

## Research on Dynamic Three-Dimensional Terrain Correction Methods for Quantitative Inversion in Airborne Gamma-Ray Spectrometry

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### Abstract

Aerial survey is a dynamic and continuous process, and there are also different height distributions of the ground in the measurement area, which leads to problems such as overlapping of measurement areas and inaccurate altitude correction during the survey process. Commonly used terrain correction methods are based on the concept of finite elementization of ground surface radioactive sources, using GPS coordinates, radar altitude, and ground elevation distribution information from aerial surveys, combined with the sourceless efficiency calibration method to construct a response matrix, and then inverted for surface nuclide content. But most of the sourceless efficiency calibration methods used are numerical calculations that consider the body detector as a point detector and do not take into account the changes in intrinsic detection efficiency under different incident directions of gamma rays. Therefore, when the altitude of the measurement area varies greatly or the flight altitude of the aerial survey is relatively low, such sourceless efficiency calibration method calculations tend to have a large bias, which affects the accuracy of the terrain correction. To address the above problems, this paper employs a novel sourceless efficiency calibration method based on the Boolean operation of the ray deposition process, and simplifies the traditional body source measurement model to the surface source measurement model to achieve fast and accurate efficiency calibration; and then, through the discretization of the measurement process, superposes the static measurement process as equivalent to the dynamic measurement process, and builds and optimizes the dynamic measurement response matrix based on the calibration method; Finally, the PSO-MLEM algorithm is used to solve the dynamic measurement response matrix to achieve the dynamic terrain correction of aerial survey data. Analysis of the Baiyun'ebo test area reveals that, after applying dynamic terrain correction, the inverted anomalies in uranium

(eU), thorium (eTh), and potassium (K) concentrations are closer to ground measurements (within 5.72%–30.79%) and exhibit clearer anomaly boundaries compared to traditional height-based corrections. However, due to the inherent statistical fluctuations and the characteristics of matrix inversion, higher measurement values tend to absorb lower ones, potentially enlarging anomalous regions. Nevertheless, the high-anomaly regions after inversion largely coincide with ground truth validation, demonstrating that the proposed method can effectively correct airborne gamma spectrometry data.

## Full Text

### Preamble

Research on Dynamic Three-Dimensional Terrain Correction Methods for Quantitative Inversion in Airborne Gamma-Ray Spectrometry

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Aerial surveying is a dynamic and continuous process complicated by varying ground elevation distributions within the measurement area, leading to issues such as overlapping measurement zones and inaccurate altitude correction during surveys. Conventional terrain correction methods employ finite element concepts for ground surface radioactive sources, utilizing GPS coordinates, radar altitude, and ground elevation distribution data from aerial surveys, combined with sourceless efficiency calibration to construct a response matrix for inverting surface radionuclide concentrations. However, most sourceless efficiency calibration methods rely on numerical calculations that treat the detector as a point detector while neglecting variations in intrinsic detection efficiency under different gamma-ray incident directions. Consequently, when altitude variations are substantial or flight altitudes are relatively low, such sourceless efficiency calibration methods tend to produce significant biases that compromise terrain correction accuracy. To address these limitations, this paper introduces a novel sourceless efficiency calibration method based on Boolean operations of the ray deposition process, simplifying the traditional volume source measurement model to a surface source model for rapid and accurate efficiency calibration. By discretizing the measurement process, the static measurement process is superimposed to approximate the dynamic measurement process, and a dynamic measurement response matrix is constructed and optimized based on this calibration method. Finally, the PSO-MLEM algorithm is employed to solve the dynamic measurement response matrix, achieving dynamic terrain correction

of aerial survey data. Analysis of the Baiyun'ebo test area reveals that after applying dynamic terrain correction, the inverted anomalies in uranium (eU), thorium (eTh), and potassium (K) concentrations are closer to ground measurements (within 5.72%–30.79%) and exhibit clearer anomaly boundaries compared to traditional height-based corrections. However, due to inherent statistical fluctuations and matrix inversion characteristics, higher measurement values tend to absorb lower ones, potentially enlarging anomalous regions. Nevertheless, the high-anomaly regions after inversion largely coincide with ground truth validation, demonstrating that the proposed method can effectively correct airborne gamma spectrometry data.

