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Authors: Zhang Zhaowei, Han Wenbiao

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Full Text

Preamble

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ZHANG Zhaowei[1,2], HAN Wenbiao[1,2]

(1. Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China; 2. School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China)

Abstract

Future space-based gravitational wave detectors (such as LISA and Taiji) will detect gravitational wave events from the mergers of supermassive binary black holes. In addition to emitting traditional inspiral-merger-ringdown gravitational wave signals, strongly precessing supermassive binary black holes can also produce gravitational wave burst signals from their spin precession that are observable by space-based detectors. By simulating gravitational wave signals from several precessing binary black hole systems, we investigate the capability for parameter estimation from gravitational wave burst signals radiated by the black holes' intrinsic quadrupole moments, providing a feasibility analysis for testing the no-hair theorem. Using the Fisher matrix method, we calculate confidence intervals for parameter estimation. Our results demonstrate that if such gravitational wave burst signals can be detected, this would directly confirm the existence of quadrupole moments in black holes. Cross-validation with gravitational wave signals from the inspiral-merger-ringdown phases can further test the no-hair theorem.

Keywords: supermassive binary black holes; spin precession; gravitational-wave bursts; parameter estimation; no-hair theorem

1 Introduction

The rise of gravitational wave astronomy has provided new opportunities to study extreme celestial objects and strong gravitational fields in the universe. Since the Laser Interferometer Gravitational-Wave Observatory (LIGO) made the first direct detection of gravitational waves in 2015, the Virgo interferometer and the Kamioka Gravitational-Wave Detector (KAGRA) have observed and analyzed multiple black hole merger events, as well as gravitational wave signals from binary neutron stars, black hole-neutron star systems, and binary black hole systems [**?**, **?**]. These observations have not only verified the predictions of general relativity but also provided important observational data for understanding the physical properties of black holes and gravitational theories.

The no-hair theorem in general relativity states that isolated black holes in steady state are completely described by only three parameters: mass, angular momentum, and charge, with all other multipole moments uniquely determined by these three parameters \cite{3–5}. The validity of this theoretical prediction in the strong-field limit must be tested through observations. If observed black hole multipole moments deviate from the predictions of general relativity, this would have profound implications for gravitational theory.

Observations from LIGO and Virgo have made it possible to test the no-hair theorem. By analyzing the ringdown phase of gravitational wave signals, one can measure the quasi-normal mode frequencies and damping times of the post-merger black hole, thereby obtaining information about the black hole' s multipole moments [?, ?]. Specifically, using the frequencies and damping times of quasi-normal modes, one can infer the mass and spin of the black hole and test whether these parameters are consistent with results obtained from the inspiral and merger phases. However, due to the low signal-to-noise ratio of ringdown signals from stellar-mass black holes observed by ground-based detectors, particularly the even lower signal-to-noise ratio of higher-order modes, strict tests of the no-hair theorem have been limited.

Furthermore, ground-based detectors have limited sensitivity in the low-frequency band, making it difficult to detect low-frequency gravitational wave signals produced by supermassive black hole (SMBH) merger events. The merger of supermassive black holes is considered an ideal astrophysical laboratory for testing the no-hair theorem because the ringdown phase signals are stronger and last longer, providing higher precision measurements. Future space-based gravitational wave detectors, such as the Laser Interferometer Space Antenna (LISA) and China' s Taiji program, aim to detect low-frequency gravitational wave signals from supermassive binary black hole systems. Space-based detectors cover a frequency range of $10^{-4} \sim 1$ Hz, which corresponds exactly to the gravitational wave frequencies of supermassive black hole mergers. These detectors can observe supermassive black hole merger events with high signal-to-noise ratios, particularly in the ringdown phase, enabling precise measurements of black hole quasi-normal mode parameters and more stringent tests of the no-hair theorem [?].

