Current Distribution Rules in Parallel Superconducting Circuits

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Abstract

In a parallel circuit composed of normal-state conductors, the distribution of current is inversely proportional to the resistance of each branch. However, superconductors have no resistance; how is current distributed in a parallel circuit composed of superconductors? Due to the complexity of superconducting current measurement, this fundamental question has long been neglected. This paper analyzes the factors affecting current distribution in parallel circuits composed of normal-state conductors, and concludes that current distribution in parallel superconducting circuits should be inversely proportional to the inductance of each branch. To experimentally verify this conclusion, this study designs an experimental apparatus for measuring current distribution in parallel superconducting circuits, in which Hall effect sensors are employed to measure current intensity, thereby avoiding disruption of the zero-resistance characteristic of the parallel superconducting circuit. Experimental results confirm that current distribution in parallel superconducting circuits is inversely proportional to the inductance of each branch.

Full Text

Current Distribution Rule in Parallel Superconducting Circuits

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Abstract

In parallel circuits composed of normal-state conductors, current distribution is inversely proportional to the resistance of each branch. However, superconductors have zero resistance, raising the question: how is current distributed in parallel superconducting circuits? Due to the complexity of measuring superconducting currents, this fundamental question has long been neglected. This paper analyzes the factors affecting current distribution in parallel circuits composed of normal-state conductors and concludes that current distribution in parallel superconducting circuits should be inversely proportional to the inductance of each branch. To experimentally verify this conclusion, we designed a measurement apparatus for current distribution in parallel superconducting circuits, employing Hall effect sensors to measure current intensity without disrupting the zero-resistance property of the parallel superconducting circuit. Experimental results confirm that current distribution in parallel superconducting circuits is inversely proportional to the inductance of each branch.

Keywords: parallel superconducting circuits, current distribution, Hall sensor, cuprate superconductor

1. Introduction

Since Onnes discovered superconductivity in 1911 [1], more than a century has passed, yet superconductivity remains one of the most active and mysterious phenomena in physics. Breakthroughs in searching for superconducting materials continue to emerge, with cuprate superconductors [2], iron-based superconductors [3], and nickelate superconductors [4] being discovered successively. A greater variety of superconductors with different forms, compositions, and structures can help us understand more about the properties of superconducting currents and expand our knowledge of electromagnetic theory. In terms of superconducting electromagnetic phenomena, superconducting currents exhibit many novel characteristics distinct from normal currents. In 1933, Meissner and Ochsenfeld [5] discovered perfect diamagnetism in superconductors. In 1956, Cooper [6] proposed that superconducting carriers are electron pairs bound as Cooper pairs, which behave as bosons. Many Cooper pairs form Bose-Einstein condensation at low temperatures, leading to collective electron motion; thus, superconducting current is a macroscopic quantum phenomenon. According to the Ginzburg-Landau equations [7], superconducting current is distributed on the surface of superconductors. These studies demonstrate significant differences in transmission characteristics between superconducting and normal currents.

This study focuses on the difference in current distribution mechanisms between superconducting and normal currents in parallel circuits. We have innovatively designed an experimental apparatus to measure the differences in current distribution between superconducting and normal currents in parallel circuits, verifying the rule that current distribution in parallel superconducting circuits is inversely proportional to the inductance of each parallel branch.

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2. Experimental Principle and Setup

2.1 Experimental Principle

First, let us analyze the differences between parallel circuits in the normal state and the superconducting state.

$$I = I_L + I_R$$

[Figure 1: see original paper] Equivalent circuit diagram of a parallel circuit in the normal state, where I_L, R_L, and L_L denote the current intensity, resistance, and inductance of the left branch, and I_R, R_R, and L_R denote the current intensity, resistance, and inductance of the right branch. (In this paper, subscripts L and R are used to indicate parameters of the left and right circuits, respectively; the same applies hereinafter).