**Keywords:** Airborne gamma-ray spectrum; Dynamic three-dimensional; Terrain correction

## Introduction

Airborne gamma-ray spectrometry enables characterized or quantitative analysis of radionuclides through processing of environmental gamma-ray measurement data [?]. As a critical step in airborne gamma-ray data processing, the inversion method for surface radionuclide content profoundly impacts the accuracy of aerial measurement results [?, ?]. According to China's nuclear industry standards, altitude correction of energy window counts is typically performed by combining ground standard source calibration data with ray attenuation principles and elevation data during flight [?]. Conventional altitude correction uses only radar height as a parameter, without considering the influence of actual ground undulation or non-uniform radionuclide distribution, resulting in eU content errors exceeding 30% in rough terrain [?] and maximum eTh content errors of 21.5% across nine typical terrain types [?]. Xiong Shengqing established a 2D terrain influence coefficient correction model by dividing the undulating surface into multiple oblique sources and accumulating their contributions [?], while Wan Jianhua obtained optimal results for the terrain influence coefficient method through segmentation of the volume model after in-depth analysis [?], and Bai Yunfeng performed terrain correction under 3D spatial information subdivision of the aerial survey process using Google Earth [?]. Subsequently, Brian Minty proposed using response matrices for three-dimensional quantitative inversion, employing finite element concepts to correct interactions between survey areas and constructing airborne gamma-ray spectrometry response matrices through radar altitude and terrain height, yielding significantly improved inversion results [?]. This method was also validated by Md Moudud for inverting  $^{137}\text{Cs}$  activity in the environment [?]. However, actual airborne gamma-ray surveys are continuous processes, and the energy spectrum at a recording point represents a synthesis of contributions from a continuous survey area segment during the measurement time. For this reason, Sun Kun and Liu Qiushi proposed a method combining aerial flight paths with sourceless efficiency calibration to construct dynamically terrain-corrected response matrices, validating this approach through aerial surveying experiments in the LSS area of Gansu

and the EGRY area of Inner Mongolia \cite{11–13}.

Through the research of numerous scholars, dynamic terrain correction theory has gradually matured, making the primary challenge of terrain correction methods dependent on detection efficiency calculation for radioactive finite elements [?]. Sourceless efficiency calibration methods can be divided into three categories: numerical calculation methods, Monte Carlo simulation methods, and hybrid approaches [?]. Wu Hexi applied numerically calculated sourceless efficiency calibration to airborne gamma-ray spectrometry, finding good agreement with measured values after comparison [?]. Xiong Chao derived a sourceless efficiency calibration method based on numerical integration through axial symmetry of the detector and object geometry and gamma field superposition principles, demonstrating accuracy in NaI and HPGe detector measurement experiments [?]. Zhang Jian constructed a numerical ray deposition model by studying ray deposition processes in the detector sensitive volume combined with reaction probabilities, validating it through point source experiments with a CeBr3 detector [?]. Numerical calculations offer speed and convenience for sourceless efficiency calibration, yet struggle to account for multiple scattering effects on photopeaks. For complex measurement systems, numerical methods often fail to provide analytical equations or require significant model optimization, reducing credibility and complicating calculations.

The second category, Monte Carlo simulation-based sourceless efficiency calibration, is frequently applied to simulate challenging measurement conditions due to its modeling flexibility [?]. Kalus Noack explored reducing errors from deep penetration in Monte Carlo simulations, finding that bias methods can effectively reduce simulation error [?]. Ghassoun investigated relationships between point source and surface source detection efficiency and measurement distance/detector size using Monte Carlo simulation, providing theoretical basis for optimizing large-volume source simulations [?]. Zhao Jun proposed a geometric transformation method converting volume sources to line sources for Monte Carlo simulation of large source detection efficiency, useful for reducing variance in large source simulation results [?]. Bao-Lu Yang tested the accuracy of LabSOCS and ANGLE sourceless efficiency calibration methods under 241Am sources, validating their effectiveness for HPGe gamma-ray measurements [?]. Frosio optimized the ISOCS/LabSOCS model by analyzing each geometry's effect on radioactive source measurement uncertainty, demonstrating uncertainty reduction by up to 8 times [?]. Although Monte Carlo simulation can eliminate multiple scattering effects on photopeaks, its computational efficiency struggles to meet practical needs when simulating complex measurement systems.

The third hybrid method leverages numerical computation advantages for solving incident ray penetration distances at different angles and Monte Carlo simulation capabilities for handling complex particle transport processes in detectors, achieving complementary strengths. Qingxian Zhang obtained angular injection distributions from infinite volume sources through numerical computation, combining them with Monte Carlo simulation to model detection efficiency under

different angular distributions, energies, and heights. This approach demonstrated high accuracy for natural nuclides such as U-series, Th-series, and K after ground and flight calibration experiments in Shijiazhuang [?]. Building upon this method, we improved the Monte Carlo simulation component by simulating gamma-ray deposition processes in detectors using Geant4 combined with Boolean operations to rapidly obtain detection efficiency for different incidence modes, achieving at least 162 times efficiency improvement over traditional Monte Carlo simulation while maintaining errors within 5% for point and volume source validation experiments with a CeBr3 detector [?].