Precessing binary black hole systems represent important research objects in gravitational wave astronomy due to the rich physical information contained in their signals. Spin-orbit coupling causes spin precession when the spin axes of the binary black holes are misaligned with the orbital angular momentum. This precession leads to complex modulation of the amplitude and phase of gravitational wave signals [?]. In traditional precessing binary black hole models, research has typically focused on the modulation effects of orbital precession on gravitational waves, without fully considering the role of changes in the black holes' intrinsic quadrupole moments. If variations in black hole quadrupole moments can directly generate gravitational wave burst signals, this would provide new possibilities for testing the no-hair theorem in general relativity. Precisely capturing these gravitational wave burst signals would help verify this fundamental property of black holes and deepen our understanding of their internal structure and evolutionary processes.

Gravitational wave bursts arise in precessing binary black hole systems because the mass quadrupole moment of a black hole changes with time due to its spin precession, radiating brief, transient gravitational wave signals. This effect differs from traditional waveform models that only consider precession' s modulation of orbital gravitational waves; instead, the quadrupole moment variation caused by black hole spin precession itself becomes a unique gravitational wave source. The frequency of these bursts is comparable to that of the late inspiral stage, and under certain conditions, their intensity is sufficient for detection by LIGO. In next-generation detectors such as LISA, the signal-to-noise ratio can reach even higher values. Observations of such precessing gravitational wave bursts would provide important information about black hole structure [?].

This paper employs the Fisher matrix method to evaluate parameter estimation capabilities for supermassive binary black holes, including their masses, spins, and quadrupole moments. By extracting information about black hole quadrupole moments carried by gravitational wave signals, we can test the no-hair theorem' s prediction that a black hole' s quadrupole moment is uniquely determined by its mass and spin [?].

The structure of this paper is as follows: Section 1 introduces the development of gravitational wave astronomy and research background on the no-hair theorem, clarifying our research motivation; Section 2 details gravitational waves from precessing binary black holes, including waveform characteristics during the inspiral-merger-ringdown phases and gravitational wave burst phases; Section 3 describes our data analysis methods, focusing on the application of Fisher matrices in gravitational wave parameter estimation and no-hair theorem testing; Section 4 presents data analysis results and discusses parameter estimation for supermassive precessing binary black hole signals and testing of the no-hair theorem; finally, we summarize the main content and outline future research directions.

2 Gravitational Waves from Precessing Binary Black Holes

Precessing binary black hole systems are important research targets in gravitational wave astronomy. When the spin axes of a binary black hole system are misaligned with the orbital angular momentum, spin-orbit coupling effects cause the system to precess. This precession makes the orbital plane and black hole spin directions vary with time, resulting in complex amplitude and phase modulation of gravitational wave signals throughout the inspiral phase.

During the inspiral phase, black holes gradually approach each other as gravitational wave radiation reduces the orbital radius. Precession effects cause periodic or quasi-periodic modulation of gravitational wave amplitude and phase. These modulated signals contain information about black hole spin magnitudes, orientations, and masses, providing an important basis for precise black hole parameter measurement. Gravitational wave signals exhibit asymmetry and rapid frequency evolution. Numerical relativity simulations show that precession affects the spin of the post-merger black hole, thereby altering gravitational wave radiation characteristics.

In the ringdown phase, the merged black hole radiates gravitational waves through quasi-normal mode oscillations, gradually settling into a stable state.

Precession may lead to multi-mode characteristics in ringdown signals, providing possibilities for testing the no-hair theorem of general relativity and measuring black hole multipole moments.

Furthermore, precessing black holes can produce transient gravitational wave events—gravitational wave bursts—that are generated directly by the black holes' quadrupole moments. Traditional binary black hole inspiral-merger-ringdown models do not consider this gravitational wave burst component, which we can use to test the accuracy of the no-hair theorem. The following sections discuss in detail the gravitational wave characteristics of precessing binary black holes at various stages and the signal features of gravitational wave bursts.

