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Spatial Curvature, Spacetime Transformation, and Cosmic Redshift —Applications of Spacetime Structure Geometry on Large Spatial Scales

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Abstract

Based on the spacetime structural geometry established by the author, this paper employs precise mathematical models to provide a novel interpretation of spatial curvature in the macroscopic world, derives the curvature formula for light path curves in natural spacetime and elucidates its physical significance; demonstrates the spacetime transformation relationship between the geometric scene of natural spacetime and the observer' s visual imagery, and presents the spacetime transformation equations; discusses the natural properties of cosmic redshift and derives the redshift differential formula; and proposes experimental methods for verifying the new spacetime relationship of photons, among other aspects.

Full Text

Space Curvature, Space-Time Transformation, and Cosmic Redshift —Applications of Geometry of Space-Time Structures at Large Spatial Scales

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Abstract

Based on the Geometry of Space-Time Structures founded by the author, this paper employs precise mathematical models to provide novel explanations for space curvature problems in the macroscopic world. We derive the curvature formula for optical path curves in natural space-time and elucidate its physical significance; demonstrate the space-time transformation relationship between

the geometric scene of natural space-time and the observer' s visual image, presenting the space-time transformation equation; discuss the natural properties of cosmic redshift and derive the redshift differential formula; and propose experimental methods to verify the new space-time relationships of photons.

1. Introduction

Through over 30 years of unremitting effort, the author has established the Geometry of Space-Time Structures, which takes natural space-time and the structure and motion of matter within it as its object of study. This framework aims to dynamically reveal the most fundamental laws of motion and intrinsic relationships in nature from a broader perspective using modern analytical geometry methods [1-3]. This paper endeavors to demonstrate several applications of space-time structure geometry at macroscopic and ultra-macroscopic spatial scales through concise logic. The main contributions include: clarifying the distinction and space-time relationship between the natural geometric scene of space-time and the visual geometric image observed by observers; defining the distance differential element in natural space-time and the corresponding distance differential element in the observer' s visual image space, and establishing their relationship; deriving the optical path curvature equation and quantitative relationships of space-time curvature with new physical interpretations; deriving and analyzing the matrix algebraic relationships of space-time transformation; elucidating the physical essence of cosmic redshift, deriving the cosmic redshift differential formula, and providing experimental methods to verify the new space-time relationships of photons.

2. Logical Foundations of Space-Time Structure Geometry and the Distance Differential Element dS

Space-Time Structure Geometry takes natural space-time and all objects within it as its research subject. Its foundational principles include: time, space, and matter are integrated forms of energy motion that all possess minimal quantized units; the speed of light is maximal and invariant; the time quantum is the light quantum; time always progresses at the speed of light; complete natural space has three degrees of freedom; any connection between two completely separated material points in natural space is mediated by photons, with the distance between points being the optical path; and the structure, form, and motion state of any object in natural space are forms of material energy existence, fundamentally consistent and logically equivalent. This framework dynamically, holographically, and directly characterizes the morphological structure and motion changes of objects, revealing the physical significance of geometric forms and their temporal variations [1-3]. Space-Time Structure Geometry is a geometry of motion. Simple, regular static geometric figures and simple, ordered periodic geometric forms are merely special cases in the morphological changes of matter in nature. Under the action of time, the morphological structure of objects is constantly changing. For any specific or relatively independent individual or group, this can be viewed as a generalized topological transformation of its geometric form (i.e., temporal transformation), corresponding to a specific topological transformation group that may also be called a self-similarity group.

In Space-Time Structure Geometry, points have "size," lines have "width," and surfaces have "thickness." There are no absolute straight lines or planes. Line segment lengths, surface areas, morphological structures, and their enclosed spatial volumes are all measures and manifestations of material energy. Determining the magnitudes and calculation rules for these fundamental geometric quantities constitutes the basic logical relationships of any analytical geometry. Large-scale spatial morphological structures are composed of relatively smaller structural subsets, making composite structures unsuitable as universal structural units. Broadly speaking, selecting coordinate differential elements such as dx, dy, dz, dt as the determined quantities for space-time measurement, with corresponding macroscopic quantities being superimposed and combined from these differential elements through summation and multiplication logic, represents a reasonable foundational mathematical assumption and a fundamental logical choice in the sense of natural philosophy. (This explains why differential geometry is relatively accurate when handling fundamental physical problems.)