In summary, this paper introduces a novel sourceless efficiency calibration method based on Boolean operations of the ray deposition process to calibrate the static airborne gamma-ray survey model. Based on this foundation, multiple static measurement models are superimposed to approximate the dynamic measurement model, upon which the dynamic terrain response matrix is constructed. This approach provides a solid theoretical foundation for accurate inversion of surface radionuclide concentrations in aerial surveys.

## 2.1 Sourceless Efficiency Calibration Method

This paper employs a sourceless efficiency calibration method based on Boolean operations of the ray deposition process to calculate detection efficiency for airborne gamma-ray spectrometers [?]. Analyzing the interaction between rays and matter, the process from radioactive source emission to photopeak recording by the detector can be divided into two stages: (1) attenuation produced by each shielding layer (including detector packaging) before rays enter the detector sensitive volume, termed the AE part; and (2) the probability of complete energy deposition (intrinsic detection efficiency) due to Compton scattering, electron-positron pair production, and photoelectric effect when rays enter the detector sensitive volume, termed the E, , S part. Therefore, the detection efficiency formula for a radioactive source can be expressed by equation (1):

MATH\_1

where  $E, V$  is the photopeak detection efficiency of the radiation source,  $\Omega$  is the angular range of ray emission, and  $V$  is the spatial range of the radiation source.

Parameters involved in the shielding process include: point source location distribution  $V$ , shield geometry and material information (including air, aircraft floor, detector mounting box, detector housing, vibration damping layers, etc.), and ray angular distribution  $\Omega$  (elevation and azimuth angles). To solve the AE part quickly and flexibly, continuous variables in the measurement environment can be discretized (Fig. 1 [Figure 1: see original paper] shows a schematic diagram of the discretization for the GR820 airborne gamma spectrometer detector sensitivities, consisting of 15 NaI(Tl) crystals measuring  $10.16 \text{ cm} \times 10.16 \text{ cm} \times 40.64 \text{ cm}$ ).

After similar discretization of each shielding component, the angle and distance relationships between each scattering point coordinate and the ray equation in a specific emission direction are used to obtain the ray penetration distance  $d_l$  in each shielding body. Thus, equation (1) can be converted to:

MATH\_2

where  $\mu_{E,l}$  is the linear absorption coefficient of shielding body  $l$  for rays of energy  $E$ , obtained through winXcom software calculations.

For the  $E, \theta, S$  part, Geant4 can be used to obtain and record the deposition process of NE particles with energy  $E$  in an infinitely large sensitive volume at specific incidence angles and positions as original coordinates  $D_n$ . These coordinates are then spatially transformed according to the ray incidence direction ( $\theta, \phi$ ) and incidence point coordinates  $S_{in}$ , as shown in equation (3):

MATH\_3

where  $D'_n$  are the transformed deposition coordinates, and  $R_z, R_x$  are rotation matrices for coordinate system rotations about the  $z$ -axis and  $x$ -axis, respectively.

Subsequently,  $D'_n$  is intersected with the sensitive volume through graphical Boolean operations, as illustrated in Fig. 2 [Figure 2: see original paper] (using a rectangular detector from the GR820 as an example). Points located inside the sensitive volume are assigned a value of 1, while those outside are assigned 0, stored in column vector  $N_p$  in subscript order. Thus, the intrinsic detection efficiency  $E, \theta, S$ , where all deposition processes occur within the sensitive volume, can be computed using equation (4):

MATH\_4

where  $N_{in,E, \theta, S}$  represents the number of rays fully deposited within the detector,  $\rho_n$  is the medium density (2.2 g/cm<sup>3</sup> with SiO<sub>2</sub> substrate, for example),  $N_p$  is the number of particles with full energy deposition during the  $n$ -th reaction,  $\mathbf{1}$  is a unit column vector with the same length as  $N_p$ , and  $NE$  is the total number of incident particles. Finally, detection efficiency is obtained by combining equation (4) with equation (2). During efficiency calibration for narrow-beam gamma rays in the energy range of 59.54–2620 keV, this method's computational efficiency is approximately 900–7200 times higher than conventional Monte Carlo simulations, with further improvements for longer ray path lengths (CPU @ R5800H).

