. Illustrated in the lower panels of Figs.

5 and 6, we detail the ΔE , 1BC, and 2BC contributions to the energy differences in the low-lying mirror states of $^{18}\text{Ne}/^{18}\text{O}$ and $^{19}\text{Na}/^{19}\text{O}$ mirror pairs. Our GSM results indicate that the energy differences in the ground states of $^{18}\text{Ne}/^{18}\text{O}$ primarily stem from Coulomb interactions, with ΔE making a minimal contribution. Furthermore, for $^{19}\text{Na}/^{19}\text{O}$, the ΔE contribution is noted to be around 100 keV. Interestingly, we find varying contributions of ΔE , 1BC, and 2BC across different mirror states within each nucleus. For instance, the 3+1 states in $^{18}\text{Ne}/^{18}\text{O}$ exhibit a higher ΔE contribution and lower Coulomb interactions relative to their ground states. The heightened ΔE values underscore the distinct nuclear interaction contributions to mirror symmetry breaking in these systems, showcasing the complex interplay of forces that shape the energy landscapes of mirror nuclei.

In evaluating the MED, the ground state energy of mirror nuclei serves as the baseline, with MED being determined by the discrepancy in excitation energies of corresponding mirror states. Adopting the energy difference of the ground states of mirror nuclei as a reference—illustrated by red dashed lines in Figs. 5 and 6—the difference between the values of ground and excited mirror states corresponds to the MED, highlighted by the red arrows in these figures. The results reveal that both the ΔE values and the Coulomb interaction exhibit significant variations across different mirror states, both contributing to the MED.

Furthermore, to validate our conclusion, we performed Gamow shell model calculations using an optimized Hamiltonian fitted to reproduce a series of selected experimental data for sd-shell nuclei. This optimized Hamiltonian has been employed to investigate the low-lying states in ^{21}Al [?] and ^{22}Si [?], as well as isospin symmetry breaking in those nuclei. While the calculated excitation energies exhibit slight differences, the MED results align with both the current GSM calculations and the established conclusions regarding the MED mechanism.

Fig. 5 [Figure 5: see original paper]. Upper panel: calculated energies (GSM), energies minus the two-body Coulomb contribution (GSM-2BC), and energies minus one- and two-body Coulomb total contributions (GSM-1BC-2BC) of low-lying states of ^{18}Ne using GSM, along with the energies of mirror states in ^{18}O , with respect to the ^{16}O inner core. The GSM results are also compared with experimental data. Lower panel: the contribution for the calculated energy difference between the mirror states.

Fig. 6 [Figure 6: see original paper]. Similar to Fig. 5, but for low-lying states in $^{19}\text{Na}/^{19}\text{O}$.

IV. SUMMARY

Based on GSM calculations, in which both inter-nucleon correlation and continuum coupling are properly treated, we deduce that significant mirror symmetry breaking in mirror states, leading to large MED values, arises from the occupa-

tion of weakly bound or unbound $s_{1/2}$ partial waves in the proton-rich nucleus, while its counterpart in the neutron-rich nucleus remains deeply bound. This dichotomy culminates in a more expansive radial density distribution for states within the proton-rich nucleus, as opposed to their mirror counterparts. Additionally, the difference in radial density distributions between mirror states implies disparate contributions from nuclear interactions, underscored by significant ΔE values, which further highlight the presence of mirror symmetry breaking. Moreover, states with an extended radial density distribution tend to yield smaller Coulomb contributions compared to ground states characterized by more localized distributions. This factor chiefly accounts for the reduced excitation energies in states influenced by the Thomas-Ehrman shift effect, thereby engendering substantial negative MED values in mirror states. Our GSM calculations corroborate that both nuclear and Coulomb interactions play crucial roles in manifesting the significant mirror symmetry breaking associated with large MED values.

