# Effect of Tensor Force on Deformation of Octupole Nuclei in the Skyrme-Hartree-Fock Approach

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## Abstract

The influence of the tensor force on reflection-asymmetric deformation of Ba isotopes is studied within the Skyrme-Hartree-Fock framework. For $^{144}$Ba, the noncentral tensor couplings ($\alpha_T$, $\beta_T$) enhance octupole deformation by strengthening the coupling between the $\Delta N = 1$, $\Delta l = \Delta j = 3$ orbital pairs located on opposite sides of the Fermi surface, whereas the central terms ($\alpha_C$, $\beta_C$) weaken it. The nucleon localization function reveals pronounced surface clustering ($\mathcal{C} \approx 0.9$), with tensor-induced variations consistent with changes in the octupole parameter $\beta_3$. These results demonstrate that tensor interactions play a key role in shaping octupole couplings and localization, offering microscopic insight into the interplay among single-particle structure, tensor-force components, and reflection-asymmetric degrees of freedom in medium-heavy nuclei.

## Full Text

## Preamble

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central tensor couplings ($\alpha$_T, $\beta$_T) enhance octupole deformation by strengthening the coupling between the $\Delta N = 1$, $\Delta l = \Delta j = 3$ orbital pairs located on opposite sides of the Fermi surface, whereas the central terms ($\alpha$_C, $\beta$_C) weaken it. The nucleon localization function reveals pronounced surface clustering (C 0.9), with tensor-induced variations consistent with changes in the octupole parameter $\beta_3$. These results demonstrate that tensor interactions play a key role in shaping octupole couplings and localization, offering microscopic insight into the interplay among single-particle structure, tensor-force components, and reflection-asymmetric degrees of freedom in medium-heavy nuclei.

## Introduction

The study of equilibrium shapes and shape transitions has long been a central topic in nuclear structure physics [1, 2]. The equilibrium shape of a nucleus results from the delicate interplay between single-particle shell effects and effective nucleon–nucleon interactions [3]. While most medium-heavy and heavy nuclei exhibit quadrupole deformations with axial and reflection symmetry, there are localized regions in the nuclear chart where reflection asymmetry develops due to strong octupole correlations [4], such as $^{228}$Th [5], $^{224}$Ra, $^{220}$Rn [6], $^{146}$Ba [7], $^{144}$Ba [8], and $^{96}$Zr [9]. These correlations are particularly enhanced when valence nucleons occupy pairs of orbitals with opposite parity and quantum numbers differing by $\Delta l = \Delta j = 3$ near the Fermi surface [10, 11], for which the octupole ($Y_3{}^0$) matrix elements reach their maximum. In such cases, moderate changes in the single-particle level spacings can alter the stability of reflection-asymmetric shapes.

Prototypical examples of this mechanism are the neutron (proton) orbital pairs ( $1i_{13}/_2$, $2f_7/_2$) and ($ 1h{11}/{2}$, $ 2d{5}/{2}$), which are present in regions around (N 88, Z 56). The strong repulsion between these $\Delta l = \Delta j = 3$ partners across the Fermi energy can generate octupole-deformed minima in the potential energy surface, providing a microscopic origin for static reflection asymmetry [10, 12]. This sensitivity of octupole correlations to the single-particle structure naturally raises the question of how specific components of the effective interaction, such as the tensor force, might alter these orbital spacings and hence the stability of octupole-deformed minima.

Recent experimental advances have brought this question into sharper focus. In the actinide region, Coulomb excitation experiments on $^{224}$Ra and $^{220}$Rn have revealed extremely large electric octupole transition strengths, directly signaling stable pear-shaped configurations [6]. In the rare-earth region, precision lifetime and $\gamma$-ray spectroscopy of neutron-rich $^{144}$,$^{146}$Ba have demonstrated enhanced B(E3) values consistent with strong octupole collectivity [7, 8], in agreement with earlier theoretical predictions [11, 12]. Among these, $^{144}$Ba stands out with one of the largest measured octupole deformation parameters ($\beta_3$ $0.17^{+4}{}_6$) [8], making it an ideal candidate for probing the microscopic drivers of reflection-asymmetric shapes.

