Reexamined Mass of 22C via Constraints from Recent Experimental Extraction of Its Radius

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Abstract

The neutron-rich nucleus 22C, located at the neutron dripline, exhibits intriguing structural properties, such as its Borromean nature and potential two-neutron halo configuration. Despite experimental advancements, uncertainties persist in the energy of two-neutron separation S2n and the radius of matter for this attractive nucleus 22C. In this work, we employ the three-body Faddeev approach to investigate the ground-state properties of 22C, constrained by the recently deduced matter radius. By optimizing the neutron-core and three-body interactions to reproduce the experimental radius, the two-neutron separation energy S2n is redetermined, revealing a weakly bound system dominated by s-wave configuration. Additionally, an excited state, exhibiting an Efimov-like pattern, is identified through analyzing the specific density distributions and relative distances in the three-body system, highlighting the geometric similarity between the ground and excited states.

Full Text

Preamble

Reexamining the Mass of ^{22}C via Constraints from Recent Experimental Determinations of its Radius

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The neutron-rich nucleus ^{22}C, located at the neutron dripline, exhibits intriguing structural properties, including its Borromean nature and potential two-neutron halo configuration. Despite experimental advances, uncertainties persist regarding its two-neutron separation energy S_{2n} and matter radius. In this work, we employ the three-body Faddeev approach to investigate the ground-state properties of ^{22}C, constrained by the recently deduced matter radius. By optimizing the neutron-core and three-body interactions to reproduce the experimental radius, we redetermine the two-neutron separation energy S_{2n}, revealing a weakly bound system dominated by s-wave configuration. Additionally, an excited state exhibiting an Efimov-like pattern is identified through analysis of specific density distributions and relative distances in the three-body system, highlighting the geometric similarity between ground and excited states.

Keywords: Two-neutron separation energy, Nuclear radius, Three-body approach, Neutron-rich nucleus

Introduction

With the advent of radioactive ion beam technology and facilities, the nuclear landscape has been extended to encompass $Z = 118$ and reach toward both the proton and neutron driplines [1–3]. Contrary to the proton-rich side of the nuclear chart, experimental exploration of neutron-rich nuclei remains quite limited due to their short lifetimes and tiny cross sections [4–7]. Among these dripline nuclei, the heaviest experimentally determined carbon isotope, ^{22}C, is of particular interest not only in nuclear physics but also in Efimov physics, owing to its Borromean nature [8–11]. Approximately two decades ago, ^{22}C was predicted to be an ideal s-wave two-neutron halo nucleus with a dominant $\nu(s_{1/2})^2$ configuration [12,13], a prediction subsequently confirmed through reaction cross-section measurements despite an unexpectedly large matter radius [14]. Furthermore, this dripline nucleus plays a crucial role in understanding other exotic structural properties, such as possible new magicity at $N = 16$ and shape decoupling [15–20].

As mentioned previously, ^{22}C is a typical Borromean nucleus, forming a bound three-body (^{20}C + n + n) system while none of its two-body subsystems (^{20}C + n or n + n) are bound. However, significant uncertainties and puzzles remain regarding its nuclear properties, particularly the separation energy and matter radius [2,21–23]. Since the pioneering reaction experiment with ^{22}C on a liquid hydrogen target [14], extensive efforts have been undertaken to clarify the ambiguities surrounding the bulk properties of this intriguing nucleus [9,21,22]. For instance, interaction cross sections (σ_I) of ^{22}C on a carbon target were accurately measured at 235 MeV/nucleon, yielding $\sigma_I = 1.280 \pm 0.023$ b [9]. Within a four-body Glauber reaction model, the root-mean-squared matter radius of ^{22}C was deduced to be 3.44 ± 0.08 fm, significantly smaller than previous extractions. Additional experiments, including neutron removal reactions from carbon isotopes and reconstruction of the ^{20}C + n decay-energy spectrum, have further contributed to our understanding of ^{22}C structure [8,21,22].