REFERENCES

- [1] I. Tanihata, H. Savajols, R. Kanungo, Recent experimental progress in nuclear halo structure studies. *Prog. Part. Nucl. Phys.* 68, 215-313 (2013). <https://doi.org/10.1016/j.ppnp.2012.07.001>
- [2] L. Zhou, D. Q. Fang, S. M. Wang, et al., Structure and 2p decay mechanism of ^{18}Mg . *Nucl. Sci. Tech.* 35, 107 (2024). <https://doi.org/10.1007/s41365-024-01479-1>
- [3] X. Q. Du, C. W. Wang, D. Y. Tao, et al., Dineutron and diproton correlations in the exotic nuclei ^6He and ^6Be . *Nucl. Sci. Tech.* 36, 205 (2025). <https://doi.org/10.1007/s41365-025-01778-1>
- [4] E. Lunderberg, P. A. DeYoung, Z. Kohley, et al., Evidence for the ground-state resonance of ^{26}O . *Phys. Rev. Lett.* 108, 142503 (2012). <https://doi.org/10.1103/PhysRevLett.108.142503>
- [5] A. H. Wuosmaa, J. P. Schiffer, K. E. Rehm, et al., Structure of ^7He by proton removal from ^8Li with the $(d, ^3\text{He})$ reaction. *Phys. Rev. C* 78, 041302 (2008). <https://doi.org/10.1103/PhysRevC.78.041302>
- [6] M. T. Wan, L. Ou, M. Liu, et al., Properties of the drip-line nucleus and mass relation of mirror nuclei. *Nucl. Sci. Tech.* 36, 26 (2025). <https://doi.org/10.1007/s41365-024-01633-9>
- [7] L. Zhou, S. M. Wang, D. Q. Fang, et al., Recent progress in two-proton radioactivity. *Nucl. Sci. Tech.* 33, 105 (2022). <https://doi.org/10.1007/s41365-022-01091-1>
- [8] R. G. Thomas, An analysis of the energy levels of the mirror nuclei, ^{13}C and ^{13}N . *Phys. Rev.* 88, 1109-1125 (1952). <https://doi.org/10.1103/PhysRev.88.1109>
- [9] J. B. Ehrman, On the displacement of corresponding energy levels of ^{13}C and ^{13}N . *Phys. Rev.* 81, 412-416 (1951). <https://doi.org/10.1103/PhysRev.81.412>
- [10] K. A. Chipps, D. W. Bardayan, J. C. Blackmon, et al., First direct measurement of the $^{17}\text{F}(p, \gamma)^{18}\text{Ne}$ cross section. *Phys. Rev. Lett.* 102, 152502 (2009). <https://doi.org/10.1103/PhysRevLett.102.152502>