Theoretically, octupole deformation has been explored with self-consistent mean-field calculations based on nuclear energy density functionals (EDFs) [12–15], macroscopic–microscopic models [16, 17], geometrical collective models [18, 19], the interacting boson model [20, 21], and the interacting boson model combined with nuclear energy density functionals [22–25]. While these frameworks have mapped octupole energy surfaces and explained spectroscopic observables, the explicit role of the tensor force in stabilizing or weakening octupole minima has been discussed far less systematically.

The tensor force can modify the spin–orbit potential and shift the relative energies of spin–orbit partners j_> and j_<. Such modifications can in turn affect a wide range of nuclear properties, including both ground-state and excited-state structures [26–29]. In the Skyrme EDF, its isoscalar ($\alpha$_T) and isovector ($\beta$_T) couplings govern the tensor interaction between like nucleons and between neutrons and protons, respectively. Because octupole correlation is controlled by the spacing of $\Delta l = \Delta j = 3$ orbitals near the Fermi surface, tensor-induced shifts in single-particle energies may directly influence reflection-asymmetric stability. Recent relativistic Hartree–Fock (RHF) studies, including a dedicated analysis for $^{144}$Ba [30], have shown that tensor terms may significantly modify octupole minima. This motivates a systematic investigation with independent variation of $\alpha$_T and $\beta$_T in the Deformed Skyrme–Hartree–Fock (DSHF) framework, although the direction and magnitude of the effect depend on the interplay between $\alpha$_T and $\beta$_T that needs to be studied carefully.

In addition to their known impact on shell evolution, spin–orbit splittings, and magnetic/charge-exchange modes [31–33], tensor terms can also reshape the spatial localization of nucleons. Within SHF, the non-central tensor component enters the spin–orbit potential via the spin-current density $\mathbf{J}$_q($\mathbf{r}$), where q denotes the nucleon isospin. By shifting spin–orbit partners and altering occupation patterns near the Fermi surface, the tensor force can reorganize the spatial distribution of nucleons with specific spin and isospin. The nucleon localization function (NLF) [34, 35], which is sensitive to both spatial and spin–isospin degrees of freedom, is therefore a powerful microscopic probe for tensor-induced structure changes. In octupole-deformed nuclei, where $\Delta l = \Delta j = 3$ pairs dominate the shape-driving mechanism, NLF analysis can reveal how tensor-driven single-particle rearrangements couple to reflection-asymmetric correlations.

In this work, we use the DSHF+BCS (Bardeen-Cooper-Schrieffer) approach to investigate the tensor-force effects on octupole deformation and the associated localization structures. We focus on the microscopic pathway by which $\alpha$_T and $\beta$_T independently modify $\Delta l = \Delta j = 3$ orbital spacings and thereby tune octupole stability. Furthermore, we employ the NLF to quantify tensor-induced changes in the spatial localization patterns in $^{144}$Ba as a representative case. This combined analysis provides a unified mean-field perspective on the interplay between tensor interactions, single-particle structure, and reflection-asymmetric shapes.

This paper is organized as follows. Section II briefly reviews the formalism of

the SHF model, the nuclear localized functions (NLF), and tensor force in the Skyrme functional; results and discussions are presented in Section III; finally, summary and outlook of our work are provided in Section IV.

## II. Theoretical Framework

In the DSHF approach, the total energy of a nucleus is given by [36, 37]

$$E = \int d^3\mathbf{r}\, \varepsilon(\mathbf{r}),$$

where the energy-density functional is

$$\varepsilon = \varepsilon_N[\rho_n, \rho_p, \tau_n, \tau_p, \mathbf{J}_n, \mathbf{J}_p].$$