The binding energy or two-neutron separation energy of ^{22}C represents another critical quantity for dripline nuclei, yet it remains uncertain. To date, only one direct mass measurement of ^{22}C exists, establishing an upper limit of $S_{2n} \leq 320$ keV [23]. According to the recent atomic mass evaluation (AME20), this value was determined to be 35 ± 20 keV [2]. From a theoretical perspective, different approaches within the shell model and three-body frameworks yield varying predictions for separation energies [12,24–26]. Additionally, extensive efforts have been devoted to studying this exotic nucleus and its isotopes via ab initio methods [27–29]. Nevertheless, debate continues regarding our understanding of ^{22}C.

A well-established correlation exists between neutron separation energy and matter radius for neutron-rich nuclei [12,24,30,31]. This relationship allows us to constrain the structural properties of ^{22}C, even if absolute values remain uncertain. Based on extracted matter radii from reaction measurements, the two-neutron separation energy can be obtained through empirical formulas relating energy and radius [23,24,32]. For example, following this procedure, S_{2n} of ^{22}C would be approximately 10 keV from the large radius reported decades ago [24]. Very recently, reaction cross sections of neutron-rich carbon isotopes on ^{12}C have been systematically measured over a wide incident energy range from 30 to 950 MeV/nucleon [33]. The matter distributions and radii of carbon isotopes, including the exotic ^{22}C, were then determined using the finite-range Glauber model with Coulomb correction. Impressively, the simultaneously obtained charge radii are in excellent agreement with those directly extracted from charge-change cross sections [33,34].

Given these developments, it is particularly significant to determine what can be learned from these new high-accuracy data. Against this background, we employ the three-body Faddeev method to further understand the binding mechanism of ^{22}C through its two-neutron separation energy, with the constraint of reproducing its matter radius. Theoretical approach details and parameterization choices are presented in the next section, with specific results and discussions on nucleon density distributions given in Section III. A summary is provided in the final section.

II. Theoretical Approach

The Faddeev equations provide a rigorous mathematical framework for three-body systems, whose essence lies in decomposing the total wavefunction into three Faddeev components (Ψ_1, Ψ_2, Ψ_3), each corresponding to and describing the pairwise interaction between particles [35]. For instance, in the ^{22}C ($n + n + ^{20}$C) system, Ψ_1 may correspond to the interaction between the two neutrons, while Ψ_2 and Ψ_3 correspond to neutron-core interactions. Each component is associated with a different Jacobi coordinate system (as illustrated in Figure 1 [Figure 1: see original paper]). The equation takes the form:

$$(T_i + h + V_i - E)\psi^i_{JM} = -V_i(\psi^j_{JM} + \psi^k_{JM})$$

The left-hand side represents the independent dynamics of the subsystem, including the kinetic energy T_i, the core Hamiltonian $\widehat{h}$, and the two-body interaction potential V_i (encompassing both nuclear and Coulomb forces). The right-hand side represents how the superposition of the other two components feeds back into the evolution of the current component through potential energy terms, reflecting the three-body coupling that constitutes the root cause of complexity in three-body problems.

Through this decomposition, the Faddeev equations transform the three-body problem into a coupled system of integro-differential equations, thereby avoiding the complexities of directly solving the high-dimensional Schrödinger equation. However, even with this simplified form, numerical solution presents significant challenges due to multi-dimensional coordinate systems. To further reduce computational dimensionality and achieve a unified description of dynamical behavior across different Jacobi coordinates, the introduction of hyperspherical coordinates becomes essential.

