Neutron Magic Numbers in sd Shell from Nuclear Charge Radii within Relativistic Hartree Bogoliubov Model

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Date: 2025-10-23T10:57:13+00:00

Abstract

Charge radii are sensitive indicators for identifying nuclear structure phenomena throughout the whole nuclide chart. In particular, the shrinking trend of charge radii along a long isotopic chain is intimately associated with the shell quenching effect. In this work, the systematic evolutions of charge radii along the proton numbers $Z = 8$, 10, 12, 14, and 18 isotopes are investigated by using a relativistic Hartree Bogoliubov model. An ansatz regarding neutron-proton correlation around the Fermi surface is considered to describe the abnormal behavior of nuclear charge radii. Our results show that the neutron-proton pairing corrections around the Fermi surface lead to a sudden increase in the charge radii of these isotopic chains at $N = 8$, 20, and 28, reflecting the fact that this correction enhances the shell closure at $N = 8$, 20, and 28. The reproduction of the $N = 14$ charge radius in Mg isotopes is sensitive to the treatment of pairing correlations. The BCS approach overestimates the shell effect, while the Bogoliubov quasiparticle transformation method, which suggests stronger pairing near the proton Fermi surface, yields better agreement with experiment. The deviations between theoretical and experimental values of 25 even-even nuclei show that the neutron-proton pairing correction significantly improves charge radii calculated with meson-exchange interactions. However, it does not lead to improvement for those from density-dependent effective interactions.

Full Text

Preamble

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Charge radii serve as sensitive indicators for identifying nuclear structure phenomena throughout the entire nuclide chart. In particular, the shrinking trend of charge radii along long isotopic chains is intimately associated with shell quenching effects. In this work, we investigate the systematic evolution of charge radii along isotopic chains with proton numbers Z = 8, 10, 12, 14, and 18 using a relativistic Hartree-Bogoliubov model. An ansatz regarding neutron-proton correlations around the Fermi surface is employed to describe the anomalous behavior of nuclear charge radii. Our results demonstrate that neutron-proton pairing corrections around the Fermi surface lead to sudden increases in the charge radii of these isotopic chains at N = 8, 20, and 28, reflecting the fact that these corrections enhance shell closure at N = 8, 20, and 28. The reproduction of the N = 14 charge radius in Mg isotopes is sensitive to the treatment of pairing correlations. The BCS approach overestimates the shell effect, whereas the Bogoliubov quasiparticle transformation method, which suggests stronger pairing near the proton Fermi surface, yields better agreement with experiment. The deviations between theoretical and experimental values for 25 even-even nuclei show that the neutron-proton pairing correction significantly improves charge radii calculations with meson-exchange interactions, but does not lead to improvement for those from density-dependent effective interactions.

Introduction

Shell structure represents a distinctive feature of nuclear many-body systems, characterized by the existence of proton and neutron magic numbers. These nuclear magic numbers reveal the chemical stability and internal intrinsic structure of atomic nuclei, which is of profound significance for understanding nuclear physics, element formation, and practical applications [1–5]. Magic nuclei manifest through various phenomena, such as sudden increases in binding energy [6], high excitation energies, reduced E2 transition probabilities [7], localized abrupt changes in charge radius [8, 9] and proton radius [10, 11], and reduced neutron/proton capture cross sections [12] compared to neighboring nuclei. Modern radioactive beam facilities and detection systems have further extended studies to extreme nuclides, potentially revealing new magic numbers or revising traditional shell models [1, 13–15].

The systematic evolution of bulk properties from oxygen to argon isotopic chains carries rich information about shell structure, particularly regarding the disappearance of traditional magic numbers and the emergence of new ones. Along the oxygen isotopes, high excitation energies and low B(E2) values of the ${2^+}_1$ state [16, 17], small quadrupole transition parameters β_2 [18], reduced proton radii [11], large inclusive cross sections, and wide momentum distributions from quasifree (p, pN) scattering [12] provide strong evidence for the existence of the N = 14 subshell closure in ^{22}O. Additionally, a large shell gap is clearly identified for N = 16 in ^{24}O from various measurements [11, 17, 19–24], demonstrating that ^{24}O is a doubly magic nucleus. Measurements of masses, charge radii, and Coulomb excitation of neutron-rich Ne, Na, and Mg isotopes [9, 25–27] have suggested the breakdown of the traditional magic number N = 20 [28–31]. Higher in mass, the observation of a low-lying ${2^+}_1$ state in ^{42}Si provides transparent evidence for the collapse of the N = 28 shell closure [32].