## 2.2 Simplification of the Calibration Model

The ground gamma-ray injection rate model typically simplifies the ground gamma-ray source as an infinite cylindrical volume source and the detector as a point detector for injection rate calculations [?]. However, as a 3D model, the volume source model increases detection efficiency computation time. Therefore, based on the concept of radioactive spectrum equilibrium, the volume source

model can be simplified to a surface source model to improve computational efficiency.

In quantitative inversion of airborne gamma-ray spectra, the primary focus is on counts within the characteristic peak ranges of U-series, Th-series, and  $^{40}\text{K}$ . Consequently, the simplification process only analyzes the equilibrium process for characteristic gamma rays of each nuclide.

Gamma rays are emitted to the surface from underground rock with uniformly distributed radionuclides and density  $\rho$ . The characteristic gamma-ray injection at point O can be considered as the superposition of gamma rays emitted from spherical shell rock with radius  $r = 0 \sim R$ . The injection at point O can be expressed by equation (5):

MATH\_5

In reality, the natural gamma-ray surface injection rate reaches 95% of the infinite volume source injection rate before  $R = 50$  cm. Since the range of lithological variations and aerial survey coverage are generally much larger than 50 cm, this paper simplifies the aerial gamma-ray volume source model to a surface source model and performs dynamic terrain correction based on surface sourceless efficiency calibration. The relationship between natural radionuclide content on the surface and the surface injection rate can be expressed by equation (6):

MATH\_6

where the rock is considered a homogeneous medium with density  $\rho$  ( $\text{g}/\text{cm}^3$ ), radionuclide content  $q$  ( $\text{g}/\text{g}$ ), gamma-ray absorption coefficient  $\mu$  ( $\text{cm}^{-1}$ ), gamma-ray emission probability  $P$ , and decay constant  $\lambda$ .

From equation (5), when  $R$  approaches infinity, the injection rate at point O equals that of an infinite volume source and becomes isotropic in spatial distribution. Thus, the surface injection under infinite radioactive volume source conditions is numerically equal to that at point O, with each point theoretically conforming to the  $2\pi$  steradian angular distribution of a point source. In equation (5), all parameters are constants except  $R$ .

Taking the surface injection rate under infinite volume source conditions as a benchmark, the saturation curves of natural gamma-ray surface injection rate versus  $R$  for energies of 1.46, 1.76, and 2.62 MeV are shown in Fig. 3 [Figure 3: see original paper] (with a medium density of  $2.2 \text{ g}/\text{cm}^3$  and  $\text{SiO}_2$  substrate, for example).

## 2.3 Dynamic Terrain Correction Method

**2.3.1 Dynamic Terrain Aerial Survey Forward Model** In the aerial survey process, surface radioactivity distribution can be divided into finite elements according to influencing factors such as measurement point location, time, and measurement distance (as shown in Fig. 4 [Figure 4: see original paper], with

$n_j$  divisions in latitude and  $n_k$  divisions in longitude). When the spectrometer flies from recording point  $t_{i-1}$  to  $t_i$ , its measurement range moves dynamically with the spectrometer. Therefore, all radioactive finite elements within the total measurement range of a single flight segment contribute to the spectrometer counts, with each finite element's detection efficiency changing with spectrometer coordinates.

Combining the simplified model from Section 2.2, the counts in a single spectrum can be considered as the superposition of contributions from all radiation field injection rates within the measurement range during that flight segment. Moreover, since each geological body has different radioactivity content, the spectrum count rate  $C_i$  of the  $i$ -th measurement can be expressed by equation (7):

MATH\_7

where  $t$  is the measurement time,  $[x, y]$  is the measurement range of the  $i$ -th measurement,  $A$  is the ray injection rate of a single radioactive finite element, and  $\eta$  is the detection efficiency at the corresponding measurement coordinates at time  $t$ .

To ensure inversion accuracy in aerial surveys, the range of radioactive finite elements is often larger than map elevation accuracy, resulting in terrain relief within each radioactive finite element. Furthermore, the intersection region between the measurement range and the corresponding radioactive finite element changes dynamically with time. Therefore, after discretizing the single measurement time  $t$  and converting the inversion object into a radioactive finite element  $A(j, k)$  containing height distribution, the contribution of the radioactive finite element to the spectrometer counts at the  $i$ -th measurement can be expressed by equation (8):

MATH\_8

where  $S(t_i, j, k)$  is the intersection area of the  $(j, k)$  radioactive finite element with the measurement range during the  $i$ -th measurement, and  $\eta(t_i, j, k)$  is the average detection efficiency of this intersection area. Further, the spectrometer count  $C(i)$  at the  $i$ -th measurement can be converted from equation (7) to equation (9):

MATH\_9

Thus, the entire measurement process from the first to the  $n$ -th measurement can be expressed in matrix form as equation (10):

MATH\_{10}

where  $B$  is the response of the radioactive finite element to the spectrometer count rate.