Since there are no absolute straight lines, the Euclidean Cartesian coordinate system is merely an ideal reference coordinate system. At microscopic scales, the uniform orthogonal coordinate relationships of the Cartesian system remain applicable. The self-inner product of a vector is the square of the vector' s "length," which allows for a generalized definition of spatial distance differential dS. In three-dimensional natural space, any physical point can similarly be uniquely determined by (x, y, z) coordinates. In the neighborhood of point (x, y, z), a local 3D unit orthogonal coordinate system (which may be called a "frame") can be established. The line vector differential dr = dx + dy + dz, and dr dr = dr^2. This also represents the distance differential dS between any two points in the infinitesimal neighborhood subspace of point (x, y, z) when the subspace is uniform. Since physical points in natural space are correlated—for instance, two points may be connected by a certain curve or surface—the more general vector differential in a unit orthogonal coordinate system is the partial derivative of the vector with respect to orthogonal independent variables. The corresponding distance differential dS is:

$$dS = \sqrt{dr \cdot dr} = \sqrt{dx^2 + dy^2 + dz^2} \quad (1)$$

According to differential geometry, equation (1) can determine the geometric properties of corresponding curves and surfaces. Of course, for any n-dimensional space where the coordinate system is not necessarily orthogonal, with point coordinates represented as (x_1, , x , , x), Riemannian geometry defines the distance differential as:

$$dS = \sqrt{g_{ij} dx_i dx_j}$$

where g are metric coefficients, elements of the metric tensor matrix.

In Space-Time Structure Geometry, the distance S between any two points in three-dimensional natural space is the optical path length between them [2]. Let n(x, y, z) represent the generalized refractive index tensor of the space medium. The distance differential dS is:

$$dS = \sqrt{n_x^2 dx^2 + n_y^2 dy^2 + n_z^2 dz^2} = \sqrt{(n_i dx_i)^2} \quad (2)$$

The distance S between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in natural space is:

$$S = \int_1^2 (n_i dx_i) \quad (3)$$

where the integration path is the optical path. For any three-dimensional spatial coordinate system, equation (2) becomes the Riemannian geometry distance differential form.

3. Natural Space-Time: Differential Equations and Curvature Equations of Optical Path Curves, and Space Curvature

Time is an independent free variable that advances uniformly at the speed of light, equivalently acting on any point in natural space. The time dimension is a dimension of motion. Natural space-time is defined [1-2] as the real space of nature, mathematically formulated as a quasi-four-dimensional space composed of a static three-dimensional natural space plus a quasi-one-dimensional time dimension advancing at speed c. Time only acts on the present moment, meaning only the current "time point" is significant. There is no independently existing time-space; the time dimension is "virtual."Therefore, the quasi-four-dimensional space coordinate system of natural space-time can be represented by a complex space coordinate system (x, y, z, ict).

The distance differential in quasi-four-dimensional natural space-time is:

$$dS = \sqrt{dx^2 + dy^2 + dz^2 - c^2 dt^2} \quad (4)$$

When dt = 0, equation (4) reduces to equation (2), indicating that optical path curves are isochronous. Photons are space-time-matter integrated quanta, serving as the medium for measuring material space-time size and "filling"space-time itself. The distance (space-time size) of a photon' s own quasi-four-dimensional space-time should be zero. For free particles with non-zero rest mass, changes in their 3D material form result from time-driven photon absorption. Therefore, for distance as a geometric invariant in 4D space, space-time variable effects should cancel through subtraction. This is the essential geometric logic of the complex space coordinate system (x, y, z, ict), and relationship (4) is logically correct.

For any observer, the scene of the natural world is an observed image. For distant or microscopic objects, this is a naturally scaled-down or magnified detected image. The visual image observed or detected by the observer is denoted as P, while the actual natural scene is represented by P. The transformation P → P is a space-time transformation (denoted by T). Due to different positional distances and varying amounts of optical-matter information observed by the observer, at large scales the visual image P exhibits characteristics of light-induced contraction where material scenes shrink with increasing distance. In geometric theory, an object' s angular size decreases with distance, but its actual spatial size value does not decrease—this represents completely different mathematical logic. However, for any observer, the observable natural scene is precisely the "incarnation" and reflection of P, which is P. Using coordinate representation: assuming any material point has coordinates (x, y, z) in the natural scene P coordinate system (referred to as "P-space"), and its coordinates in the visual image P coordinate system (referred to as "P-space") are (x̂, ŷ, ẑ), with a one-to-one correspondence between (x, y, z) and (x̂, ŷ, ẑ), but the unit length of the P coordinate system contracts as coordinate values increase. Space-Time Structure Geometry defines the distance differential in visual image space (dŜ), expressed using the natural space-time space (i.e., P-space) coordinate system, as:

$$dS = \frac{dr}{\sqrt{1+\left(\frac{r}{S_0}\right)^2}} = \frac{S_0}{\sqrt{S_0^2+r^2}}dr \quad (5)$$

where S_0 is the space-time transformation characteristic length. This paper only discusses the case represented by equation (5), where S_0 = (n_i dx_i) represents the optical path from point (x, y, z) to the coordinate origin.

The geometric form described by differential equation (5) is precisely the natural scene observed by humans. The integral along any curve r(x(t), y(t), z(t)) gives the curve length, with the optical path line being the shortest between any two points. Broadly speaking, equation (5) is a dynamic matrix equation for matter-energy information transformation that can also be applied to artificial intelligence 3D dynamic recognition algorithms. Through repeated training and learning with large amounts of real scene data, relatively accurate and complete visual matrix sets for different times and material states can be obtained. The corresponding space-time transformation of equation (5) is a light-induced transformation that preserves the straightness of straight line segments. In this sense, it may be viewed as a new type of projective geometry, but its logic more accurately reflects objective reality than simple projective geometry.

Below, we use equation (5) to derive the relationship between second derivatives and curvature for arbitrary optical path lines, and explain the curvature properties of natural space-time [1-2]. A smooth curve is represented by r(x(t), y(t), z(t)).

Using equation (5), let:

$$dr = \frac{S_0}{\sqrt{S_0^2 + r^2}}(n_x dx\mathbf{e}_x + n_y dy\mathbf{e}_y + n_z dz\mathbf{e}_z) = \frac{S_0}{\sqrt{S_0^2 + r^2}} n_i dx_i \mathbf{e}_i$$

$$\frac{dr}{dt} = \frac{S_0}{\sqrt{S_0^2 + r^2}}\left(n_x \frac{dx}{dt}\mathbf{e}_x + n_y \frac{dy}{dt}\mathbf{e}_y + n_z \frac{dz}{dt}\mathbf{e}_z\right) = \frac{S_0}{\sqrt{S_0^2 + r^2}} \frac{dS_0}{dt}\mathbf{e}_i \quad (6)$$

Assuming n__x = n__y = n__z = 1, equation (8) becomes:

$$\frac{d^{2r}}{dt^2} = \frac{S_0}{\sqrt{S_0^2 + r^2}}\left(\frac{d_0^{2S}}{dt^2} - \frac{r}{S_0^2 + r^2}\left(\frac{dS_0}{dt}\right)^2\right)\mathbf{e}_i - \frac{S_0^2 r}{(S_0^2 + r^2)^2}\left(\frac{dS_0}{dt}\right)^2 \mathbf{e}_i \quad (9)$$

According to formula [5], we obtain:

$$\kappa = \frac{\left|\frac{dr}{dt} \times \frac{d^{2r}}{dt^2}\right|}{\left|\frac{dr}{dt}\right|^3} = \frac{\sqrt{S_0^2 + r^2}}{S_0 r} \quad (10)$$

and through vector cross product operation rules:

$$\kappa = \frac{\sqrt{S_0^2 + r^2}}{S_0 r}$$

The natural space-time scene observed by humans contracts to a dimension D $\rightarrow 0$ at infinite distance ($S_0 \rightarrow \infty$), becoming a space-time point without change, but its curvature $\rightarrow \infty$ and internal interaction energy becomes infinite. Infinite cosmic space contracts into a strongly rotating quantum with infinite energy-matter. From a symmetric imaging perspective, the infinite distant point can be regarded as the "self-image" of the cosmic space centered on the observer, with the radius equal to the observer' s distance to the far point. The so-called large amount of "dark matter" in cosmic space may be related to this.

When the generalized refractive indices are all constant 1, the same contraction curvature property of natural space-time exists. This is the inherent material image property of natural space mediated by light, fundamentally different from the curvature meanings of hyperboloid and ellipsoid geometries. This natural curvature of visual image space in natural space-time is caused by holographic contraction changes of dynamic space with distance, centered on the observer, driven by independent time, and mediated by light. Of course, natural space-time also includes the curvature of curves and surfaces.