A variety of nuclear structure models have been employed to unveil the underlying mechanisms governing the emergence of new magicity and shell quenching phenomena. Shell model results suggest that the appearance of new magic numbers N = 14 and N = 16 is attributed to strong neutron-proton tensor interactions [10, 11, 21, 33]. Ab initio calculations with novel versions of two- and three-nucleon forces lead to considerable improvement in simultaneously describing binding energies, charge and matter radii for stable O isotopes, though deficiencies are encountered for the most neutron-rich systems [34]. The collapse of the N = 20 shell closure is attributed to population of the neutron pf shell in the presence of sd orbitals at considerable prolate deformation, a phenomenon known as the island of inversion [35, 36].

The coupled-cluster method based on nucleon-nucleon and three-nucleon potentials qualitatively reproduces evolutionary trends in charge radii and the neutron magic number N = 14 in Ne and Mg isotopes after accounting for angular momentum projection, though isotope shifts remain challenging [37]. In mean-field theories, the ground state of ^{32}Mg is spherical and becomes deformed only after a 20% increase in spin-orbit strength based on the Skyrme SLy4 force [38]; a deformed ground state can be achieved by adjusting neutron and proton pairing gaps, but the magic number N = 20 persists for the ground state of ^{32}Mg in relativistic mean-field (RMF) theory [39]. Angular momentum projection approaches based on the HFB model [40, 41] and RMF model [42] transform the spherical mean-field ground state of ^{32}Mg into a deformed state with β_2 close to the measured value. A similar picture is obtained from the projected shell model [43].

As mentioned above, the shrinking trend of charge radii along an isotopic chain serves as a signature for identifying shell closure effects. Charge radii are influenced by various mechanisms, including pairing correlation [44–48], deformation [49, 50], cluster structure [51, 52], shell evolution [53–55], and center-of-mass correlation [56–58]. The modified RMF plus BCS equation ansatz

(RMF(BCS)* model), which incorporates neutron-proton pairing correlations around the Fermi surface into the charge radius formula [59, 60], successfully describes odd-even staggering and inverted parabolic-like behavior in Ca isotopes. This method also provides a good description of charge radii for most O, Ne, and Mg isotopes. However, it predicts odd-even staggering in the charge radii of the O isotopic chain with a sudden increase at N = 14 and underestimates charge radii for N = 14 and N > 18 nuclei in Mg isotopes.

Since the BCS approximation for pairing correlation is not suitable for nuclei far from the β-stability line [61, 62], and the magic numbers 14, 16, and 20 in the sd shell appear in neutron-rich regions and even at the drip line, it is necessary to further study charge radii using the same ansatz but treating pairing correlation with a more appropriate method.

The Bogoliubov transformation provides a more reliable treatment of pairing correlations than the BCS method for unstable nuclei far from the β-stability line. Based on this, we recently introduced neutron-proton pairing correlation extracted from quasiparticle states around the Fermi surface into the multidimensionally-constrained relativistic Hartree-Bogoliubov (MDC-RHB) model [63]. This approach successfully reproduces the charge radii of Ca and Ni isotopes and N = 28, 30, 32, and 34 isotones.

In this work, we revisit the problems encountered in the RMF(BCS)* approach and examine the shell closure effect around neutron numbers N = 14, 16, and 20 from the perspective of nuclear charge radii, elucidating the relationship between shell structure in these isotopes and neutron-proton pairing.

This paper is organized as follows. In Sec. II, we briefly introduce the MDC-RHB model and the neutron-proton pairing correlation extracted from quasiparticle states around the Fermi surface. In Sec. III, we investigate charge radii for Z = 8, 10, 12, 14, and 18 isotopes using this method and discuss shell structure phenomena around N = 14, 16, and 20. A summary is presented in Sec. IV.

