Analysis of Clock Difference Prediction Algorithms for Domestic Optically Pumped Cesium Atomic Clocks (Postprint)

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Abstract

Cesium atomic clocks are the critical core devices in modern atomic timekeeping. In recent years, the domestically produced optically pumped cesium atomic clock TA1000 has been widely applied in timekeeping operations. The clock difference prediction algorithm for atomic clocks is one of the factors affecting timekeeping performance. Different types of atomic clocks exhibit different noise characteristics, and noise affects the stability and accuracy of clock difference prediction. To investigate clock difference prediction algorithms suitable for TA1000, three classical clock difference prediction algorithms applied to cesium clocks are analyzed, including the first-order linear regression (Liner Regression, LR) model, the autoregressive integrated moving average (Autoregressive Integrated Moving Average model, ARIMA) model, and the Kalman (Kalman) model. By utilizing clock difference sequences with different sampling durations for modeling, the clock difference data for the next four different durations are predicted. Based on this, the clock difference prediction effects of the three models are analyzed and compared, and the advantages and disadvantages of each clock difference prediction model when applied to TA1000 are summarized. Experiments show that among the three models, the autoregressive integrated moving average model is more suitable for short-term clock difference prediction of TA1000.

Full Text

Preamble

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Analysis of Clock Difference Prediction Algorithm for Domestic Optically-pumped Cesium Atomic Clock

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Abstract

Cesium atomic clocks are critical core equipment in modern atomic timekeeping. In recent years, the domestic optically-pumped cesium atomic clock TA1000 has been widely applied in timekeeping operations. The clock difference prediction algorithm for atomic clocks is one of the factors affecting timekeeping performance. Different types of atomic clocks exhibit different noise characteristics, which influence the stability and accuracy of clock difference predictions. To investigate suitable clock difference prediction algorithms for the TA1000, we analyzed three classical prediction algorithms applied to cesium clocks: the first-order linear regression (LR) model, the autoregressive integrated moving average (ARIMA) model, and the Kalman model. By utilizing clock difference sequences with different sampling durations for modeling, we predicted the next four clock difference datasets of varying durations. Based on this, we analyzed and compared the clock difference prediction performance of the three models, summarizing their respective advantages and disadvantages when applied to the TA1000. Experiments demonstrate that among the three models, the ARIMA model is more suitable for short-term clock difference prediction of the TA1000.

Keywords time, methods: statistical, methods: data analysis

1 Introduction

Time and frequency signals are widely used in major national defense projects such as navigation and positioning, weapon systems, precision strike, and coordinated operations, as well as in national economic and livelihood fields including 5G communications, finance, power grid synchronization, and seismic monitoring. Time and frequency standard signals are typically generated and maintained by timekeeping systems [**?**, **?**]. The quality of time-frequency signals largely depends on the composition of the timekeeping system. Small cesium clocks are one of the core devices in modern timekeeping, utilizing quantum transitions of cesium atoms to generate standard time-frequency signals. Small cesium clocks exhibit excellent frequency accuracy and stability performance, with compact size and light weight, making them the most numerous and widely used timekeeping atomic clocks internationally.

Many countries have established independent standard time-frequency service systems, with each timekeeping laboratory utilizing multiple atomic clocks to form a clock ensemble for establishing and maintaining a time reference. The National Time Service Center of the Chinese Academy of Sciences (NTSC) has established a timekeeping system to independently maintain local Coordinated Universal Time (UTC). Historically, timekeeping laboratories worldwide have employed the American magnetic state-selection small cesium clock 5071A. In recent years, due to tense international situations and to meet the strategic demand for independently generating national standard time, the number of domestic optically-pumped cesium clocks TA1000 has been continuously increasing, gradually replacing the 5071A.

An important task in timekeeping is clock difference prediction, which involves predicting the future performance of atomic clocks based on historical clock difference data. Clock difference prediction algorithms form the foundation for comprehensive atomic time scale algorithms [**?**] and master clock frequency steering algorithms. The International Bureau of Weights and Measures has already adopted weighting of atomic clocks from various timekeeping laboratories worldwide based on their predictability. Different types of atomic clocks have different noise characteristics [**?**, **?**]. Therefore, during the gradual application of domestic optically-pumped cesium clocks in timekeeping operations, it is necessary to study suitable short-term clock difference prediction algorithms for them, although relevant research is still lacking. To ensure the continuity, stability, and reliability of UTC(NTSC), we selected clock difference prediction algorithms that have been applied in practical operations, including the univariate linear regression (LR) model [**?**, **?**], the autoregressive integrated moving average (ARIMA) model [**?**, **?**], and the Kalman model [**?**, **?**].