In radioactive measurements, radionuclide decay is a random process. Consequently, even under constant source and measurement conditions, the spec-

trometer count rate is not fixed but exhibits statistical fluctuations. If a nucleus decays with probability rate  $\tau$ , then the probabilities of decay and non-decay within time interval  $t$  are  $(1 - e^{-\tau t})$  and  $e^{-\tau t}$ , respectively. When the spectrometer has detection efficiency  $\eta$ , the probability  $p$  that a decay is both emitted and detected, and the probability  $q$  that it does not produce a count, can be defined as shown in equation (11):

$$\text{MATH}_{\{11\}}$$

Thus, radionuclides obey a binomial distribution. Given measurements on  $N_0$  radioactive nuclei, the probability that the spectrometer count  $= n$  at time  $t$  can be expressed by equation (12):

$$\text{MATH}_{\{12\}}$$

During aerial surveys,  $N_0$  is much greater than  $p$ . By applying a limiting approximation to equation (12), it can be shown that  $P(= n)$  follows a Poisson distribution with mean  $\lambda = N_0 p$ , as given in equation (13):

$$\text{MATH}_{\{13\}}$$

In summary, after incorporating background count rate  $\delta$  and Poisson statistical fluctuations, the dynamic terrain aerial survey forward model can be expressed by equation (14):

$$\text{MATH}_{\{14\}}$$

Therefore, the PSO-MLEM algorithm [?] based on Poisson statistical fluctuations is employed to solve the matrix.

**2.3.2 Construction of the Dynamic Terrain Response Matrix** As derived in Section 2.3.1, construction of the dynamic terrain response matrix  $B$  is achieved by superimposing contributions from each radioactive finite element under static measurement conditions to obtain their total contributions under dynamic measurement conditions.

As illustrated in Fig. 5 [Figure 5: see original paper], during static measurements, the intersection area between each radioactive finite element and the measurement range varies with detector position. Moreover, terrain variations within each finite element significantly impact detection efficiency. Therefore, the main challenges in constructing the response matrix include: (1) determining the static measurement range and obtaining spatial information for all radioactive finite elements within it; (2) rapidly computing surface source detection efficiency after acquiring finite element spatial distributions; and (3) determining the minimum number of convergent segments required to approximate dynamic measurements through static measurement superposition.

Among naturally occurring radionuclides, the 2.62 MeV gamma ray from the Th-series nuclide  $^{208}\text{Tl}$  undergoes less attenuation in matter compared to the

1.46 MeV gamma ray from  $^{40}\text{K}$  and the 1.76 MeV gamma ray from the U-series nuclide  $^{214}\text{Bi}$ . Consequently, its detection efficiency is more significantly affected by measurement range and incident angle within the detector sensitive volume. Since the response matrix construction process is largely consistent across different gamma-ray energies, this section focuses on constructing the response matrix using the 2.62 MeV gamma ray as an example, with the GR820 airborne gamma spectrometer at a flight altitude of 100 meters.

**1. Spatial information of radioactive finite elements within the static measurement range.** First, the static measurement range of the detector is determined. Based on the previously described sourceless efficiency calibration method, a model is constructed assuming a measurement altitude of 100 meters. Surface detection intensity for 2.62 MeV gamma rays is calculated over radial distances ranging from 100 m to 1000 m in 100 m increments. The resulting detection efficiency trend as a function of measurement radius is shown in Fig. 6 [Figure 6: see original paper], with the 98% saturation point indicated by a red asterisk.

After determining the measurement range at a given flight altitude, it is necessary to identify the area and spatial altitude distribution of radioactive finite elements within this range that contribute to spectrometer counts. As shown in Fig. 8 [Figure 8: see original paper], each radioactive finite element exhibits an altitude distribution. To account for altitude variation within each finite element, a vertical correction is applied by subdividing the element into altitude sub-elements—regions with different elevations but identical radioactive activity. The detection efficiency of each altitude sub-element is calculated individually based on its intersection with the measurement range. Contributions from all altitude sub-elements belonging to the same radioactive finite element are then aggregated to determine the total contribution of that element to the spectrometer, as illustrated in Fig. 7 [Figure 7: see original paper].