II. Theoretical Framework

In the MDC-RHB model, the RHB equation in coordinate space can be written as follows [64, 65]:

$$\begin{pmatrix} h-\lambda & -\Delta^* \\ \Delta & -h+\lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

where λ is the Fermi energy, Δ is the pairing field, E_k is the quasiparticle energy, and $(U_k(r), V_k(r))^T$ is the quasiparticle wave function. The single-particle Hamiltonian h can be expressed as:

$$h = \alpha \cdot p + \beta[M + S(r)] + V(r) + \Sigma_R(r)$$

where M is the nucleon mass, and $S(r)$, $V(r)$, and $\Sigma_R(r)$ are the scalar, vector, and rearrangement potentials, respectively.

For meson-exchange interactions:

$$\begin{aligned}
S(r) &= g_\sigma \sigma, \\
V(r) &= g_\omega \omega^0 + g_\rho \rho^0 \cdot \tau_3 + e\frac{1-\tau_3}{2}A^0, \\
\Sigma_R(r) &= \frac{\partial g_\sigma}{\partial \rho_V}\rho_S \sigma + \frac{\partial g_\omega}{\partial \rho_V}\rho_V \omega^0 + \frac{\partial g_\rho}{\partial \rho_V}\rho_{V\tau_3}\rho^0,
\end{aligned}$$

where g_σ, g_ω, and g_ρ are coupling constants of σ, ω^0, and ρ^0 meson fields, A^0 is the time-like component of the Coulomb field mediated by photons, and e is the charge unit for protons. The quantities $\tau_3 = 1$ and -1 distinguish neutron and proton components.

For point-coupling interactions:

$$\begin{aligned}
S(r) &= \alpha_S \rho_S + \alpha_{TS}\rho_{TS}\tau_3 + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta\rho_S + \delta_{TS}\Delta\rho_{TS}\tau_3, \\
V(r) &= \alpha_V \rho_V + \alpha_{TV}\rho_{TV}\tau_3 + \gamma_V \rho_V^3 + \delta_V \Delta\rho_V + \delta_{TV}\Delta\rho_{TV}\tau_3 + e\frac{1-\tau_3}{2}A^0, \\
\Sigma_R(r) &= \frac{\partial \alpha_V}{\partial \rho_V}\rho_V^2 + \frac{\partial \alpha_{TV}}{\partial \rho_V}\rho_{TV}^2\tau_3,
\end{aligned}$$

where α_S, α_V, α_{TS}, α_{TV}, β_S, γ_S, γ_V, δ_S, δ_V, δ_{TS}, and δ_{TV} are coupling constants for different channels. The subscripts S, V, and T indicate the symmetries of the couplings: S stands for scalar, V for vector, and T for isovector.

The densities are defined as:

$$\begin{aligned}
\rho_S &= \sum_k V_k^\dagger(r)\gamma^0 V_k(r), \\
\rho_V &= \sum_k V_k^\dagger(r) V_k(r), \\
\rho_{TS} &= \sum_k V_k^\dagger(r)\tau_3 \gamma^0 V_k(r), \\
\rho_{TV} &= \sum_k V_k^\dagger(r)\tau_3 V_k(r),
\end{aligned}$$

representing isoscalar density, time-like components of isoscalar current, isovector density, and time-like components of isovector current, respectively.

The pairing field Δ is calculated from the effective pairing interaction V and the pairing tensor κ as:

$$\Delta(r_1\sigma_1, r_2\sigma_2) = \int d^3r_1' d^3r_2' V(r_1\sigma_1, r_2\sigma_2, r_1', r_2')\kappa(r_1', r_2')$$

where the pairing tensor is:

$$\kappa(r_1\sigma_1, r_2\sigma_2) = \sum_k V_k^*(r_1\sigma_1)U_k(r_2\sigma_2).$$

In this work, we adopt a separable pairing force of finite range. The effective pairing interaction is:

$$V(r_1\sigma_1, r_2\sigma_2, r_1', r_2') = -G\delta(R - R')P(r)P(r')(1 - P_\sigma),$$

where G is the pairing strength, $R = (r_1 + r_2)/2$ and $r = r_1 - r_2$ are the center-of-mass and relative coordinates, respectively. The function $P(r)$ represents a Gaussian distribution:

$$P(r) = (4\pi a^2)^{-3/2} e^{-r^2/4a^2}.$$

We use a pairing strength $G = 728$ MeV $\cdot$ fm^3 and an effective range of the pairing force $a = 0.644$ fm [66].