2 Optically-pumped Cesium Clock TA1000

The domestic demand for small cesium clocks is substantial, but they have primarily relied on imports. In recent years, domestic research institutions have successively conducted research on cesium atomic clocks to avoid technological bottlenecks. Significant progress has been made in key technology development, with several prototypes produced. However, constrained by core components such as electron multipliers, there remains a considerable gap toward engineering and mass production. With advances in laser technology, laser-pumped cesium beam tubes completely avoid the extremely difficult technologies and processes of magnetic state selection and electron multipliers, gradually showing a trend to replace traditional magnetic state-selection small cesium clocks. The adoption of optically-pumped schemes has accelerated the domestic production of small cesium clocks, and the TA1000 has now achieved mass production. Compared with foreign magnetic state-selection small cesium clocks, optically-pumped cesium clocks utilize lasers for atomic state preparation and transition detection, achieving higher atomic utilization efficiency.

Theoretical calculations indicate that the frequency stability of optically-pumped small cesium clocks is approximately one order of magnitude higher than that of traditional magnetic state-selection small cesium clocks. [Figure 1: see original paper] shows a schematic diagram of the TA1000 appearance. The TA1000 consists of three parts: the electronic system, optical system, and physical system. The laser system unit is responsible for quantum state preparation and detection of the atomic beam. The cesium beam tube unit serves as the physical system of the cesium clock, providing a vacuum environment for laser-microwave-atom interactions and generating Ramsey transition signals of the cesium atomic beam. The electronic system processes the atomic transition signals, generates locking signals, and ultimately outputs high-stability time-frequency reference signals.

3 Clock Difference Prediction Algorithms

A good atomic clock should be a predictable one. For atomic clocks with good predictability, the deviation between predicted and actual measured clock difference values is small. During clock difference data preprocessing, missing data can be filled in or abnormal data can be promptly detected based on clock difference prediction algorithms. Maintaining a continuous, stable, and reliable time reference requires timely detection of atomic clock anomalies through clock difference data, followed by reasonable adjustment of atomic clock weights according to the anomaly conditions to minimize the impact of anomalies on the time reference.

Within a timekeeping clock ensemble system, sub-clocks and reference clocks are mutually compared through time interval counters to obtain clock difference data, which are equally spaced phase measurements. The greatest advantage of cesium atomic clocks is their near-zero drift. Analysis of TA1000 clock difference data trends reveals that clock difference data generally exhibit linear trends. Additionally, relevant studies [?] have shown that fluctuations in atomic clock differences result from linear superposition of various noise types, all of which provide feasibility for clock difference prediction. Through mathematical modeling and analysis of clock difference data, the accuracy of atomic clock difference prediction can be improved.

3.1 Univariate Linear Regression Model

Linear regression [?] refers to the process of learning a pattern from given data using computational methods, where the regression coefficients of this pattern are required to be linear, and then using the learned model to predict new data. The univariate linear regression model [?] has only one independent variable x and dependent variable y, without non-first-order terms such as squares. Its expression is:

$$y = kx + b \tag{1}$$

The objective of the univariate linear regression model is to determine the regression coefficient k and constant term b such that the model predictions are as close as possible to the true observed values. The univariate linear regression model allows for errors between predicted and true values. Individual true values may lie above or below the regression line. As the amount of data increases, the average of these errors approaches zero, meaning the residuals follow a normal distribution with zero mean. The parameters k and b are obtained through the least squares method. The optimization objective of the univariate linear regression model is to minimize the mean squared error, expressed as:

$$J(k; b) = \sum_{i=1}^{m} (y_i - \hat{y}_i) \tag{2}$$

where $J(k; b)$ is the total sum of squared deviations between all observed values and their corresponding regression estimates, y_i represents observed values, $\hat{y}_i$ represents corresponding regression estimates, and m is the sample size. Taking partial derivatives of the mean squared error function with respect to k and b and setting them equal to zero yields the optimal solution.

3.2 Autoregressive Integrated Moving Average Model

The autoregressive integrated moving average (ARIMA) model [**?**, **?**] is one of the most common statistical models used for time series forecasting, which can be divided into three components: the autoregressive model, the moving average model, and differencing. The ARIMA model has three parameters and can be expressed as ARIMA(p, d, q), where p is the number of autoregressive terms, q is the number of moving average terms, and d is the number of differencing operations required to make the sequence stationary.