If the generalized refractive indices are all constant 1, with space being uniform and isotropic, optical path lines are straight lines. For light rays passing through the origin, the optical path S = S_0 = r, and from equations (2) and (5), λ_0 = 1. Therefore:

$$S = r, \quad \lambda_0 = 1 \quad (11)$$

4. Matrix Algebra of Space-Time Transformation of Natural Scene P

As discussed above, the natural scene P of natural space-time and the visual image P observed by the observer have a one-to-one corresponding space-time transformation relationship. For any observer, the visual image P represents the natural scene P. The coordinate origins of the P-space coordinate system (x, y, z, ict) and the P-space coordinate system (x̂, ŷ, ẑ, ict̂) coincide, with coordinate axes aligned but different length scales. The unit length of the P-space coordinate axes contracts and decreases as coordinate values increase.

The P-space is not an ordinary vector space; ordinary vector translation and inner/outer product operations do not exist. However, since material point coordinates (x, y, z) → (x̂, ŷ, ẑ) remain an orthogonal coordinate system, the distance differential dŜ in infinitesimal space still follows vector operation logic similar to dS. Generally, the transformation T from distance differential dS or (dx, dy, dz, icdt) to distance differential dŜ is a one-to-one light-induced transformation, with matrix representation being a 4th-order square matrix. Because the observer' s visual image P is simultaneous (t̂ = 0) at any moment, the space-time transformation matrix simplifies to a 3$×$4 matrix, with the 3D visual image presenting dynamic natural scenes.

The space-time transformation of P is essentially the transformation from quasi-4D space-time (x, y, z, ict) to 3D space (x̂, ŷ, ẑ). For any point (x, y, z) in P-space at distance S from the observer, the corresponding time is t = 0 - s/c = -s/c, and its 4D coordinates are (x, y, z, -is). Therefore:

$$\begin{pmatrix} x \\ \\ \end{pmatrix} = \begin{pmatrix} \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 & 0 & 0 \\ 0 & \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 & 0 \\ 0 & 0 & \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ -is \end{pmatrix} \quad (12)$$

where zero matrix elements are determined by the orthogonal coordinate component relationships corresponding to space-time transformation, and the transformation from ict to -is is zero. Equation (12) is the space-time transformation equation from the geometric scene P of natural space-time to the observer' s visual image ℙ.

According to Space-Time Structure Geometry, the quasi-4D space-time geometric distance differential element dS = √(dx^2 + dy^2 + dz^2 - c^2dt^2) represents material energy, with the corresponding quantity observed by the observer being dŜ = dr/√(1 + $(r/S_0)^2$). From equations (4) and (6), taking S_0 = (n_i dx_i) and setting $\lambda_0 = 1$, equation (12) becomes:

$$\begin{pmatrix} dx \\ d \\ d \end{pmatrix} = \begin{pmatrix} \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 & 0 & 0 \\ 0 & \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 & 0 \\ 0 & 0 & \frac{S_0}{\sqrt{S_0^2+r^2}} & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \\ dz \\ -icdt \end{pmatrix} \quad (13)$$

Setting the fixed time differential element (ic dt) to zero in equation (13) yields equation (5) expressed in the natural space-time coordinate system, including the coordinate differential relationships. This transformation P → ℙ is equivalent to a holographic, non-simple linear scale transformation. At this point, the space-time transformation P → ℙ is a concomitant, definite correspondence. For any observer, the observed real natural scene ℙ is the definite, unique "incarnation" of P. This transformation, combined with translation and Lorentz transformations, constitutes the complete set (group) of space-time transformations, with P at different positions, times, or for different observers having corresponding relationships. This space-time transformation preserves the invariance of forms (straight lines, geometric structures, etc.).