The modified root-mean-square (rms) charge radius r_{ch} is given by [67]:

$$r_{\text{ch}}^2 = \langle r_p^2 \rangle + 0.7056 \text{ fm}^2 + \Delta_D \text{ fm}^2 + \frac{a_0}{A^\delta},$$

where the first term represents the charge distribution of point-like protons, the second term accounts for the finite size of protons, and the third term represents short-range correlations from the difference in Cooper pair fractions between neutrons and protons [59]. A is the mass number and a_0 is a normalization constant. The term $\Delta_D = |D_n - D_p|$ is defined as:

$$D_{n,p} = \sum_k u_{n,p}^k v_{n,p}^k,$$

where $u_{n,p}^k$ and $v_{n,p}^k$ are the occupation amplitudes of the kth quasiparticle orbital for neutrons or protons. D_n (D_p) measures the surface diffuseness encoded by the neutron (proton) eigenfunctions [68]. In practice, quasiparticle levels satisfying $|E_k - \lambda| < 20$ MeV are included in the sum. The last term in Eq. (9) represents neutron-proton correlation arising from simultaneously unpaired neutrons and protons. The parameters $a_0 = 0.561$ and $\delta = 0.355$ (0.000) for odd-odd (even-even, odd-even, and even-odd) nuclei are taken from Ref. [67].

In the MDC-RHB model, multipole moments can be included under V4 symmetry [69, 70]. In this work, we restrict calculations to axial and reflection symmetries, meaning only quadrupole deformation β_{20} is considered. To avoid parameter-dependent results, we employ the effective interactions PK1 [56], NL3 [71], DD-ME2 [72], and DD-PC1 [73].

[Figure 1: see original paper] (Color online) Charge radii of O, Ne, Mg, Si, and Ar isotopes calculated by the RHB (left) and RHB* (right) models with DD-ME2, NL3, DD-PC1, and PK1 effective interactions. Experimental data are taken from Refs. [8, 27, 74, 75] (solid squares). Gray bands indicate neutron numbers N = 14 and 20.

III. Results and Discussion

As mentioned in the Introduction, N = 14 and 16 shell quenching effects are observed in O, N, Ne, and Mg isotopes. To investigate these characteristic magicities, we calculate the systematic evolution of charge radii along O, Ne, Mg, Si, and Ar isotopic chains using the MDC-RHB model. Results obtained with and without neutron-proton correlations around the Fermi surface (the third and fourth terms in Eq. (9)) are denoted as RHB* and RHB models, respectively.

The charge radii along O, Ne, Mg, Si, and Ar isotopic chains are depicted in Fig. 1 using PK1, DD-ME2, NL3, and DD-PC1 effective interactions. RHB calculations yield charge radii for $^{16-22}$O of the same order of magnitude, as shown in Fig. 1(a). Among these, the PK1 force provides the most accurate estimate for ^{16}O. However, all four forces underestimate the charge radius of ^{18}O. In Fig. 1(b), the correction term increases the charge radius of ^{18}O, thereby improving the description provided by the PK1 and NL3 parameter sets. The double-magic nature of ^{16}O makes it largely free of surface correlations, maintaining its charge radius close to the original value. The systematic trend of charge radius changes is suppressed at N = 8 and 16 in the RHB* model, implying that shell quenching at N = 8 and 16 is enhanced after implementing corrections around the Fermi surface.

Notably, our calculations do not reproduce the odd-even staggering in O isotopes or the sudden increase at N = 14 predicted by the RMF(BCS)* model [59]. The charge radius of ^{22}O does not decrease relative to ^{20}O in our calculations, whereas Ref. [11] reported significantly smaller proton radii for both ^{22}O and ^{24}O. We attribute the kink at N = 16 to the third term in Eq. (9).

To illustrate this, we plot the single-particle levels of the ground states of $^{22-26}$O as a function of occupation probability v^2 in Fig. 2 [Figure 2: see original paper]. For O isotopes, the proton magic number Z = 8 implies a fully occupied proton shell, resulting in $D_p = 0$. For neutrons in $^{22-26}$O, pairing correlations cause broadening of single-particle levels around the Fermi surface, enabling nucleon occupation of states above it. We depict the $u_k v_k$ values as olive solid lines alongside occupation numbers for each single-particle level. Larger $u_k v_k$ values are associated with partially occupied levels near the Fermi surface. The calculated D_n and D_p values from these $u_k v_k$ (see Table 1) show that D_n is largest in ^{26}O and relatively small in ^{24}O, consistent with the kink at N = 16. Thus, the N = 16 magic character is reinforced by enhanced pairing correlations among neutrons at N = 18. Similarly, the D_n value for ^{18}O increases suddenly from that of ^{16}O, leading to the kink at N = 8 in this isotopic chain. New data from planned laser spectroscopy experiments by the COLLAPS and CRIS collaborations [13] are expected to measure charge radii of neutron-rich and proton-rich O isotopes, providing tests of our predictions.