2.1 Inspiral-Merger-Ringdown Gravitational Waves from Precessing Binary Black Holes

For full parameter estimation, we employ frequency-domain waveforms for the inspiral-merger-ringdown phases of binary black holes [**?**]. The two polarization states of gravitational waves, $\tilde{h}_+(f)$ and $\tilde{h}_\times(f)$, are expressed in the frequency domain as [**?**, **?**]:

$$\tilde{h}_+(f) = A(f) \cdot \frac{1+\cos^2\iota}{2} \cdot \cos(\Phi(f))$$
$$\tilde{h}_\times(f) = A(f) \cdot \cos\iota \cdot \sin(\Phi(f))$$

where $A(f)$ is the amplitude and ι is the inclination angle. The expressions for amplitude $A(f)$ and phase $\Phi(f)$ are \cite{11-13}:

$$A(f) = \begin{cases} A_{\mathrm{INS}}(f), & f < f_{\mathrm{match}} \\ A_{\mathrm{MR}}(f), & f \geq f_{\mathrm{match}} \end{cases}$$

$$\Phi(f) = \begin{cases} 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi \mathcal{M}_c f)^{-5/3} + \delta\Phi_{\mathrm{PN}}, & f < f_{\mathrm{match}} \\ \Phi_{\mathrm{MR}}(f), & f \geq f_{\mathrm{match}} \end{cases}$$

where f_{match} is the piecewise matching frequency for the amplitude/phase expressions, t_c is the merger time, c is the speed of light, ϕ_c is the phase at merger, $\mathcal{M}_c$ is the chirp mass, $\delta\Phi_{\mathrm{PN}}$ is the post-Newtonian correction term, and $A = \frac{c}{D_L \pi^{-2/3}}$ with D_L being the luminosity distance from source to Earth. Both the amplitude $A_{\mathrm{MR}}(f)$ and phase $\Phi_{\mathrm{MR}}(f)$ for the merger and ringdown phases are fitting functions based on numerical relativity simulations \cite{11-13}.

2.2 Precessing Gravitational Wave Bursts from Binary Black Holes

When the spin axis of a black hole is misaligned with the orbital angular momentum direction, spin precession occurs during binary black hole merger through spin-orbit coupling. Although spin-spin coupling can also cause precession, its contribution to orbital dynamics is much smaller than that of spin-orbit coupling. Since orbital angular momentum is typically several times larger than spin angular momentum (for black holes, the spin parameter $\chi < 1$) and the spin-orbit coupling coefficient is larger, the effect of spin-spin coupling can be neglected. For simplicity, we consider only the dominant spin-orbit coupling effect [?]. The gravitational wave burst signal can be expressed as [?]:

$$\tilde{h}_{+,1} = -\frac{\pi^{6/5} f^{7/10} G^{27/10} m^3 M^{1/5} F^{4/5}}{2^{33/10} c^{71/10} \mu^{1/2} D_L} \delta e^{i\psi_{+,1}} \sin 2\alpha \sin 2\iota$$

Through similar treatment of $\tilde{h}_{+,2}$, we obtain the waveform for the dominant 2Ω component ($f_2 = 2f$):

$$\tilde{h}_{+,2} = -\frac{\pi^{6/5} f^{7/10} G^{27/10} m^3 M^{1/5} F^{4/5}}{c^{71/10} \mu^{1/2} D_L} \delta e^{i\psi_{+,2}} \sin^2 \alpha \cos^4 \frac{\iota}{2}$$

$$\psi_{+,2} = 2\pi f t_2 - \Phi_c - \pi \frac{25 c^{9/5} M^{2/5} F^{8/5}}{192 \mu (\pi f_2 G)^{3/5}}$$

where ι is the angle between the line of sight and the initial (non-precessing) spin direction, α is the spin rotation angle, $\mu = m_1 m_2 / M$ is the reduced mass, and F_1, F_2 are related to the mass ratio of the binary black holes. When the precession effect is strong ($\alpha \geq \pi/4$), the 2Ω component is significantly enhanced, being almost several times stronger than the Ω component [?]. In this case, $\tilde{h}_{+,2}$ is the dominant component, and we use the $\tilde{h}_{+,2}$ signal to evaluate parameter estimation capabilities.