To efficiently record intersection information between each altitude sub-element and the measurement range (including intersection area and geographic coordinates), the measurement range is simplified as a square with side length  $2R$  and center coordinates  $(O_x, O_y)$ . Using equation (15), the vertex coordinates of each altitude sub-element rectangle  $(x_{m,n,l}, y_{m,n,l})$  are processed to compute their intersection with the square measurement range. The resulting intersection regions are all rectangles (where  $l = 1$  or  $2$  indicates minimum and maximum  $x$  or  $y$  values). Consequently, for each altitude sub-element within the measurement range, the intersection area  $S_{m,n}$  and the coordinates of the four rectangle vertices  $(V_{x_{m,n,l}}, V_{y_{m,n,l}})$  are recorded. These serve as essential input parameters for subsequent detection efficiency calculations.

MATH\_{15}

In the equation, sort refers to arranging elements in ascending order. The two middle values from the sorted set are assigned as the minimum and maximum  $x$  and  $y$  coordinates of the intersection rectangle's vertices.

In addition to obtaining intersection information for altitude sub-elements within the measurement range, elements with smaller angles can still be hidden, though this requires very rugged terrain, as illustrated in Fig. 8. Since gamma-ray attenuation in rock and soil is significantly greater than in air, contributions from obstructed altitude sub-elements during each measurement are set to zero. To determine whether an altitude sub-element is obstructed, the angle between the sub-element and spectrometer is first calculated. Then all other altitude sub-elements along the line of sight between the sub-element and spectrometer are evaluated. If any intermediate sub-element has an angle to the spectrometer greater than  $\theta_c$ , the original sub-element is considered obstructed and its contribution is set to zero. If all intermediate angles are smaller than  $\theta_c$ , the sub-element is considered unobstructed.

**2. Fast computation of surface source detection efficiency during aerial surveys.** After obtaining spatial information for each altitude sub-element contributing to spectrometer counts, it is necessary to calculate surface source detection efficiency for each valid altitude sub-element. In practical aerial surveys, thousands of measurement records are typically generated. Each record must be decomposed into multiple static measurement superpositions, with each static measurement involving dozens of altitude sub-elements. This results in an extremely large computational workload when constructing the dynamic terrain response matrix.

To address this issue, the characteristics of detection efficiency distribution within the measurement range during aerial surveys are analyzed to optimize the efficiency computation process. The distribution within the first quadrant of a  $500 \times 500$  m area is sufficient to represent the detection efficiency pattern within a 500 m radius at a flight altitude of 100 meters. With a grid resolution of 1 meter, a total of 250,000 points are calculated. The resulting detection efficiency distribution is shown in Fig. 9 [Figure 9: see original paper].

Due to spatial symmetry, the overall detection efficiency distribution resembles a two-dimensional Gaussian distribution. However, closer inspection of local regions (e.g., the area within [100–160, 60–140] in Fig. 9) reveals that the distribution is approximately planar. Within such regions, the detection efficiency at the geometric center can effectively approximate the average detection efficiency for the entire planar area. To identify sub-regions that can be approximated as planar segments, a profile line is drawn outward from the center and piecewise linear fitting is performed, as shown in Fig. 10 [Figure 10: see original paper]. Segments with coefficient of determination  $R^2 > 0.99$  are retained. Using this method, the 500 m-radius measurement range is divided into 12 linearly fitted segments, each corresponding to a region where detection efficiency exhibits approximately linear variation. The geometric center of each segment is then used to approximate the average detection efficiency within that region. Consequently, the entire measurement range is partitioned into 12 quasi-linear regions, as illustrated in Fig. 10 [Figure 10: see original paper].

**3. Minimum number of discrete segments for dynamic measurement.**

According to equation (14), the count rate within a characteristic energy window statistically follows a Gaussian distribution determined by the product of radioactive intensity and detection efficiency. Even when flying over the same area at different speeds (i.e., with different measurement times but the same flight distance), detection efficiency for a given ground region remains constant as it depends only on geometry. Therefore, discretizing a dynamic measurement into multiple static measurements is primarily related to flight altitude and distance rather than measurement duration. For example, if a flight distance of 10 m requires  $n$  discrete segments, a flight distance of 20 m would require  $2n$  segments by linear superposition.

As shown in Figs. 6 and 9, the spectrometer count rate is mainly contributed by a small region directly beneath the detector, and this region is most sensitive to variations in flight distance. Thus, as illustrated in Fig. 11 [Figure 11: see original paper], this section analyzes the influence of discrete segment number  $n$  on detection efficiency within the left and right  $50 \times 50$  m zones for a GR820 spectrometer flying at 100 m altitude.