For the nucleonic functional _N, we use the standard Skyrme forces SLy5 [38], SAMi [39], and SAMi-T [40]. The one-body density _q, kinetic density $\tau$_q, and spin-current density $\mathbf{J}$_q read

$$\langle \rho_q, \tau_q, \mathbf{J}_q \rangle = \sum_k \langle \phi_k^q | \hat{O} | \phi_k^q \rangle,$$

with the operator $\mathbf{O}$ representing 1, - $^2$, or ( $\times\ \sigma$)/i, respectively. The SHF Schrödinger equation for the single-particle wavefunctions $\phi$_k^q is derived by varying the total energy functional:

$$\left[ -\frac{\hbar^2}{2m}\nabla^2 + V_q(\mathbf{r}) - i\mathbf{W}_q(\mathbf{r}) \cdot (\nabla \times\ ) \right] \phi_k^q(\mathbf{r}) = \epsilon_k \phi_k^q(\mathbf{r}),$$

where V_q($\mathbf{r}$) is the central part of the mean field depending on the densities, while $\mathbf{W}$_q($\mathbf{r}$) is the spin–orbit interaction part [36, 37].

Based on the nucleonic mean field, pairing forces (between like nucleons only) are taken into account using the BCS approximation with a density-dependent $\delta$ force [41, 42]

$$V_q(\mathbf{r}_1, \mathbf{r}_2) = V_0' \left( 1 - \frac{\rho_N((\mathbf{r}_1 + \mathbf{r}_2)/2)}{0.16\ \text{fm}^{-3}} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2),$$

where pairing strengths V_0' = -1146 MeV $\cdot$ fm$^3$ for protons and V_0' = -999 MeV $\cdot$ fm$^3$ for neutrons are used for medium-mass and heavy nuclei [43]. A smooth energy cutoff is employed in the BCS calculations [44].

In the present calculations, the deformed SHF Schrödinger equation is solved in cylindrical coordinates (r, z), under the assumption of axial symmetry of

the mean field. The geometric deformation parameters of the nuclear core are expressed as

$$\beta_{\lambda 0} \equiv \frac{4\pi}{3AR_0^{\lambda}} Q_{\lambda 0}, \quad R_0 = 1.2 \text{ fm},$$

with multipole moments

$$Q_{20} = \sqrt{\frac{5}{16\pi}} \int d^{3r}\, \rho(\mathbf{r})(2z^2 - r^2),$$

$$Q_{30} = \sqrt{\frac{7}{16\pi}} \int d^{3r}\, \rho(\mathbf{r}) z(2z^2 - 3r^2).$$

### A. Nuclear Localization Function

The localization measure was originally introduced in atomic and molecular physics to characterize chemical bonds in electronic systems [34] and was subsequently introduced to nuclear systems [35, 45–47]. The localization function is related to the spatial two-body correlation between two like-spin fermions of the same kind and can be applied to different fermion systems, including nuclei and hypernuclei [45, 46, 48, 49]. It can be written as the probability of finding the second particle located within a shell of small radius around the assumed particle at $\mathbf{r}$ with the same spin $\sigma$ ($\uparrow$ and $\downarrow$) and isospin q (= n, p) [34, 35]:

$$C_{q\sigma}(\mathbf{r}) = \frac{\rho_{q\sigma}(\mathbf{r})\tau_{q\sigma}(\mathbf{r}) - \mathbf{j}^2_{q\sigma}(\mathbf{r}) - \frac{1}{4}[\nabla\rho_{q\sigma}(\mathbf{r})]^2}{\rho_{q\sigma}(\mathbf{r})\tau^{\mathrm{TF}}_{q\sigma}(\mathbf{r})},$$

where *$\phi k\hat{}\{q\sigma\}$ ($k$ = 1, ⋯, N_q) are the single-particle (s.p.) wavefunctions of the k-th occupied states for the different particles q = n, p. The Thomas–Fermi kinetic energy density is* $\tau\{\mathrm{q}\sigma\}\hat{}\{\mathrm{TF}\} = (6\pi^2)\hat{}\{2/3\}$ _$\{\mathrm{q}\sigma\}\hat{}\{5/3\}$.

In the present work, we focus on the even-even Ba isotopes, for which both time-reversal symmetry and signature symmetry are preserved in the calculations. As a consequence, the physical quantities for a pair of time-reversed single-particle states (i.e., with opposite spin orientations but otherwise identical quantum numbers) are identical. Therefore, the localization function for one spin projection (e.g., spin-up) is sufficient to represent that of the opposite spin. In addition, the current density $\mathbf{j}$ vanishes in the static case [35]. Note that in systems with N $\neq$ Z the localization functions of neutron and proton must be treated as distinct quantities, and neither can any longer be approximately regarded as an indicator of $\alpha$ clustering. They can nevertheless still be used to quantify the formation of localized, cluster-like arrangements of nucleons in space.