- [11] J. Lee, X. X. Xu, K. Kaneko, et al., Large isospin asymmetry in $^{22}\text{Si}/^{22}\text{O}$ mirror Gamow-Teller transitions reveals the halo structure of ^{22}Al . *Phys. Rev. Lett.* 125, 192503 (2020). <https://doi.org/10.1103/PhysRevLett.125.192503>
- [12] M. Z. Sun, Y. Yu, X. P. Wang, et al., Ground-state mass of ^{22}Al and test of state-of-the-art ab initio calculations. *Chinese Phys. C* 48, 034002 (2024). <https://doi.org/10.1088/1674-1137/ad1a0a>
- [13] H. H. Li, Q. Yuan, J. G. Li, et al., Investigation of isospin-symmetry breaking in mirror energy difference and nuclear mass with ab initio calculations. *Phys. Rev. C* 107, 014302 (2023). <https://doi.org/10.1103/PhysRevC.107.014302>
- [14] C. X. Yuan, C. Qi, F. R. Xu, et al., Mirror energy difference and the structure of loosely bound proton-rich nuclei around $A = 20$. *Phys. Rev. C* 89, 044327 (2014). <https://doi.org/10.1103/PhysRevC.89.044327>
- [15] I. Stefan, F. de Oliveira Santos, O. Sorlin, et al., Probing nuclear forces beyond the drip line using the mirror nuclei ^{16}N and ^{16}F . *Phys. Rev. C* 90, 014307 (2014). <https://doi.org/10.1103/PhysRevC.90.014307>
- [16] J. G. Li, N. Michel, H. H. Li, et al., One-neutron halo structure of ^{29}Ne . *Phys. Lett. B* 832, 137225 (2022). <https://doi.org/10.1016/j.physletb.2022.137225>
- [17] X. Mao, J. Rotureau, W. Nazarewicz, et al., Gamow-shell-model description of Li isotopes and their mirror partners. *Phys. Rev. C* 102, 024309 (2020). <https://doi.org/10.1103/PhysRevC.102.024309>
- [18] N. Michel, J. G. Li, L. H. Ru, et al., Calculation of the Thomas-Ehrman shift in ^{16}F and $^{15}\text{O}(p, p)$ cross sections within the Gamow shell model. *Phys. Rev. C* 106, L011301 (2022). <https://doi.org/10.1103/PhysRevC.106.L011301>
- [19] K. Way, Elementary Theory of Nuclear Shell Structure. *Science* 122, 603 (1955). <https://doi.org/10.1126/science.122.3170.603.b>
- [20] E. Caurier, G. Martínez-Pinedo, F. Nowacki, et al., The shell model as a unified view of nuclear structure. *Rev. Mod. Phys.* 77, 427–488 (2005). <https://doi.org/10.1103/RevModPhys.77.427>
- [21] F. Ajzenberg-Selove, Energy levels of light nuclei $A = 5-10$. *Nucl. Phys. A* 490, 1–225 (1988). [https://doi.org/10.1016/0375-9474\(88\)90124-8](https://doi.org/10.1016/0375-9474(88)90124-8)
- [22] P. Campbell, I. D. Moore, M. R. Pearson, Laser spectroscopy for nuclear structure physics. *Prog. Part. Nucl. Phys.* 86, 127–180 (2016). <https://doi.org/10.1016/j.pnpnp.2015.09.003>
- [23] J. G. Li, B. S. Hu, S. Zhang, et al., Unbound ^{28}O , the heaviest oxygen isotope observed: a cutting-edge probe for testing nuclear models. *Nucl. Sci. Tech.* 35, 21 (2024). <https://doi.org/10.1007/s41365-024-01373-w>
- [24] C. Angulo, G. Tabacaru, M. Couder, et al., Identification of a new low-lying state in the proton drip line nucleus ^{19}Na . *Phys. Rev. C* 67, 014308 (2003). <https://doi.org/10.1103/PhysRevC.67.014308>
- [25] S. Zhang, Y. Z. Ma, J. G. Li, et al., The roles of three-nucleon force and continuum coupling in mirror-symmetry breaking of the oxygen mass region. *Phys. Lett. B* 827, 136958 (2022). <https://doi.org/10.1016/j.physletb.2022.136958>
- [26] M. A. Bentley, S. M. Lenzi, Coulomb energy differences between high-spin states in isobaric multiplets. *Prog. Part. Nucl. Phys.* 59, 497–561 (2007). <https://doi.org/10.1016/j.pnpnp.2006.10.001>
- [27] A. P. Zuker, S. M. Lenzi, G. Martínez-Pinedo, et al., Isobaric multiplet