Based on static measurement parameters derived in the previous sections, the number of discrete segments is varied from 1 to 20. The corresponding variations in detection efficiency within the left and right side regions under different discretization numbers are shown in Fig. 12 [Figure 12: see original paper]. When the discretization number is 1, the detection efficiency corresponds to the static measurement value at the detector position (25, 25, 100), resulting in lower initial detection efficiency on the left side compared to the right side. As the discretization number increases, detection efficiencies in both regions gradually converge. Using a  $\pm 2\%$  deviation as the convergence criterion, the minimum discretization number required for convergence at a flight altitude of 100 m is determined to be 4.

In summary, after obtaining the spatial distribution of radioactive finite elements within the measurement range (Step 1) and developing a fast surface source detection efficiency calibration method based on linear segment midpoint approximation (Step 2), the static response matrix construction method is established. Then, using the minimum discretization number for dynamic measurement obtained in Step 3, static response matrices are superimposed to construct the dynamic terrain response matrix. The complete workflow is illustrated in Fig. 13 [Figure 13: see original paper], with each step's relative error remaining within 2%. According to error propagation principles, the overall error is expected to remain within 3.5%.

### 3.1 Airborne and Ground Gamma Spectrometry Measurements

The experimental area is located in the Baiyun'ebo iron ore mining district, Darhan Muminggan United Banner, Baotou City, Inner Mongolia Autonomous Region. The site features abundant mineral samples and low vegetation cover, making it suitable for subsequent ground gamma-ray measurements and geolog-

ical mapping to validate airborne survey data. Measurements were conducted using the GR820 airborne gamma spectrometer, strictly following the “Specifications for Airborne Gamma-Ray Spectrometry” (EJ/T 1032-2005). The selected survey area spans from 110°00 11 E to 110°04 11 E and 41°37 29 N to 41°53 31 N, covering approximately 165 km<sup>2</sup>.

The survey line layout is shown in Fig. 14 [Figure 14: see original paper]. A total of 20 main survey lines were arranged longitudinally (east-west) with approximately 350-meter spacing, and 5 transverse tie lines were arranged latitudinally (north-south) with an average spacing of 7.4 kilometers, yielding 10,085 measurement points. To ensure flight safety, the flight altitude was slightly elevated, with approximately 81.40% of the survey conducted between 80–120 meters and an average flight altitude of about 100 meters. The test zone includes 10 main survey lines and 1 tie line, comprising 987 measurement points.

To verify the effectiveness of the dynamic terrain correction algorithm, a test area was selected within the airborne survey region, as shown in the red box in Fig. 14(a). This area is located near the Baiyun’ebo iron ore deposit, covering coordinates 110°01 05 E to 110°03 04 E and 41°47 45 N to 41°50 50 N, with approximate dimensions of 5.7 km × 2.7 km. Satellite imagery indicates relatively flat terrain with clearly visible strip-shaped ore belts in the central and upper portions. The lower left corner adjoins the Baiyun’ebo mining zone. During the survey, ground conditions were dry with low vegetation coverage, making the area well-suited for ground-based gamma spectrometry (sampling locations marked by red lines in the figure). The ground investigation section includes 10 airborne gamma survey lines comprising 2,083 measurement points. Measurements were conducted using the GR-320 ground gamma spectrometer, strictly following the “Technical Specifications for Ground Gamma-Ray Spectrometry” (DZ/T 0205–1999).

### 3.2 Evaluation and Analysis of the Method

Based on spatial information from the airborne survey system, the dynamic terrain response matrix (with detection efficiency corrected to sensitivity) is constructed using the method described in the previous chapter. The PSO-MLEM inversion algorithm is then applied to background-corrected airborne gamma data for eU, eTh, and K to estimate their surface concentrations. Matrix construction parameters are summarized in Table 1, and inversion results are presented as contour maps in Figs. 15, 16, and 17.

**TABLE 1. Inversion parameters for dynamic terrain correction**

No.	Parameter	Indicator
1	Number of Discretizations	4
2	Measurement Range	500 × 500 m
3	Finite Element Size	30 × 30 m

No.	Parameter	Indicator
4	Altitude Sub-element Size	$5 \times$ Flight Altitude

As shown in Fig. 15 [Figure 15: see original paper], four anomalous zones were delineated based on eU content from ground measurements. After applying conventional altitude correction to airborne survey results, anomalous contours were observed in all zones except Zone 2. Although eU content values were relatively low, the spatial variation pattern indicated good airborne survey accuracy. After dynamic terrain correction, all anomalous zones became directly observable through contour maps. A comparison of average eU content in each anomalous zone (Table 2 ) shows that values obtained after dynamic terrain correction were generally closer to ground measurements. Compared with altitude correction, dynamic terrain-corrected inversion results showed accuracy improvements ranging from 3.12% to 25.79%.