### B. A Brief Introduction to Tensor Force

The tensor term, also known as the $J^2$ term, originates from both the zero-range central and tensor forces contribution to the energy functional. By considering these two types of forces, _{Tensor} can be written as

$$\varepsilon_{\text{Tensor}}(\mathbf{r}) = \frac{1}{2}\alpha_T \mathbf{J}_q^2(\mathbf{r}) + \frac{1}{2}\beta_T \mathbf{J}_q(\mathbf{r}) \cdot \mathbf{J}_{q'}(\mathbf{r}),$$

with the like-particle tensor term governed by the coupling constant $\alpha$_T, which is proportional to the square of the spin-current density of the same nucleon species, and the neutron-proton tensor term governed by $\beta$_T, which couples the spin-current densities of neutrons and protons [28, 50]. The subscript label q denotes neutrons (protons) and q' represents protons (neutrons).

The like-particle and proton-neutron coupling constants $\alpha = \alpha$_C + $\alpha$_T and $\beta = \beta$_C + $\beta$_T can again be separated into contributions from central and noncentral forces:

$$\alpha_C = -\frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_{1x}1 + t_{2x}2), \quad \beta_C = -\frac{1}{8}(t_1 - t_2) + \frac{1}{8}(t_{1x}1 + t_{2x}2),$$

$$\alpha_T = \frac{5}{12}U, \quad \beta_T = \frac{5}{24}(T + U).$$

The values of $\alpha$ and $\beta$ are obtained from various Skyrme energy density functionals, which can be grouped into three categories: (1) Functionals without explicit tensor terms, i.e., these functionals omit the tensor contributions $\alpha$_T and $\beta$_T, but do account for the $J^2$ term arising from the central interaction. Examples include SkP [51], SLy5 [38], SkO' [52], BSk9 [53], and SAMi [39]. (2) Functionals with perturbative tensor inclusion. In this group, tensor terms are added in a perturbative manner without re-adjusting the original parameter sets. Representative cases are SLy5+T [54] and SIII+T [55]. (3) Functionals with fully fitted tensor terms, namely tensor contributions are treated on the same level as other terms during the fitting process. This category includes Skxta and Skxtb [56], the TIJ family (with I, J ranging from 1 to 6) [57], SkP-T, SLy4-T, SkO-T [58], and SAMi-T [40].

For the TIJ family, the tensor effect can be shown by the evolution of nuclear structure with I or J, and the $\alpha$, $\beta$ parameters are given by $\alpha$ = 60(J - 2), $\beta$ = 60(I - 2).

## III. Results and Discussion

Figure 1 [Figure 1: see original paper] compares theoretical and experimental binding energies of Ba isotopes, where the theoretical results are calculated with

different NN interactions. The grey shaded area represents the results with 36 sets of interaction from the TIJ family. SLy5b denotes SLy5 without central and noncentral tensor contributions, SLy5 is the standard SLy5 NN interaction that includes the central component of the tensor force, and SLy5+T represents the inclusion of both central and noncentral tensor contributions. The figure shows that all NN interactions exhibit reasonably good predictive ability for the binding energy of Ba isotopes. Nonetheless, SAMi predictions in neutron-rich regions tend to overbind, but this phenomenon is partially mitigated in SAMi-T by the inclusion of tensor forces.

To further illustrate the differences in predictive performance among various NN interactions, the two-neutron separation energies S_{2n} = B(N, Z) - B(N - 2, Z) of Ba isotopes are presented in Figure 2 [Figure 2: see original paper]. As shown in the figure, the SAMi and SAMi-T interactions exhibit noticeable deviations from experimental values in regions with $^{128}$-$^{132}$Ba and $^{142}$-$^{154}$Ba, yielding significantly less accurate predictions for S_{2n}. In contrast, SLy5 and its two variants (SLy5b and SLy5+T) produce results for the entire chain of Ba isotopes that are much closer to the experimental data.