[Figure 2: see original paper] (Color online) Single-particle levels of the ground state of $^{22-26}$O as a function of occupation probability v^2 calculated with the RHB model using the NL3 effective interaction. Black dashed lines indicate Fermi surfaces. Blue dashed-dotted lines correspond to the BCS formula with an average pairing gap. Olive solid lines below each single-particle level show corresponding $v_k u_k$ values.

The D_n, D_p, and Δ_D values in Eq. (10) for O and Ne isotopes calculated with the NL3 effective interaction.

The charge radii for Ne isotopes are shown in Figs. 1(c) and 1(d). RHB results agree with data from Ref. [8]. Specifically, density-dependent effective interactions DD-ME2 and DD-PC1 yield results closer to experimental values, whereas meson-exchange interactions NL3 and PK1 slightly underestimate them. Furthermore, kinks are observed at N = 8 and N = 14. After including pairing corrections, charge radii increase and those calculated with meson-exchange effective interactions align better with experimental results [8]. For comparison, revised charge radii from Ref. [74] are included as grey squares. RHB* model predictions for ^{26,28}Ne using DD-ME2 and DD-PC1 effective interactions agree with these revised data. For both RHB and RHB* models, the charge radius is smallest at N = 14 along the isotopic chain calculated with DD-ME2, NL3, and PK1 effective interactions. However, DD-PC1 yields the smallest value at N = 16. With increasing neutron number, the RHB model predicts no abrupt change in charge radii across N = 20. In contrast, the RHB* model anticipates that proton-neutron pairing results in a sudden increase in the charge radius of ^{32}Ne, thereby manifesting the magicity of N = 20. We list D_n and D_p values for Ne isotopes in Table 1. Our results show that N = 20 shell quenching can be attributed to enhanced pairing correlations around the neutron Fermi surface in ^{32}Ne.

For the Mg isotopic chain, as illustrated in Fig. 1(e), RHB calculations reproduce experimental charge radii for proton-rich isotopes but underestimate those for neutron-rich isotopes. Kinks are observed at N = 14 and N = 20. In Fig. 1(f), surface correlation between protons and neutrons leads to increased charge radii, particularly for isotopes with neutron numbers greater than 14. Consequently, charge radii calculated with the RHB* model agree better with data than those from the RHB model. Therefore, on the neutron-rich side of Mg isotopes, this neutron-proton pairing surface vibration may serve as the underlying mechanism responsible for observed changes in charge radii.

[Figure 3: see original paper] (Color online) Charge radii for Mg isotopes with (a) BCS and (b) Bogoliubov quasiparticle transformation treatment. Results are calculated with the NL3 effective interaction based on the same single-particle Hamiltonian. Experimental data are taken from Ref. [27] (solid squares). Gray bands indicate neutron numbers N = 14 and 20.

In RMF(BCS)* calculations [59], the same correction is employed for charge radii, but the BCS approach is used for the pairing channel. The resulting depression of the charge radius at N = 14 is too large. In contrast, the charge radius obtained by the RHB* model for ^{26}Mg (N = 14) in this work using NL3 and PK1 effective interactions are 3.0434 fm and 3.0415 fm, respectively, very close to the experimental value of 3.0340(26) fm [27]. This finding indicates that characterization of N = 14 magicity exerts significant influence on the treatment of pairing correlations.

Since both the pairing correction treatment and pairing force form differ between Ref. [59] and this work, it is necessary to investigate the pairing force within the same mean-field Hamiltonian and analyze the microscopic mechanism behind the N = 14 shell closure. Therefore, we calculate charge radii of Mg isotopes by switching the pairing correction from the Bogoliubov quasiparticle transformation to the BCS approximation. Fig. 3 shows the results. As illustrated in Fig. 3(a), the charge radius determined by BCS pairing correlation exhibits a pronounced kink at ^{26}Mg (N = 14), both before and after neutron-proton surface correction. This observation aligns with literature findings [59], suggesting strong magicity for N = 14. In Fig. 3(b), application of the Bogoliubov quasiparticle transformation to treat pairing correlation reveals that charge radii of $^{24-29}$Mg agree well with data after including the additional neutron-proton correction. Thus, this correction facilitates reproduction of the charge radius at neutron number N = 14.