- yrast energies and isospin nonconserving forces. *Phys. Rev. Lett.* 89, 142502 (2002). <https://doi.org/10.1103/PhysRevLett.89.142502>
- [28] K. Kaneko, Y. Sun, T. Mizusaki, et al., Variation in displacement energies due to isospin-nonconserving forces. *Phys. Rev. Lett.* 110, 172505 (2013). <https://doi.org/10.1103/PhysRevLett.110.172505>
- [29] Y. H. Lam, N. A. Smirnova, E. Caurier, Isospin nonconservation in sd-shell nuclei. *Phys. Rev. C* 87, 054304 (2013). <https://doi.org/10.1103/PhysRevC.87.054304>
- [30] A. Magilligan, B. A. Brown, New isospin-breaking “USD” Hamiltonians for the sd shell. *Phys. Rev. C* 101, 064312 (2020). <https://doi.org/10.1103/PhysRevC.101.064312>
- [31] P. Ba_łczyk, J. Dobaczewski, M. Konieczka, et al., Isospin-symmetry breaking in masses of $N = Z$ nuclei. *Phys. Lett. B* 778, 178–183 (2018). <https://doi.org/10.1016/j.physletb.2017.12.013>
- [32] R. D. O. Llewellyn, M. A. Bentley, R. Wadsworth, et al., Establishing the maximum collectivity in highly deformed $N = Z$ nuclei. *Phys. Rev. Lett.* 124, 152501 (2020). <https://doi.org/10.1103/PhysRevLett.124.152501>
- [33] J. G. Li, N. Michel, W. Zuo, et al., Resonances of $A = 4$, $T = 1$ isospin triplet states within the ab initio no-core Gamow shell model. *Phys. Rev. C* 104, 024319 (2021). <https://doi.org/10.1103/PhysRevC.104.024319>
- [34] M. S. Martin, S. R. Stroberg, J. D. Holt, et al., Testing isospin symmetry breaking in ab initio nuclear theory. *Phys. Rev. C* 104, 014324 (2021). <https://doi.org/10.1103/PhysRevC.104.014324>
- [35] E. Caurier, P. Navrátil, W. E. Ormand, et al., Ab initio shell model for $A = 10$ nuclei. *Phys. Rev. C* 66, 024314 (2002). <https://doi.org/10.1103/PhysRevC.66.024314>
- [36] J. G. Li, H. H. Li, S. Zhang, et al., Double-magicity of proton drip-line nucleus ^{22}Si with ab initio calculation. *Phys. Lett. B* 846, 138197 (2023). <https://doi.org/10.1016/j.physletb.2023.138197>
- [37] X. Y. Xu, S. Q. Fan, Q. Yuan, et al., Progress in ab initio in-medium similarity renormalization group and coupled-channel method with coupling to the continuum. *Nucl. Sci. Tech.* 35, 215 (2024). <https://doi.org/10.1007/s41365-024-01585-0>
- [38] S. Uthayakumaar, M. A. Bentley, E. C. Simpson, et al., Spectroscopy of the $T = 3/2$, $A = 47$ and $A = 45$ mirror nuclei via one- and two-nucleon knockout reactions. *Phys. Rev. C* 106, 024327 (2022). <https://doi.org/10.1103/PhysRevC.106.024327>
- [39] H. H. Li, J. G. Li, M. R. Xie, et al., Ab initio calculations of mirror energy difference in sd-shell nuclei. *Chin. Phys. C* 47, 124101 (2023). <https://doi.org/10.1088/1674-1137/acf035>
- [40] R. Id Betan, R. J. Liotta, N. Sandulescu, et al., Two-particle resonant states in a many-body mean field. *Phys. Rev. Lett.* 89, 042501 (2002). <https://doi.org/10.1103/PhysRevLett.89.042501>
- [41] N. Michel, W. Nazarewicz, M. Płoszajczak, et al., Gamow shell model description of neutron-rich nuclei. *Phys. Rev. Lett.* 89, 042502 (2002). <https://doi.org/10.1103/PhysRevLett.89.042502>
- [42] N. Michel, W. Nazarewicz, M. Płoszajczak, et al., Shell model in the complex energy plane. *J. Phys. G: Nucl. Part. Phys.* 36, 013101 (2009). <https://doi.org/10.1088/0954-3899/36/1/013101>