According to the no-hair theorem, a black hole' s mass multipole moments are completely determined by its mass and spin. The relationship between δ and the spin parameter χ is:

$$\delta = 2\left(1 - \sqrt{1-\chi^2}\right)\left(1 - \sqrt{1-\chi^2}^2 - 1\right)$$

For $0 \leq \chi \leq 1$, we obtain $0 \leq \delta \leq 2(\sqrt{2} - 1)$. For $\chi = 0.9$, $\delta \approx 0.52$. According to general relativity, the parameter δ has specific predicted values. However, in non-general relativistic scenarios, the value of δ may deviate from this prediction, making δ a variable parameter. By quantitatively analyzing the precision of δ measurements, we can assess the sensitivity for testing general relativity. Therefore, precision analysis of parameter estimation provides a research method for validating the effectiveness of general relativity.

3 Data Analysis

The Fisher information matrix is a key mathematical tool in gravitational wave data analysis. By quantifying the sensitivity of gravitational wave signals to source physical parameters, it has important applications, particularly in analyzing precessing binary black hole systems. Based on the Fisher matrix method, we investigate confidence intervals for key physical parameters. This method demonstrates unique advantages when analyzing non-stationary and non-Gaussian background noise, providing more precise quantitative analysis of parameter calculation accuracy.

3.1 Signal-to-Noise Ratio (SNR)

The performance of gravitational wave detectors is first evaluated through the signal-to-noise ratio (SNR). The SNR calculation formula is:

$$\rho = \sqrt{\sum_{n=1}^{N_d} \rho_n^2}$$

where N_d is the number of interferometers and ρ_n is the SNR of the n-th interferometer, defined as:

$$\rho_n = \sqrt{\langle \tilde{h}_n, \tilde{h}_n \rangle}$$

where $\tilde{h}_n$ represents the Fourier transform of the gravitational wave signal in the n-th interferometer, and $\langle \cdot, \cdot \rangle$ denotes the inner product defined under the noise power spectral density $S_n(f)$ as [**?**]:

$$\langle a, b \rangle = 4 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \frac{a^*(f) b(f)}{S_n(f)} df$$

3.2 Fisher Information Matrix and Parameter Uncertainties

For the parameter vector to be estimated $\theta = (\theta_1, \theta_2, \cdots, \theta_k)$, the Fisher information matrix is defined as:

$$F_{ij} = \sum_{n=1}^{N_d} \left\langle \frac{\partial \tilde{h}_n}{\partial \theta_i}, \frac{\partial \tilde{h}_n}{\partial \theta_j} \right\rangle$$

where $\tilde{h}_n$ is the Fourier transform of the gravitational wave signal in the n-th interferometer and θ_i is the i-th component of the source parameters. The element F_{ij} of the Fisher information matrix measures the sensitivity of the signal to changes in parameters θ_i and θ_j.

By solving the inverse of the Fisher information matrix $\Sigma = \mathbf{F}^{-1}$, we obtain the covariance matrix Σ calculated from the parameters, where Σ_{ij} represents the covariance between parameters θ_i and θ_j. Parameter uncertainties are given by the diagonal elements of the covariance matrix:

$$\Delta\theta_i = \sqrt{\Sigma_{ii}}$$

The Fisher matrix is an effective parameter estimation tool that can evaluate parameter uncertainties by calculating the Fisher information matrix [**?**, **?**]. It has wide applications in estimating gravitational wave source physical parameters (such as mass, spin, distance, etc.), particularly when analyzing signals from precessing binary black hole systems. This paper employs the Fisher matrix method, which combines high-precision precessing waveform models (such as IMRPhenomPv3) with gravitational wave burst signals. Using this method, we calculate confidence intervals for source parameters and conduct targeted verification of the no-hair theorem.