[Figure 4: see original paper] (Color online) Single-particle levels of the ground state of ^{26}Mg as a function of occupation probability v^2 calculated with BCS and Bogoliubov quasiparticle transformation treatments for pairing correction. The NL3 effective interaction is adopted. Black dashed lines indicate Fermi surfaces. Blue dashed-dotted lines correspond to the BCS formula with an average pairing gap.

In Fig. 4, we analyze the microscopic mechanism of charge radius depression at N = 14 from the single-particle levels of ^{26}Mg. Figures 4(a) and 4(b) show neutron and proton single-particle energy levels calculated using the BCS method. These figures clearly show that neutrons and protons predominantly occupy single-particle states below the Fermi surface, with nearly zero occupation above it. This indicates that pairing gaps for both N = 14 and Z = 12 are relatively small, suggesting greater nuclear stability associated with a smaller charge radius. According to Eq. (9), neutron-proton pairing correlation around the Fermi surface is determined by Δ_D, which in turn depends on non-integer occupation numbers of protons and neutrons. BCS calculations indicate predominantly integer occupation, resulting in small Δ_D. Single-particle energy levels for neutrons obtained with the RHB model (see Fig. 4(c)) are consistent with those from the BCS method. However, fractional occupations in several single-particle energy levels appear around the proton Fermi surface in RHB calculations (Fig. 4(d)). Consequently, the charge radius of ^{26}Mg calculated by the RHB* model is substantially larger than that from the RHB model, with this modified result being more consistent with experimental data.

The observed absence of N = 20 shell closure in Mg isotopes is typically attributed to the island of inversion. Energy level inversion results in substantial deformation in the ground state of ^{32}Mg, leading to dissolution of the N = 20 shell closure. However, the RHB model fails to reproduce this phenomenon, as all four parameter sets yield spherical ground states. Beyond-mean-field calculations that produce reasonably large deformation in the ground states of ^{32}Mg [40–42] can address the observed discrepancy between calculated and measured charge radii. In the present study, we demonstrate that RHB* calculations also yield more accurate charge radii for ^{32}Mg compared to RHB calculations, particularly when using density-dependent effective interactions DD-PC1 and DD-ME2. Beyond-mean-field calculations based on the present framework can be found in Refs. [76, 77], and further study in this direction is needed. Additionally, we show charge radii of ^{19,20}Mg in Fig. 3. A sudden change is observed, revealing the persistence of N = 8 shell closure near the proton drip line, consistent with recent invariant-mass reconstruction [78].

Charge radii of Si isotopes are displayed in Figs. 1(g) and 1(h). The charge radius of ^{28}Si (N = 14) calculated by the RHB model is larger than that of ^{30}Si (N = 16), opposite to experimental values. Including neutron-proton correlation around the Fermi surface brings the charge radii of these two nuclei into agreement with data. Among these isotopes, theoretical values from PK1 and NL3 effective interactions are comparable to experimental values, including the new value for ^{32}Si determined by collinear laser spectroscopy [75]. For N = 20 and N = 28, kinks in charge radii become more pronounced after considering the correction term, indicating enhanced shell closures at N = 20 and 28.

Figures 1(i) and 1(j) show results for Ar isotopes. Charge radii calculated by the RHB model exhibit a parabolic form as a function of neutron number, showing larger deviations from data. Charge radii calculated by the RHB* model basically agree with experimental results on the neutron-rich side, except for N = 22. On the proton-rich side, the charge radius of the N = 14 isotope deviates significantly from the experimental value after including the correction term.

To facilitate understanding of N = 14 magicity, we present in Fig. 5 [Figure 5: see original paper] the charge radius difference between adjacent even-even nuclei, defined as $\Delta r_{\rm ch}(Z,N) = r_{\rm ch}(Z,N) - r_{\rm ch}(Z-2,N)$, along N = 14 isotones. Calculations with NL3 and PK1 forces are essentially identical.