Figure 1 shows a parallel circuit in the normal state. Since the two branches of the parallel circuit form a closed loop, changes in the branch currents alter the magnetic flux through the closed loop. This induces a current that opposes the change in magnetic flux, which is precisely the effect of inductance. Therefore, inductance is always present in parallel circuits. The relationship between the left and right current intensities satisfies:

$$I_L R_L - L_L \frac{dI_L}{dt} = I_R R_R - L_R \frac{dI_R}{dt}$$

In general, when measuring current distribution in parallel circuits under normal conditions, measurements are performed when the current is in a steady state, and the induced current is dissipated by resistance. Consequently, the effect of inductance can be neglected, and only the influence of resistance on current distribution needs to be considered. Therefore, we can conclude that current distribution in normal-state parallel circuits follows the rule of being inversely proportional to resistance:

$$I_L R_L = I_R R_R$$

For parallel circuits composed of superconducting materials, resistance disappears. However, inductance does not vanish, as inductance depends only on the current distribution and not on material properties. In this case, the effect of resistance can be ignored, and current distribution is determined by inductance. Since no resistance exists, induced current is not dissipated. In equation (1), neglecting the terms containing R_L and R_R, we obtain:

$$L_L \frac{dI_L}{dt} = L_R \frac{dI_R}{dt}$$

Integrating both sides yields:

$$L_L I_L = L_R I_R \quad (2)$$

That is, current distribution in parallel superconducting circuits is inversely proportional to their respective inductances. This result is consistent with Poole' s conclusion [8]. If we define $\Delta\Phi = L_L I_L - L_R I_R$, we can see that $\Delta\Phi$ represents the change in magnetic flux through the closed loop caused by current distribution. From equation (2), we know $\Delta\Phi = 0$. This means that a parallel circuit composed of superconductors forms a closed loop, and when current flows through this parallel circuit, the magnetic flux inside the closed loop does not change. This is consistent with Zhang Yuheng' s result [9].

To verify the current distribution rule for I_L/I_R in the superconducting state, it is necessary to construct a parallel circuit where the resistance ratio and inductance ratio of the two branches are different (i.e., $R_R/R_L \neq L_R/L_L$). By measuring the current ratio I_L/I_R in both the normal and superconducting states and comparing it with R_R/R_L, we can determine the difference in current distribution between the normal and superconducting states.

2.2 Sample Preparation

Based on the composition of YBa2Cu3O7-δ, raw materials of Y2O3, BaCO3, and CuO totaling 40 g were weighed, thoroughly mixed, and calcined at 1133 K for 20 h. After cooling, the sample was removed and ground into powder. Fifteen grams of the powder were pressed into a disc with a diameter of 40 mm and sintered at 1203 K for 20 h. After cooling, the sample was machined into the annular shape shown in Figure 2 [Figure 2: see original paper].

[Figure 2: see original paper] Sample shape and dimensions (units: mm). For this sample, the left semicircular ring has a thickness of 1.36 mm, while the right semicircular ring has a thickness of 2.34 mm. Both left and right semicircular rings have a width of 4.38 mm.

2.3 Sample Installation

[Figure 3: see original paper] Sample installation diagram. Figure 3 shows the sample installation, with six electrodes and two Hall detectors uniformly distributed around the annular circuit. Using the line connecting electrode 1 and electrode 4 as the centerline, the left semicircle has a thickness of 1.36 mm and the right semicircle has a thickness of 2.34 mm.

When measuring current distribution, electrode 1 serves as a fixed connection point for the constant current source, while the other connection point of the current source is sequentially switched among electrodes 2, 3, 4, 5, and 6. Hall sensors are used to measure the current intensity on both sides of the parallel circuit. For each of these connection schemes, the resistance ratio and inductance ratio of the two parallel branches are different (i.e., $R_R/R_L \neq L_R/L_L$; calculation methods are provided in Section 4.1), fulfilling the circuit requirements specified in Section 2.1 of this paper. Therefore, by measuring the current ratio I_L/I_R on the left and right sides for each constant current source connection scheme in both the normal and superconducting states and comparing it with R_R/R_L, we can determine the difference in current distribution between the normal and superconducting states.

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3. Measurements

3.1 Superconducting Transition Measurement

With the installation shown in Figure 3, electrodes 2 and 6 were connected to a 20 mA constant current source. A voltmeter was connected to electrodes 3 and 5, establishing a four-probe measurement system. The sample temperature was gradually decreased, and the voltage between electrodes 3 and 5 changed accordingly. Since the output current intensity of the constant current source remained unchanged, the voltage variation between electrodes 3 and 5 reflected the resistance change between electrodes 3 and 5. The measurement results are shown in Figure 4 [Figure 4: see original paper].