By adopting the hyperspherical coordinate system, which includes the hyperradius ρ and hyperangles θ_i, the two-dimensional system of partial differential equations is transformed into a set of coupled one-dimensional equations. The transformation of Jacobi coordinates into hyperspherical coordinates takes the following form [36]:

$$\rho^2 = x_i^2 + y_i^2, \quad \theta_i = \arctan(x_i/y_i)$$

The hyperradius ρ characterizes the global scale of the three-body system, being invariant under translational and rotational transformations as well as permutations of particle pairs (i, j), while correlating with the size of the nuclear core. In contrast, the hyperangle θ_i describes the relative configuration between the particles, which exhibits radial dependence and correlates with the relative magnitudes of the two Jacobi coordinates. The wavefunction is expanded within the hyperspherical coordinates as [36,37]:

$$\Psi(x_i, y_i) = \rho^{-5/2} \sum_{K_i} \chi_{K_i}(\rho) \Phi^{l_{x_i} l_{y_i}}_{K_i}(\theta_i)$$

where the hyperangular component is given by:

$$\Phi^{l_{x_i} l_{y_i}}_{K_i}(\theta_i) = N_{l_{x_i} l_{y_i}} (\sin\theta_i)^{l_{x_i}} (\cos\theta_i)^{l_{y_i}} P^{l_{x_i}+1/2, l_{y_i}+1/2}_{n_i}(\cos 2\theta_i)$$

and the hyperradial component is expanded in terms of Laguerre polynomial basis functions:

$$\chi_{K_i}(\rho) = \sum_{n=1}^{N_b} a_{in} \rho^{K_i+5/2} e^{-\rho/2} L_n^{2K_i+5}(\rho)$$

Here, $K_i = l_{x_i} + l_{y_i} + 2n_i$ ($n_i = 0, 1, 2, ...$) represents the hyperangular momentum directly related to the order of the corresponding Jacobi polynomial, and N_i is the normalization coefficient.

Upon introducing the hyperspherical expansion into the Faddeev coupled equations, one obtains a set of simultaneous linear equations:

$$\mathbf{Ha} = E\mathbf{a}$$

Within the Faddeev equations, the matrix $\mathbf{H}$ requires several types of matrix elements. The two-body potential $\hat{V}_{jk} = \hat{V}_i(x_i)$ takes the form:

$$\hat{V}_i(x_i) = V_i^c(x_i) + \hat{SO} V_i^{so}(x_i) + \hat{Q} V_i^Q(x_i) + \hat{SS} V_i^{SS}(x_i) + \hat{T} V_i^T(x_i)$$

where $V_i^c(x_i)$ denotes the central interaction. The spin-orbit (SO) coupling is described by the operator $\hat{SO}$ and radial form factor $V_i^{so}(x_i)$. The tensor operator $\hat{Q}$ and radial form factor $V_i^Q(x_i)$ account for the multipole deformed potential. The standard tensor interaction is given by operator $\hat{T}$ with radial dependence $V_i^T(x_i)$, while the spin-spin interaction is represented by $\hat{SS}$ and its form factor $V_i^{SS}(x_i)$.

The central potential depends solely on the interparticle distance, with its operator form expressed as $V_i^c(x_i)$ and is diagonal in the angular momentum and spin quantum numbers. Within a fixed Jacobi coordinate system, the matrix elements are given by:

$$\langle i:\alpha_i' | V_i^c | i:\alpha_i \rangle = \delta_{l'_{x_i} l_{x_i}} \delta_{l'_{y_i} l_{y_i}} \delta_{s'_i s_i} \int \phi^*_{L'_i L'_{y_i}}(\theta_i) V_i^c(x_i) \phi_{L_i L_{y_i}}(\theta_i) d\theta_i$$

where $\phi_{L_i L_{y_i}}(\theta_i)$ represents the hyperspherical basis function. The spin-orbit potential takes the form $\hat{SO} V_i^{so}(x_i) = \Gamma_{ij} \mathbf{l}_{x_i} \cdot \mathbf{s}_j + \Gamma_{ik} \mathbf{l}_{x_i} \cdot \mathbf{s}_k$, with matrix elements incorporating scalar products of the orbital angular momentum with particle spins $\mathbf{s}_j$ or $\mathbf{s}_k$.