As shown by the green line in the figure, the empirical formula $1.2A^{1/3}$ fm depends only on mass number and cannot reveal shell structure. RHB calculations show very small $\Delta r_{\rm ch}(Z)$ in charge radius at Z = 8, implying that N = 14 magicity is very strong in ^{22}O, consistent with proton radius measurements [11]. After considering neutron-proton pairing corrections, the N = 14 shell closure in ^{22}O is weakened while that in ^{28}Si is enhanced. Future measurements of weakly bound nuclei such as ^{20}C, ^{22}O, and ^{32}Ar, though challenging, will be crucial for understanding the N = 14 shell closure.

[Figure 5: see original paper] (Color online) Charge radius difference between adjacent even-even nuclei ($\Delta r_{\rm ch}$) as a function of proton number for N = 14 isotones calculated with (a) NL3 and (b) PK1 effective interactions. Experimental data are taken from Refs. [8, 27] (solid squares). Empirical values with $r_{\rm ch} = r_0 A^{1/3}$ ($r_0 = 1.2$ fm) are shown as guidelines.

Finally, the average deviation $\bar{\chi}^2$ and root-mean-square deviation $\Delta_{\rm rms}$, defined as:

$$\bar{\chi}^2 = \frac{1}{N}\sum_i \left(\frac{r^{\rm exp}_{{\rm ch},i} - r^{\rm cal}_{{\rm ch},i}}{\Delta r^{\rm exp}_{{\rm ch},i}}\right)^2,$$

$$\Delta_{\rm rms} = \sqrt{\frac{1}{N}\sum_i \left(r^{\rm exp}_{{\rm ch},i} - r^{\rm cal}_{{\rm ch},i}\right)^2},$$

between 25 experimental and calculated charge radii are listed in Table 2 . Experimental charge radii for Mg and Ne isotopes and ^{32}Si are adopted from Refs. [27], [74], and [75], respectively; all other data are taken from Ref. [8]. The $\Delta r^{\rm exp}_{{\rm ch},i}$ in Eq. (12) is the experimental uncertainty of the corresponding measured charge radius $r^{\rm exp}_{{\rm ch},i}$ for the ith nucleus. For calculations with DD-ME2, NL3, and PK1 parameter sets, the $\bar{\chi}^2$ values of the RHB* model are smaller than those of the RHB model. Conversely, the result calculated using the DD-PC1 effective interaction shows the opposite trend. The calculated $\Delta_{\rm rms}$ for meson-exchange effective interactions NL3 and PK1 decrease after correction, while those for density-dependent effective interactions DD-ME2 and DD-PC1 increase slightly after correction. These results indicate the necessity of incorporating neutron-proton pairing corrections around the Fermi surface in mean-field calculations using meson-exchange effective interactions. Density-dependent effective interactions do not require this correction, as RHB calculations already yield larger charge radii compared to meson-exchange effective interactions.

The average deviation $\bar{\chi}^2$ and root-mean-square deviation $\Delta_{\rm rms}$ between experimental and calculated charge radii of 25 even-even nuclei from O to Ar isotopes.

IV. Summary

In this work, we systematically investigate the evolution of charge radii across oxygen to argon isotopic chains using the RHB framework. A refined charge radius formula incorporating neutron-proton pairing correlations near the Fermi surface is employed to describe anomalous behavior. We analyze the emergence of new magic numbers at N = 14 and 16, and the quenching of the traditional N = 20 shell closure.

Notably, inclusion of neutron-proton pairing correlations drives pronounced kinks in charge radii at N = 8, 20, and 28, revealing their role in enhancing shell closure effects. The Mg isotopic chain provides critical insights into the interplay between pairing treatments and shell structure. While conventional BCS theory artificially amplifies the N = 14 shell effect, the Bogoliubov transformation approach predicts enhanced proton pairing correlations, yielding results in closer agreement with experimental charge radii. Further validation against experimental data for 25 even-even nuclei confirms that neutron-proton pairing correction significantly improves the description of charge radii when using meson-exchange effective interactions, though it diminishes the agreement for density-dependent interactions. Extending this framework to heavier isotopic chains is in progress. Additionally, the interaction-dependent performance of the neutron-proton pairing correction calls for a unified theoretical approach to reconcile meson-exchange and density-dependent interactions.

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