4.1 Comparison of Detection Signals and Detector Sensitivity

Based on the IMRPhenomPv3 waveform model for the inspiral-merger-ringdown phases, we generate precessing waveforms. First, through numerical simulations, we calculate the frequency-domain images of gravitational wave signals from binary black holes of different masses at a distance of 5 Gpc. Then, we compare the observation precision of the observation curves within the detection ranges of LISA and TAIJI detectors. The vertical coordinate $2f|h_+|$ represents the frequency-weighted absolute value of the gravitational wave plus polarization amplitude h_+, where $|h_+|$ is the absolute amplitude and the coefficient 2 adjusts the frequency response of the amplitude, emphasizing high-frequency components. The detailed analysis of waveform features is shown in Figure 1a [Figure 1: see original paper].

In Figure 1b, we compare supermassive black hole gravitational wave burst signals with the sensitivity curves of current and future detectors, evaluating the detectability of such signals and the potential for extracting black hole physical parameters. For a binary black hole system with masses of $5.5 \times 10^5 M_\odot$ and $5 \times 10^5 M_\odot$, spins of 0.9, luminosity distance of 5 Gpc, and spin rotation angle $\alpha = \pi/4$, the signal-to-noise ratio of the gravitational wave burst signal reaches 52.64. According to the results in Figure 1a, the inspiral signals from supermassive binary black holes can be detected by current and future detectors across a broad frequency range. However, Figure 1b shows that effective detection of supermassive binary black hole gravitational wave burst signals requires specific mass and spin conditions. Black holes with masses in the range $10^5 M_\odot \sim 10^6 M_\odot$ and high spins are more likely to have detectable gravitational wave burst signals. This is because larger spins can enhance the intensity of gravitational wave bursts, increase the signal-to-noise ratio, and improve the likelihood of detection by detectors.

4.2 Parameter Estimation Capability Assessment Using Inspiral-Merger-Ringdown Waveforms

We performed full parameter estimation using gravitational wave signals from the inspiral-merger-ringdown phases. During the inspiral phase, spin precession causes variations in the mass quadrupole moment, leading to waveform modulation in the gravitational wave signals. In binary black hole systems, when the black hole spin axis is misaligned with the orbital angular momentum direction, the precession effect from spin-orbit coupling causes periodic changes in the orbital plane, altering the system' s mass quadrupole moment and thereby modulating the phase and amplitude of gravitational waves, affecting the evolution of the entire waveform. Through full parameter estimation of precessing gravitational wave signals in the inspiral phase, we obtained key parameters such as black hole mass and spin [**?**, **?**].

We have nine parameters: $D_L, M_1, M_2, \text{spin}_{1x}, \text{spin}_{1y}, \text{spin}_{1z}, \text{spin}_{2x}, \text{spin}_{2y}, \text{spin}_{2z}$. The values are: distance $D_L = 5$ Gpc, masses $M_1 = 5.5 \times 10^5 M_\odot$, $M_2 = 5 \times 10^5 M_\odot$, spins $\text{spin}_{1x} = 0.7$, $\text{spin}_{1y} = 0.2$, $\text{spin}_{1z} = 0.1$, $\text{spin}_{2x} = 0.4$, $\text{spin}_{2y} = 0.4$, $\text{spin}_{2z} = 0.7$. The uncertainties are: $\Delta D_L = 10^{-4}$, $\Delta M_1 = 10^{-5}$, $\Delta M_2 = 10^{-5}$. The results are shown in Figure 2 [Figure 2: see original paper]. In Figure 2, we present the parameter estimation results for the inspiral-merger-ringdown signals. The error estimates for all nine parameters meet expectations, with mass parameters showing the best estimation accuracy. After estimating the basic black hole parameters, we combine them with parameter estimation results from gravitational wave bursts to complete the final no-hair theorem verification.

4.3 Parameter Estimation Capability Assessment for Gravitational Wave Burst Signals

To test the no-hair theorem, we first independently estimate the parameter δ from gravitational wave burst signals. Gravitational wave burst signals primarily originate from changes in the mass quadrupole moment of Kerr black holes. When the black hole spin axis is misaligned with the orbital angular momentum direction, the precession effect causes variations in the mass quadrupole moment, thereby affecting the phase and amplitude of gravitational waves [**?**, **?**].