Furthermore, these NN interactions exhibit significant discrepancies in describing nuclear deformations, particularly octupole deformations. As shown in Figure 3 (upper panel), all interactions underestimate the quadrupole deformations of $^{134}$,$^{136}$,$^{138}$Ba, while SLy5, SLy5b, and SLy5+T demonstrate excellent agreement with experimental values for other isotopes. In neutron-rich regions where experimental data are unavailable, SLy5 predictions show reasonable consistency with theoretical calculations from relativistic models (using DD-PC1, NL3*) [4] and the axially octupole-deformed relativistic Hartree-Fock model (for example PKO2) [30].

The SAMi interaction maintains comparable accuracy to earlier forces in describing properties of finite nuclei near saturation density and infinite nuclear matter. It has successfully reproduced the Gamow-Teller Resonance (GTR) in $^{48}$Ca, $^{90}$Zr, and $^{208}$Pb, in addition to isobaric analog resonances and spin-dipole resonances in $^{90}$Zr and $^{208}$Pb [39]. However, its theoretical predictions for quadrupole deformations of Ba isotopes exhibit notable discrepancies from experimental data [60]. For instance, from $^{124}$Ba to $^{134}$Ba, SAMi calculations significantly differ from experimental values. In comparison, the SAMi-T interaction partially improves this situation by including tensor forces, yet its agreement remains inferior to that of SLy5. For neutron-rich isotopes, SAMi/SAMi-T exhibit slightly divergent quadrupole deformation trends compared to relativistic models and SLy5. Nonetheless, conclusive evaluation remains impossible due to the lack of experimental validation in this region. Conversely, macroscopic-microscopic (mic+mac) predictions show significant discrepancies. Furthermore, the 36 TIJ parameters demonstrate satisfactory agreement with experimental quadrupole deformation ranges, suggesting relative insensitivity to tensor forces.

Predictions for octupole deformation display more pronounced variations. Most remarkably, the TIJ parameters span a broad range that encompasses the pre-

dictions of nearly all other interactions. Notably, octupole deformations demonstrate much higher tensor-force sensitivity than quadrupole deformations. The SLy5 interactions (particularly SLy5+T) as well as the RHF-PKO2 yield larger octupole deformation values, in closest agreement with $^{144}$Ba experimental data. In contrast, predictions obtained with other non-relativistic EDFs such as UNEDF0, UNEDF1, UNEDF2, SV-min, and SLy4 [59], as well as with relativistic functionals including DD-PC1 and NL3* [4] and the mic+mac approaches, generally exhibit poorer agreement with experiment than SLy5. In particular, SAMi and SAMi-T yield almost vanishing octupole deformations. A comparative analysis of the three SLy5 variants indicates that the central tensor contribution tends to suppress octupole deformation, whereas the inclusion of noncentral tensor terms enhances it.

Figure 4 [Figure 4: see original paper] presents the potential energy surfaces of Ba isotopes calculated using the SLy5 interaction as functions of $\beta_2$ and $\beta_3$. The plot clearly demonstrates that isotopes between $^{142}$Ba and $^{150}$Ba exhibit significant octupole deformation. As shown in Figure 5 [Figure 5: see original paper] (using $^{144}$Ba as an example; data for other nuclei are listed in Table I), by comparing the potential energy surfaces obtained from SLy5b, SLy5, and SLy5+T, one observes that both the central and the noncentral components of the tensor force not only influence the magnitude of octupole deformation but also significantly affect the binding energy. Specifically, the central component actually weakens nuclear binding, while the tensor component enhances the bound state of nuclei.

To better understand the origin of this octupole deformation, we next examine the underlying shell structure and orbital couplings. To illustrate this mechanism, we take $^{144}$Ba as an example to analyze the dominant factors driving octupole deformation in Ba isotopes. The necessary condition for the presence of octupole correlation is the existence, near the Fermi level, of pairs of orbitals strongly coupled by the octupole interaction. For normally deformed systems, the condition for strong octupole coupling is satisfied for particle numbers associated with the maximum $\Delta N = 1$ interaction between the intruder subshell (l, j) and the normal-parity subshell (l - 3, j - 3) [10, 11]. The regions of nuclei with strong octupole correlations correspond to particle numbers near 34 ($g_{9/2}$ $p_{3/2}$ coupling), 56 ($h_{11/2}$ $d_{5/2}$ coupling), 88 ($i_{13/2}$ $f_{7/2}$ coupling), and 134 ($j_{15/2}$ $g_{9/2}$ coupling).