The autoregressive moving average model combines autoregressive and moving average models, containing p autoregressive terms and q moving average terms, expressed as:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{3}$$

where y_t represents the value at time t, ϕ_0 is the constant term, ϕ_i are autoregressive coefficients, ε_t is the white noise error term at time t, and θ_i are moving average coefficients.

The autoregressive moving average model indicates that a random time series can be composed of its own historical values and random disturbance terms. If the series is stationary, meaning its trend does not change over time, future moments can be predicted using historical values of the series.

The ARIMA model combines the autoregressive moving average model with differencing. Its principle is to transform data into stationary data through differencing, and then establish a model for the differenced stationary data using only its historical values and current and lagged values of random error terms. The model' s advantages are simplicity and ease of training, requiring only endogenous variables without external variables. Its disadvantages include the requirement that time series data be stable or become stable after differencing, and that the model can only capture linear relationships, not nonlinear ones.

3.3 Kalman Model

Kalman Filtering (KF) [**?**, **?**] is an algorithm that utilizes linear system state equations to optimally estimate system states from a set of measurements containing system noise and interference noise. Kalman filtering calculates the joint distribution at different moments based on measurement values at various times to estimate unknown factors, thus providing more accurate estimates than methods based on single measurements.

The Kalman filter algorithm calculates the current state estimate using the previous moment' s state estimate and the current moment' s observation value. The algorithm consists of two parts: prediction and update, which iterate continuously to obtain optimal estimates. In the prediction phase, the estimate of the current state is made using the estimate from the previous state, including uncertainty estimates. In the update phase, the observation value of the current state and the predicted value obtained from the prediction phase are weighted and averaged for optimization, where the observation value contains certain errors such as random noise. Variables with higher certainty receive greater weight, resulting in a more accurate new estimate.

Let $\hat{X}_{t|t}$ represent the state estimate at time t. The Kalman filter state is represented by the following variables: $\hat{X}_{t|t-1}$ represents the prediction of the state at time t given past states up to time $t-1$, and $P_{t|t-1}$ represents the prediction of the error covariance matrix at time t given the error covariance matrix from past $t-1$ moments. $P_{t|t}$ represents the posterior estimate error covariance matrix at time t.

The prediction phase expressions are as follows:

$$\begin{cases} \hat{X}_{t|t-1} = F_t \hat{X}_{t-1|t-1} + B_t u_t \\ P_{t|t-1} = F_t P_{t-1|t-1} F_t^T + Q_t \end{cases} \tag{4}$$

where F_t is the state transition matrix, F_t^T is the transpose of the state transition matrix, B_t is the control matrix, u_t is the control vector, and Q_t is the process noise covariance matrix.

In the update phase, the current state estimate is calculated based on the current observation value and predicted value. This estimate is more accurate because the current observation value participates in the calculation. The error covariance matrix of the state estimate is obtained through the error covariance matrix calculated in the prediction step, the observation noise covariance matrix, and the Kalman gain. The update phase expressions are as follows:

$$\begin{cases} K_t = P_{t|t-1} H_t^T (H_t P_{t|t-1} H_t^T + R_t)^{-1} \\ \hat{X}_{t|t} = \hat{X}_{t|t-1} + K_t (Z_t - H_t \hat{X}_{t|t-1}) \\ P_{t|t} = (I - K_t H_t) P_{t|t-1} \end{cases} \tag{5}$$

where Z_t is the observation value, R_t is the observation noise covariance matrix, H_t is the measurement mapping matrix, H_t^T is the transpose of the mapping matrix, K_t is the Kalman gain, and I is the identity matrix.

4 Experiments and Results Analysis

We conducted clock difference prediction and testing using phase comparison data between four optically-pumped cesium clocks at the National Time Service Center' s reference laboratory and China' s time reference UTC(NTSC). The prediction models included the first-order linear regression model, the ARIMA model, and the Kalman filter model. The original clock difference data had a sampling interval of 1 h, and the predicted clock difference data also had a time interval of 1 h. To investigate suitable short-term clock difference prediction models for optically-pumped cesium clocks, each prediction model utilized 1 d, 10 d, and 30 d of clock difference data for modeling, then predicted clock difference data for the next 12 h, 1 d, 2 d, and 5 d (i.e., each clock had test data volumes of 12, 24, 48, and 120 points). The prediction results were compared with measured values to evaluate the prediction stability and accuracy of the models.