[Figure 4: see original paper] Voltage between electrodes 3 and 5 as a function of temperature when a constant current of 20 mA passes between electrodes 2 and 6.

As shown in Figure 4, the superconducting transition occurs at approximately 102 K, and the superconducting transition is completed upon cooling to 98 K, at which point the resistance becomes zero.

3.2 Calibration of Hall Sensors

When using Hall sensors to measure current intensity in a conductor, the Hall voltage varies with the sensor’ s position relative to the conductor and its installation angle. Therefore, it is necessary to calibrate the measurements from the two Hall detectors to ensure comparability between the two Hall sensor readings. The sample temperature was adjusted to 108 K, where the sample is in the normal state. The constant current passing between electrodes 2 and 6 was set to 180 mA. By reversing the direction of the constant current, the

Hall sensor readings changed accordingly, and this variation is proportional to the current intensity. Thus, this variation represents the Hall signal we aim to measure. When a constant current is applied between electrodes 2 and 6, both Hall sensors measure the same current intensity, so the Hall voltages from the two sensors are equivalent. Their ratio can be used for mutual calibration of subsequent measurements. Throughout the subsequent sample current measurements, the positions and orientations of the sample and Hall sensors were kept unchanged. Through this mutual calibration, inconsistencies in data caused by installation position and angle differences between the two Hall sensors can be eliminated. After this calibration procedure, the measurement values from the two Hall sensors become comparable.

3.3 Data Measurement

The sample temperature was then maintained at 108 K, with electrode 1 serving as a fixed connection point for the constant current source while the other connection point was sequentially switched among electrodes 2, 3, 4, 5, and 6 to obtain different resistance and inductance ratios between the left and right circuits. The constant current source output was maintained at 180 mA. Hall voltage variations were obtained by reversing the current direction from the constant current source. The measured Hall voltage values from the two sensors were adjusted according to the calibration data from Section 3.2. Since the Hall voltage measured by the Hall sensors is proportional to the current intensity in the conductor, the ratio of the Hall voltages from the two sensors equals the current intensity ratio I_L/I_R between the two circuit sides. The I_L/I_R values measured at this stage represent the results in the normal state.

The sample temperature was lowered to 90.5 K, where the sample is in the superconducting state. The constant current source was adjusted to 100 mA to avoid affecting the superconducting state with excessive current. The calibration and data measurement procedures described in Sections 3.2 and 3.3 were repeated. The I_L/I_R values measured at this stage represent the results in the superconducting state. These measurement results are listed in Table 1 .

Measured I_L/I_R data for the sample at temperatures of 108 K and 90.5 K. The sample is in the normal state at 108 K and in the superconducting state at 90.5 K.

Table 1 shows the measurement results for the sample in the normal state (108 K) and superconducting state (90.5 K). In the table, R_R/R_L represents the resistance ratio of the right circuit to the left circuit, while L_R/L_L represents the inductance ratio of the right circuit to the left circuit. The R_R/R_L values are calculated based on electrode positions in the normal state (calculation methods for R_R/R_L are provided in Section 4.1).

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4. Analysis

4.1 Calculation of Inductance Ratio L_R/L_L and Resistance Ratio R_R/R_L

The calculation of circuit inductance is very complex and is generally obtained experimentally. In this experiment, the geometric shape of the circuit is relatively simple, so we do not need to calculate the specific values of L_L and L_R; we only need to calculate the ratio L_R/L_L.

From the sample dimensions in Figure 2, the inductance per unit arc length of the left and right semicircular rings is in a ratio of $4.38/4.38 = 1$. This is because inductance is proportional to the magnetic flux generated by the current within the closed ring, and magnetic flux is proportional to the magnetic induction intensity. Since the widths of the left and right semicircular rings are equal, passing the same current through both semicircular rings produces the same magnetic flux within the closed loop. Therefore, the inductances of the left and right semicircular rings are equal.