Building on the previous analysis of deformation-driving orbitals, Figure 6 [Figure 6: see original paper] further illustrates the microscopic mechanism underlying octupole stabilization in $^{144}$Ba. From the figure, one observes that in $^{144}$Ba, when the octupole deformation reaches $\beta_3$ 0.13, strong octupole coupling develops between the $1i_{13/2}$ and $2f_{7/2}$ ($ 1h{11}/{2}$ and $ 2d{5}/{2}) *orbitals. As a result, the* $\Omega = 3/2(\Omega = 1/2)$ *components undergo pronounced level repulsion, with the levels above the Fermi surface pushed upward and th particle level density near the Fermi energy, thereby stabilizing the octupole — deformed minimum. While octupole correlations inevitably mix different single—*

*particlestates*, *thedominantcomponentsremainν*1*i{13}*/{2}$ and $2f_{7}/_{2}$, consistent with the $\Delta j = \Delta l = 3$ selection rule for $Y_{30}$ octupole driving configurations [10]. Such stabilization can be further enhanced or suppressed by the tensor interaction, whose coupling constants alter the relative spacing of the j_> and j_< orbitals involved in the octupole-driving pairs.

In the present work, we find that the inclusion of the Skyrme-type tensor force leads to a noticeable modification of the octupole energy surface. The Skyrme proton–neutron coupling term has a form closely analogous to the monopole tensor force proposed by Otsuka et al. [26, 27], and its mechanism can be understood within the same geometric factor framework. Specifically, depending on the j_> - j' _< configuration and the sign of $\beta T$, *the interaction can be either attractive or repulsive, thereby shifting the relative positions of key orbitals across the Fermi surface in an approximately linear manner when the occupation pattern is unchanged. In contrast, the like-particle tensor term* $\alpha$T directly modifies the spin–orbit splitting between spin partners (j>, *j*<) of the same nucleon species, leading to changes in single-particle spacings that feed back self-consistently through the square of spin–current density **J**_q. As a result, its impact on octupole deformation often exhibits pronounced nonlinearity.

Taken together, the combined action of these two tensor components determines whether the octupole minimum is enhanced or suppressed. In our numerical study, as shown in Figure 7 [Figure 7: see original paper], when $\alpha T$ *is fixed and* $\beta$T is varied from negative to positive values, the octupole deformation parameter decreases almost linearly, consistent with the monopole nature of the n–p tensor interaction and with the expected repulsion between the $ 1*h{11}*/{2}$ and $2f_{7}/_{2}$ orbitals for a j_> - j' _> configuration [27]. Conversely, when $\beta$_T is fixed and $\alpha$_T is varied, the deformation changes in a clearly nonlinear fashion, reflecting the different physical mechanism of the like-particle tensor force. Altogether, this distinct behavior supports the interpretation that the n–p tensor term primarily acts as a monopole shift, while the like-particle tensor term alters spin–orbit splittings in a self-consistent manner, leading to qualitatively different effects on the octupole degree of freedom. Building on the numerical results, our parameter-space exploration demonstrates that octupole deformation is highly sensitive to variations in the tensor couplings. In particular, the octupole minimum is strongly affected by both the sign and magnitude of $\alpha T$ *and* $\beta$T, reflecting the distinct microscopic mechanisms of the like-particle and n–p tensor terms. This sensitivity is consistent with the monopole tensor mechanism proposed by Otsuka et al. [26, 27], where the interaction between specific j> *and* *j*< orbitals drives systematic shifts in single-particle energies, and with subsequent analyses showing the impact of tensor forces on multipole response in finite nuclei [28]. However, our findings contrast with recent relativistic Hartree–Fock studies, in which the tensor force was reported to play an opposing role and reduce octupole collectivity in neutron-rich Ba isotopes (e.g., [30]). This discrepancy underscores the crucial dependence of octupole deformation on both the detailed shell structure near the Fermi surface and the chosen tensor interaction scheme.