**TABLE 2. Comparison of eU content and relative error in anomaly zones**

Anomalous Zones	Ground (ppm)	Altitude correction (ppm)	Terrain correction Err.(%) (ppm)	Err.(%)

As shown in Fig. 16 [Figure 16: see original paper], five anomalous zones were delineated based on eTh content from ground measurements. After altitude correction, anomalous contours were observed in all zones except Zones 1 and 4, though contour boundaries were relatively indistinct. Following dynamic terrain correction, more distinct contours emerged in all zones except Zone 4. Nevertheless, due to limited flight line density, Zone 1 exhibited some spatial deviation. A comparison of average Th content in each anomalous zone, summarized in Table 3 , indicates that dynamic terrain correction improved inversion accuracy by 7.50% to 48.82% compared with altitude correction. Moreover, dynamically corrected results show that inversion accuracy for eTh content was significantly higher than for eU, attributable to generally higher Th concentrations in the survey area and stronger penetration capability of characteristic gamma rays from the Th decay series, which effectively reduces statistical fluctuation impacts on inversion accuracy.

**TABLE 3. Comparison of eTh content and relative error in anomaly zones**

Anomalous Zones	Ground (ppm)	Altitude correction (ppm)	Terrain correction Err.(%) (ppm)	Err.(%)
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As shown in Fig. 17 [Figure 17: see original paper], five anomalous zones were delineated based on K content from ground measurements. After altitude correction, only Zones 3 and 5 exhibited clear anomalies, while no obvious anomalies were observed at other locations. After dynamic terrain correction, corresponding anomalies appeared at Zones 3, 4, and 5; however, positional deviations were observed at Zones 1 and 2, likely related to the sparser survey line layout of the aerial survey compared to ground measurements. A comparison of average K content in each anomalous zone (Table 4 ) indicates that dynamic terrain correction improved inversion accuracy by 9.10% to 36.52% compared with altitude correction.

**TABLE 4. Comparison of K content and relative error in anomaly zones**

Anomalous Zones	Ground (ppm)	Altitude correction (ppm)	Terrain correction Err.(%) (ppm)	Err.(%)
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Based on comprehensive analysis of Figs. 15, 16, 17 and Tables 2–4, dynamic terrain correction produced U, Th, and K anomaly zone values closer to ground measurements than altitude correction, with accuracy improvements ranging from 3.12% to 48.82%, and yielded clearer anomaly contours. However, due to statistical fluctuations and inherent matrix inversion characteristics, each survey point is influenced by neighboring points within the measurement range. Elevated measurement values can absorb lower ones, resulting in an apparent increase in anomalous zones, as shown by the yellow blocks in Fig. 17(c). Nevertheless, the locations of high-anomaly zones (represented by red and white blocks) generally align well with ground survey results.

It should also be noted that apart from unavoidable measurement errors such as radar altimeter noise, GPS positioning inaccuracies, and flight attitude variations, the aerial survey used fewer and non-corresponding survey lines compared to ground measurements. Consequently, slight positional deviations between anomalies on dynamically terrain-corrected contour maps and those from ground measurements are observed.

#### 4. Summary

Based on a novel sourceless efficiency calibration method, this study derives the static measurement model of airborne gamma spectrometry into a dynamic

measurement model, further improving inversion accuracy of surface radionuclide concentrations by constructing and solving a dynamic terrain response matrix. This approach addresses detection efficiency calibration accuracy issues affecting inversion results in traditional terrain correction methods and the low simulation efficiency of Monte Carlo programs for airborne survey modeling. Airborne and ground gamma spectrometry experiments in the Baiyun'ebo iron ore district revealed that: (1) after applying dynamic terrain correction, anomalous zones of eU, eTh, and K concentrations increased significantly; using ground measurements as reference, corrected radionuclide concentrations showed higher accuracy than conventional airborne inversion, with maximum relative error reduction of 30.58%, and anomaly boundaries became clearer and more consistent with ground survey distributions; (2) due to measurement statistical fluctuations and inherent matrix inversion characteristics, elevated values tend to absorb nearby lower values, expanding anomaly zones, although high-concentration anomaly locations remain largely consistent; (3) due to slight deviations between airborne and ground survey line layouts, inversion anomaly positions show minor discrepancies compared to ground-measured anomaly zones.

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