We employed two performance metrics, Root Mean Squared Error (RMSE) and Range (R), to evaluate the accuracy and stability of prediction results [**?**]. The specific definitions of RMSE and Range are:

$$\begin{cases} \epsilon_t = test_t - pred_t \\ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \epsilon_t^2} \\ R = |\epsilon_{max} - \epsilon_{min}| \end{cases} \tag{6}$$

where ϵ_t is the residual between the expected output $test_t$ and the predicted value $pred_t$ at time t, and ϵ_{max} and ϵ_{min} represent the maximum and minimum values in the residual sequence ϵ_t, respectively.

shows the frequency stability of four TA1000 clocks relative to the reference signal UTC(NTSC) at 1 h, 1 d, and 5 d. and display the average RMSE and

Range statistical values for four TA1000 clocks using 1 d of data for modeling to predict different durations. The RMSE and Range of all three models increase with prediction duration. The first-order linear regression model generally outperforms the ARIMA model, while the Kalman model' s errors are significantly larger than those of the first-order linear regression and ARIMA models. This indicates that the first-order linear regression and ARIMA models can learn clock difference trends with only small amounts of modeling data, whereas the Kalman model' s state estimation becomes inaccurate due to insufficient modeling data.

and show the RMSE and Range for four TA1000 clocks using 10 d of data for modeling to predict different durations. According to the RMSE and Range, increasing prediction duration still leads to larger prediction errors. Further comparison with and reveals that as the modeling data volume increases, the Kalman model' s RMSE and Range generally decrease because it can more accurately estimate clock difference state parameters. The first-order linear regression model experiences increased mean squared errors due to clock noise interference, while the ARIMA model, which first differences the observation data to eliminate non-stationary components, demonstrates more robust performance.

and present the average RMSE and Range statistical values for four TA1000 clocks using 30 d of data for modeling to predict different durations. Since the first-order linear regression model is affected by cesium clock stability factors during long-term modeling, causing the prediction starting position not to be at zero, resulting in large deviations when directly calculating RMSE, we combined the first-order linear regression model with differencing for modeling when using 30 d of data. With further increases in modeling data volume, the first-order linear regression model' s RMSE and Range significantly decrease compared to using 10 d of data, indicating that long-term modeling requires combination with differencing. The Kalman model' s RMSE and Range further decrease, related to its excellent noise suppression capability, while also demonstrating that the Kalman model has certain requirements for modeling data volume. The ARIMA model' s RMSE and Range show small fluctuations with changes in modeling data volume, indicating its good noise suppression capability for clock difference data.

[Figure 2: see original paper] shows the model fitting residuals for four cesium atomic clocks using 30 d of data for modeling. All three models inevitably have model errors. The modeling residuals of ARIMA and Kalman models fluctuate near zero, while the first-order linear regression model experiences large deviations in modeling residuals due to noise and other factors during long-term modeling.

[Figure 3: see original paper] displays the prediction residuals calculated according to equation (6) for four TA1000 atomic clocks using 30 d of modeling data to predict 1 d duration. The first-order linear regression model performs differently when applied to the four atomic clocks. It performs poorly for Clock

1, while the remaining three clocks are suitable for this model. The ARIMA and Kalman models show similar performance. As prediction duration increases, the ARIMA model fluctuates more smoothly overall than the Kalman model.

and compare the average RMSE and Range statistical values for three models under three modeling data volumes. According to the Range statistical values, the first-order linear regression and ARIMA models show similar prediction stability, both significantly outperforming the Kalman model. According to the RMSE, the ARIMA model clearly demonstrates superior prediction accuracy. The ARIMA model has the effect of eliminating non-stationary components, so changes in modeling data volume have little impact on its prediction accuracy, without the phenomenon of rapidly deteriorating prediction accuracy with increasing prediction duration. Additionally, using smaller modeling data volumes can ensure clock difference prediction accuracy.

5 Conclusion

To enable the optically-pumped cesium atomic clock TA1000 to perform better in timekeeping clock ensembles, and to ensure the continuity, stability, and reliability of China's time reference UTC(NTSC) during the process of atomic clock domestication, we analyzed three classical clock difference prediction algorithms applied to cesium clocks: the first-order linear regression model, the ARIMA model, and the Kalman model. By comparing the prediction stability and accuracy of the three models, we identified the advantages and disadvantages of each algorithm when applied to TA1000:

(1) Under the same modeling data volume conditions, the prediction error of any given model increases with prediction duration.

(2) For short-term clock difference prediction, the first-order linear regression model produces the best results with only small amounts of clock difference modeling data, though long-term modeling requires further investigation. The Kalman model requires sufficient clock difference data. The ARIMA model demonstrates more robust performance across different modeling data volumes.

(3) Based on prediction stability and accuracy, the ARIMA model is more suitable for short-term clock difference prediction of the optically-pumped cesium clock TA1000.

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Note: Figure translations are in progress. See original paper for figures.