Since the tensor force acts not only on gaps between single-particle states but also on the spin–orbit potential through the spin–current density **J**_q, its influence extends beyond deformations and energies. In particular, modifications in orbital occupancies and spin-orbit partners induced by $\alpha$_T and $\beta$_T are expected to leave clear fingerprints in the nucleon localization function, which provides a complementary probe of the microscopic structure of octupole-deformed states. As a paradigmatic example, the nucleus $^{144}$Ba serves as an ideal system to analyze how the tensor force manifests in both the energy surface and the underlying localization patterns.

In previous work [61], we investigated the robustness of $\alpha$-cluster structures against tensor forces in the axially symmetric and reflection-symmetric $^{20}$Ne. We assessed the stability of the $\alpha$-cluster structure under the influence of tensor forces, demonstrating that their inclusion significantly enhances Q_s($2^+$_1) and amplifies clustering along the z-axis. As shown in Figure 8 [Figure 8: see original paper], the maximum values of the neutron and proton localization functions in octupole-deformed $^{144}$Ba reach approximately 0.9, indicating pronounced spatial localization of nucleons. Importantly, these localization peaks are predominantly found near the nuclear surface, rather than in the high-density core region. Such surface localization is consistent with prior studies of cluster formation dynamics and nucleon localization. For example, Chang Xu et al., using their quartetting wave function approach, found that $\alpha$-cluster formation in heavy nuclei is heavily suppressed in high central density regions due to strong Pauli blocking and is enhanced in lower-density, more peripheral areas [62]. A classical survey of $\alpha$-clustering also suggested that cluster condensation tends to occur when the local nuclear density falls below central values, favoring the nuclear surface [63]. Moreover, Ebran et al. introduced a "localization parameter" which increases with mass and density variation, showing that spatial localization and hence cluster character is stronger in regions of low density (typically toward the nuclear periphery) than in the dense interior [45, 64].

## IV. Summary

In summary, this work employed the Skyrme-Hartree-Fock framework, incorporating several Skyrme-type effective nucleon-nucleon interactions with and without tensor terms, to assess their performance in describing binding energies and deformation parameters of Ba isotopes. While most interactions reproduce the binding energies with reasonable accuracy, their predictions for quadrupole and especially octupole deformations show substantial discrepancies. In particular, the calculated octupole deformations vary significantly among the interactions: some parametrizations agree well with experiment, while others predict nearly vanishing octupole deformation. For $^{144}$Ba, the SLy5 interaction provides a theoretical octupole deformation in close agreement with experiment, with further improvement obtained when the non-central tensor contribution is included. Moreover, SLy5 predicts pronounced octupole deformation for nuclei in the $^{144}$Ba–$^{148}$Ba region.

From a microscopic perspective, the emergence of octupole minima can be traced back to the strong octupole correlations between $\Delta N = 1$, $\Delta l = \Delta j = 3$ orbitals across the Fermi surface. The repulsion between these partners pushes states above the Fermi level upward and those below it downward, reducing the level density near the Fermi energy and thereby stabilizing octupole-deformed configurations. The inclusion of tensor terms further modifies the equilibrium octupole deformation, with the outcome depending sensitively on both the nature and the sign of the tensor couplings. Specifically, for the representative cases of SLy5b (without tensor), standard SLy5 (with central tensor), and SLy5+T (with non-central tensor), we find that the central tensor contribution tends to reduce octupole deformation, whereas the non-central tensor term enhances it.

Beyond deformation parameters, we further examined the spatial localization characteristics induced by tensor forces. Both the central and non-central tensor components are found to modify the localization function in different spatial regions, with the central term favoring a suppression of octupole localization and the non-central term producing the opposite effect. These changes in localization are fully consistent with the corresponding trends in the octupole deformation parameter $\beta_3$. In addition, regions with localization values close to 0.9 are predominantly located near the nuclear surface (low-density regions), in agreement with previous studies of clustering phenomena in medium-heavy nuclei [62, 64, 65].

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