5. Photon Energy Equation and Cosmic Redshift [1-2]

From equation (2), Space-Time Structure Geometry defines distance in natural space-time as optical path, with distance differential dS proportional to the space' s generalized refractive index. Assuming the space refractive index tensor is only a function of position and isotropic, the distance differential relationship for optical path curve r(x(t), y(t), z(t)) is:

$$dS = nds \quad (14)$$

where n is the refractive index and total optical path is integrated along the optical path curve. Assuming parameter t is the photon motion time variable, for photons equation (4) equals zero, so:

$$\frac{dr}{dt} = n\frac{dx}{dt}\mathbf{e}_x + n\frac{dy}{dt}\mathbf{e}_y + n\frac{dz}{dt}\mathbf{e}_z = n\frac{dS_0}{dt}\mathbf{e}_i$$

The photon velocity is:

$$v = \frac{dS}{dt} = n\frac{dS_0}{dt}$$

As mentioned repeatedly, a fundamental logical principle of Space-Time Structure Geometry is that any object' s geometric structure and form are manifestations of its material information energy. Therefore, material energy differential dE dS. For a single photon, dE = h d (where h is Planck' s constant and is frequency). Since the optical path has directionality, photon momentum is represented by p = h/λ (with p as the mass unit parameter), giving:

$$dE = \mathbf{p} \cdot d\mathbf{S} = h d\nu \quad (15)$$

$$d\mathbf{p} = \frac{h}{c} d\nu \mathbf{e}_i \quad (16)$$

Substituting equation (15) into (16) and considering photons moving toward the observer (with p and dS in opposite directions):

$$d\nu = -\frac{n}{c}\frac{dS_0}{dt}\nu dt \quad (17)$$

Equations (16) and (17) constitute the dynamic photon energy equation. For a single photon, continuous propagation always encounters resistance, causing frequency to decrease as the photon advances. If dt = 0, the photon moves at constant speed c/n.

The essence of cosmic redshift is that photons, as space-time quanta, undergo passive attenuation during propagation through interactions with other particles and vacuum fluctuation quanta. The continuously propagating photon quantum state, due to resistance from other particles and thermodynamic statistical constraints on photon energy levels E, inevitably experiences energy and frequency attenuation during long-distance propagation. The relationship between photon energy attenuation and propagation distance r can be more simply determined. Let:

$$dE = -E\frac{dr}{r_0}$$

Then:

$$E = E_0 e^{-r/r_0}, \quad h\nu = h\nu_0 e^{-r/r_0}$$

When r r_0, the exponential curve becomes a nearly linear relationship with minimal change, which is precisely the implication of Hubble' s Law. For any observer in the universe, they are at the "center." There is no Big Bang, and the natural universe has no boundary.

6. Discussion–Single Photon Frequency Decomposition and Two-Photon Entanglement

Photons are space-time quanta that mediate the characterization of natural material space-time structure states. The variation or decomposition of photon frequency during motion can be verified through space-time interaction changes of single photons or two photons in vacuum.

A single photon state is a "macroscopic" quantum state. Single photon motion in vacuum is one-dimensional, but its quantum state manifests throughout the defined space, with vacuum structure being the photon' s interaction object. Assuming a single photon emitted by a single-photon source [7] moves along the x-axis, the photon state can be expressed as $= e\hat{}\{i(kx\text{-}\omega t)\}$, where k is the wave vector (x-component) and ω is the angular frequency; assuming the vacuum cavity has planar dimensions L $\times$ L (L L) and light is visible, photon interaction with cavity walls produces standing waves satisfying k L $= 2\pi m$ (m being a positive integer), "decomposing" the single photon. Opening a small hole in the rear wall of the vacuum cavity should allow observation of photons with wave vector k k (or interference patterns). Statistically, k-wave vector photons can also be detected.

Now consider two photons moving in a slender vacuum tube (L L , where L is tube length and L is tube diameter). The macroscopic quantum state, combined with strong correlation effects between particles in low-dimensional systems [4], exhibits stronger correlation properties between the two photons. Representing the two photons by $_1 = e\hat{}\{i(k_1x\text{-}\omega_1t)\}$ and $_2 = e\hat{}\{i(k_2x\text{-}\omega_2t)\}$, or simply using wave vector algebraic superposition states, the combined state $\hat{k} = \hat{k}_1 + \hat{k}_2$ yields $\hat{k}^2 = k_1{}^2 + k_2{}^2 + 2k_1k_2\cos$. When $= 0°$, $\hat{k} = k_1 + k_2$; when $= 180°$, $\hat{k} = k_1 - k_2$. This "macroscopic" superposition quantum state composed of a few photons may be detectable throughout the long vacuum tube. This forms the basis of current quantum entanglement and quantum communication technologies, though the limitations of such techniques are obvious.

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