For the installation shown in Figure 3, since the six electrodes and two Hall sensors are uniformly distributed around the annular circuit, the arc length between adjacent electrodes 2, 3, 4, 5, and 6 can be set as s, while the arc lengths between electrode 1 and electrode 2 and between electrode 1 and electrode 6 can be set as $2s$.

For calculating L_R/L_L, let the inductance per unit arc length of the left semicircular ring be k_L and that of the right semicircular ring be k_R. Then $k_L = k_R$.

For the simple geometry of our sample, the total inductance of the left and right circuits is the sum of the inductances of each segment. Taking the case where current flows between electrodes 1 and 2 as an example, Figure 3 shows that the inductance of the right circuit is $L_R = 4s \cdot k_R + 2s \cdot k_L$, while the inductance of the left circuit is $L_L = 2s \cdot k_L$. Therefore:

$$\frac{L_R}{L_L} = \frac{4s \cdot k_R + 2s \cdot k_L}{2s \cdot k_L}$$

For current flowing between any two electrodes, L_R/L_L can be calculated in the same manner. The calculation results are listed in Table 1.

For the calculation of R_R/R_L, let the resistance per unit arc length of the left semicircular ring be r_L and that of the right semicircular ring be r_R. From the sample dimensions, we know $r_L/r_R = 1.72$, i.e., $r_L = 1.72r_R$.

Taking the case where current flows between electrodes 1 and 2 as an example, Figure 3 shows that the resistance of the right circuit is $R_R = 4s \cdot r_R + 2s \cdot r_L$, while the resistance of the left circuit is $R_L = 2s \cdot r_L$. Therefore:

$$\frac{R_R}{R_L} = \frac{4s \cdot r_R + 2s \cdot r_L}{2s \cdot r_L} = 2.16$$

For current flowing between any two electrodes, R_R/R_L can be calculated in the same manner. The results are shown in Table 1.

4.2 Experimental Data Analysis

As can be seen from Table 1, the calculated R_R/R_L values for parallel circuits formed between electrode 1 and each of the other electrodes are significantly different, meeting the requirements specified in Section 2.1 of this paper. At a sample temperature of 108 K, the sample is in the normal state, and the measured current ratio I_L/I_R between the left and right circuits is essentially the same as the calculated resistance ratio R_R/R_L. At a sample temperature of 90.5 K, the sample is in the superconducting state, and the measured current ratio I_L/I_R matches the inductance ratio L_R/L_L.

Plotting the data from Table 1 as Figure 5 [Figure 5: see original paper] provides a clearer view of the differences in current distribution between the normal and superconducting states.

[Figure 5: see original paper] Relationship between I_L/I_R (shown as dashed lines for measured values at 108 K and 90.5 K), R_R/R_L (solid line), and L_R/L_L (solid line) versus electrode position.

Figure 5 clearly shows that as one end of the parallel circuit is fixed at electrode 1 and the other end is sequentially switched to electrodes 2, 3, 4, 5, and 6, both the resistance ratio R_R/R_L and the inductance ratio L_R/L_L of the left and right sides of the parallel circuit change sequentially. However, in all cases, R_R/R_L and L_R/L_L are distinctly different. The current intensity ratios I_L/I_R on the left and right sides are significantly different between the normal and superconducting states, indicating that the current distribution mechanisms differ between the two states.

In the normal state, the measured values of I_L/I_R show close agreement with R_R/R_L, meaning that I_L/I_R varies with electrode position in accordance with R_R/R_L. This demonstrates that in the normal state, current distribution in parallel circuits is inversely proportional to the resistance of each branch, consistent with the well-known current distribution rule for normal-state circuits. In the superconducting state, I_L/I_R varies with electrode position in close agreement with L_R/L_L, indicating that current distribution in parallel superconducting circuits is inversely proportional to the inductance of each branch, consistent with the analysis results presented in Section 2.1.

In this experiment, we designed a system to measure current distribution in parallel superconducting circuits, utilizing Hall sensors to measure currents in superconducting circuits without disrupting their zero-resistance property. Measurements of current distribution in parallel circuits in the normal state agree with the known current distribution rule for normal-state circuits, confirming the accuracy and reliability of our measurement system. Measurements in the superconducting state demonstrate that current distribution in parallel superconducting circuits is inversely proportional to the inductance of each branch, consistent with the analysis in Section 2.1.

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