- [43] N. Michel, M. Płoszajczak, in *Gamow Shell Model, The Unified Theory of Nuclear Structure and Reactions* (Springer, Berlin Heidelberg, 2021), pp. 1-514. <https://link.springer.com/book/10.1007/978-3-030-69356-5>
- [44] J. G. Li, B. S. Hu, Q. Wu, et al., Neutron-rich calcium isotopes within realistic Gamow shell model calculations with continuum coupling. *Phys. Rev. C* 102, 034302 (2020). <https://doi.org/10.1103/PhysRevC.102.034302>
- [45] H. H. Li, J. G. Li, N. Michel, et al., Investigation of unbound hydrogen isotopes with the Gamow shell model. *Phys. Rev. C* 104, L061306 (2021). <https://doi.org/10.1103/PhysRevC.104.L061306>
- [46] J. G. Li, Y. Z. Ma, N. Michel, et al., Recent progress in Gamow shell model calculations of drip line nuclei. *Physics* 3, 977-997 (2021). <https://doi.org/10.3390/physics3040062>
- [47] J. G. Li, N. Michel, W. Zuo, et al., Unbound spectra of neutron-rich oxygen isotopes predicted by the Gamow shell model. *Phys. Rev. C* 103, 034305 (2021). <https://doi.org/10.1103/PhysRevC.103.034305>
- [48] S. Zhang, F. R. Xu, J. G. Li, et al., Ab initio descriptions of $A = 16$ mirror nuclei with resonance and continuum coupling. *Phys. Rev. C* 108, 064316 (2023). <https://doi.org/10.1103/PhysRevC.108.064316>
- [49] T. Berggren, On the use of resonant states in eigenfunction expansions of scattering and reaction amplitudes. *Nucl. Phys. A* 109, 265-287 (1968). [https://doi.org/10.1016/0375-9474\(68\)90149-0](https://doi.org/10.1016/0375-9474(68)90149-0)
- [50] N. Michel, H. M. Aktulga, Y. Jaganathen, Toward scalable many-body calculations for nuclear open quantum systems using the Gamow shell model. *Comput. Phys. Commun.* 247, 106978 (2020). <https://doi.org/10.1016/j.cpc.2019.106978>
- [51] Y. Suzuki, K. Ikeda, Cluster-orbital shell model and its application to the He isotopes. *Phys. Rev. C* 38, 410-413 (1988). <https://doi.org/10.1103/PhysRevC.38.410>
- [52] G. Papadimitriou, A. T. Kruppa, N. Michel, et al., Charge radii and neutron correlations in helium halo nuclei. *Phys. Rev. C* 84, 051304(R) (2011). <https://doi.org/10.1103/PhysRevC.84.051304>
- [53] Y. Jaganathen, R. M. Id Betan, N. Michel, et al., Quantified Gamow shell model interaction for psd-shell nuclei. *Phys. Rev. C* 96, 054316 (2017). <https://doi.org/10.1103/PhysRevC.96.054316>
- [54] H. W. Hammer, A. Nogga, A. Schwenk, Colloquium: Three-body forces: From cold atoms to nuclei. *Rev. Mod. Phys.* 85, 197-217 (2013). <https://doi.org/10.1103/RevModPhys.85.197>
- [55] H. W. Hammer, S. König, U. van Kolck, Nuclear effective field theory: Status and perspectives. *Rev. Mod. Phys.* 92, 025004 (2020). <https://doi.org/10.1103/RevModPhys.92.025004>
- [56] S. Binder, A. Ekström, G. Hagen, et al., Effective field theory in the harmonic oscillator basis. *Phys. Rev. C* 93, 044332 (2016). <https://doi.org/10.1103/PhysRevC.93.044332>
- [57] R. J. Furnstahl, G. Hagen, T. Papenbrock, Corrections to nuclear energies and radii in finite oscillator spaces. *Phys. Rev. C* 86, 031301(R) (2012). <https://doi.org/10.1103/PhysRevC.86.031301>
- [58] A. Bansal, S. Binder, A. Ekström, et al., Pion-less effective field theory for atomic nuclei and lattice nuclei. *Phys. Rev. C* 98, 054301 (2018).

<https://doi.org/10.1103/PhysRevC.98.054301>

[59] L. Huth, V. Durant, J. Simonis, et al., Shell-model interactions from chiral effective field theory. *Phys. Rev. C* 98, 044301 (2018).

<https://doi.org/10.1103/PhysRevC.98.044301>

[60] N. Michel, J. G. Li, F. R. Xu, et al., Description of proton-rich nuclei in the $A \approx 20$ region within the Gamow shell model. *Phys. Rev. C* 100, 064303 (2019). <https://doi.org/10.1103/PhysRevC.100.064303>

[61] <http://www.nndc.bnl.gov/ensdf>

[62] M. R. Xie, J. G. Li, N. Michel, et al., Investigation of spectroscopic factors of deeply-bound nucleons in drip-line nuclei with the Gamow shell model. *Phys. Lett. B* 839, 137800 (2023). <https://doi.org/10.1016/j.physletb.2023.137800>

[63] M. R. Xie, J. G. Li, N. Michel, et al., Spectroscopic factors of resonance states with the Gamow shell model. *Science China Physics, Mechanics & Astronomy* 67, 212011 (2024). <https://doi.org/10.1007/s11433-023-2227-5>

[64] N. Michel, J. G. Li, F. R. Xu, et al., Two-neutron halo structure of ^{31}F . *Phys. Rev. C* 101, 031301(R) (2020). <https://doi.org/10.1103/PhysRevC.101.031301>

[65] K. H. Li, N. Chen, J. G. Li, et al., Gamow shell model calculations for the Thomas-Ehrman shift in the new isotope ^{21}Al . *Phys. Rev. C* 111, 034327 (2025). <https://doi.org/10.1103/PhysRevC.111.034327>

[66] Y. M. Xing, Y. F. Luo, Y. H. Zhang, et al., $Z = 14$ magicity revealed by the mass of the proton dripline nucleus ^{22}Si . *Phys. Rev. Lett.* 135, 012501 (2025). <https://doi.org/10.1103/PhysRevLett.135.012501>

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv – Machine translation. Verify with original.