We use the Fisher matrix method to evaluate the parameter estimation precision for a series of different δ values and calculate the confidence intervals for δ, with results shown in Figure 3 [Figure 3: see original paper]. The figure shows that parameter estimation precision varies with the value of δ. When δ is larger (e.g., $\delta = 0.5$ and $\delta = 0.6$), we obtain relatively narrow confidence intervals, enabling high-precision testing. Through gravitational wave burst signals, we can estimate the parameter δ within certain confidence intervals, providing a method for testing the no-hair theorem.

4.4 Parameter Estimation and Testing of the No-Hair Theorem

After obtaining the estimated value of δ, we compare it with calculation results for black hole mass and spin parameters from the inspiral, merger, and ringdown phases. We first use gravitational wave signals from these phases to calculate the black hole mass and spin parameters, obtaining confidence intervals of $M_1 = 5.5 \times 10^5 M_\odot$, $\chi_1 = 0.7348$, and $M_2 = 5.0 \times 10^5 M_\odot$, $\chi_2 = 0.9$. According to the formula, the corresponding theoretical δ values are $\delta_1 = 0.308$ and $\delta_2 = 0.517$. We then simulate gravitational wave burst signals from these two black holes and use the Fisher matrix method to estimate δ, with results shown in Figure 4 [Figure 4: see original paper].

By comparing the estimated δ value from gravitational wave burst signals with the theoretical δ value calculated from spin parameters obtained through the no-hair theorem in the inspiral phase, and through confidence interval comparison and final observational results, we can provide theoretical verification for the no-hair theorem. Our analysis shows that combining gravitational wave signals from the inspiral-merger-ringdown phases allows accurate estimation of black hole mass and spin. The research indicates that under conditions where spin exceeds 0.5, we can verify the feasibility of the no-hair theorem within a maximum error of 10%. As the spin parameter approaches extreme values, specifically between 0.7 and 0.9, the precision of confidence intervals improves significantly, with errors reducible to within 5%, or even 2%.

These results indicate that as spin increases, the error in δ gradually decreases. If next-generation gravitational wave detectors such as LISA and Taiji provide actual observational results, we can more accurately extract black hole physical properties from gravitational wave observation data. Furthermore, high-precision confidence intervals are crucial for testing the no-hair theorem in general relativity, particularly in extreme astrophysical environments.

5 Summary and Outlook

Through in-depth analysis of precessing gravitational wave burst signals from supermassive binary black hole systems, this study finds that precessing gravitational wave burst signals can be used to test the no-hair theorem. We estimated the signal-to-noise ratio of supermassive binary black hole gravitational wave burst signals, theoretically proving that they can be observed by gravitational wave detectors such as LISA and Taiji. If we can detect gravitational wave burst signals, this would indicate that black holes' intrinsic quadrupole moments can radiate gravitational waves, enabling cross-validation with gravitational waves emitted during the inspiral-merger-ringdown phases to further verify general relativity. Using gravitational wave signals that include the inspiral-merger-ringdown phases and combining them with the Fisher matrix method, we provide capability assessments for estimating key parameters such as black hole mass and spin. By comparing parameter estimation results from gravitational wave burst signals with those from inspiral phase signals, we provide a more precise method for testing the no-hair theorem.

However, the precision of the Fisher matrix method depends on detector sensitivity and noise levels, requiring calibration with actual observational data in the future. Additionally, this analysis is based on numerical simulations and has not yet been validated with real observational data; analysis of actual gravitational wave events is needed to solidify conclusions. With the operation of next-generation space-based gravitational wave detectors like LISA and Taiji, more low-frequency gravitational wave signals from supermassive binary black hole mergers will provide opportunities for further testing the no-hair theorem and exploring black hole dynamics. Future research will incorporate more complex nonlinear effects and higher-order corrections to improve waveform model precision and systematically compare waveforms under different gravitational theories to deepen understanding of extreme astrophysical phenomena.

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Note: Figure translations are in progress. See original paper for figures.