III. Numerical Results and Discussions

In this section, we present detailed calculations of the ground-state properties for the ^{22}C nucleus using a three-body model based on the Faddeev equations. As a typical neutron-dripline nucleus, the experimentally deduced root-mean-square (r.m.s.) matter radius of ^{22}C, $r_m^{\rm exp} = 3.296 \pm 0.123$ fm [33], serves as a key constraint in this study. Based on this constraint, the parameters of the

Woods-Saxon potential for the neutron-core interaction are determined through an inverse optimization approach.

In developing the n+^{20}C interaction model, critical assumptions are implemented due to the absence of experimental data on core excitations: the ^{20}C core is treated as a rigid structure with spin-parity $J^\pi = 0^+$, enforcing Pauli blocking of the core orbitals $(1s_{1/2})^2$, $(1p_{3/2})^4$, $(1p_{1/2})^2$, and $(1d_{5/2})^6$ [38]; valence neutrons predominantly occupy the $(2s_{1/2})$ orbital with very weak binding [21].

The core-neutron interaction is described using a Woods-Saxon potential under these assumptions, expressed in Eq. (7) [12,37,39], which incorporates the first two terms from Eq. (5):

$$V_{n-\mathrm{core}}(r) = V_0^c \left[1+\exp\left(\frac{r-r_0}{a}\right)\right]^{-1} + V_{so}\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr}\left[1+\exp\left(\frac{r-r_0}{a}\right)\right]^{-1} \mathbf{L}\cdot\mathbf{S}$$

Determination of the interaction potential parameters is achieved through reproduction of the experimentally deduced r.m.s. matter radii. Reference [33] reports the experimental r.m.s. matter radius of ^{22}C as $r_m^{\mathrm{exp}} = 3.296 \pm 0.123$ fm. Achieving convergence within experimental constraints necessitates reducing the radius parameter to $r_0 = 1.13A^{1/3}$ fm (versus the conventional $1.25A^{1/3}$ fm used in Refs. [12,37]), motivated by the monotonic increase of r.m.s. matter radii with r_0.

When setting $r_0 = 1.13A^{1/3}$ fm, the minimum achievable r.m.s. matter radius falls below the experimental lower limit, thereby encompassing the experimental range. The core-neutron potential strength $V_{l=2}^c$ is incrementally adjusted until r_m^{cal} precisely matches the experimental value of 3.173 fm. The maximum attainable r.m.s. matter radius through $V_{l=2}^c$ variation remains below the critical experimental values (3.296 fm and 3.419 fm). Consequently, a systematic adjustment of r_0 is implemented to translate the theoretical radius distribution, ensuring full coverage of the experimental value range. Specifically, distinct r_0 values are sequentially fixed to ensure the adjustable range of r.m.s. matter radii—achieved by tuning the neutron-core potential—encompasses each target experimental value.

From Table 1 , it is observed that r_m^{cal} also depends on the three-body interaction potential. Entries 2 and 4 demonstrate that identical r.m.s. matter radius values can correspond to multiple distinct sets of neutron-core and three-body interaction potentials. Analysis of entries 3 and 5 reveals that the neutron-core interaction potential plays a predominant role, while the three-body interaction potential contributes non-negligibly. Unique determination of this parameter set requires incorporation of an additional constraint, namely the two-neutron separation energy S_{2n}. Comparison of entries 3 and 5 reveals that within a constrained range of neutron-core potentials, S_{2n} exhibits stronger correlation

with the three-body interaction potential. Consequently, S_{2n} provides an approximate constraint for the three-body potential, enabling subsequent precise calibration of $V^c_{l=2}$.

The two-neutron separation energy S_{2n}, obtained as an outcome of the calculation constrained by the matter radius, is then compared with theoretical and experimental values for validation. Reference [9] reports $S_{2n} = 0.56^{+0.27}_{-0.20}$ MeV, but calculations considering only two-body potentials exhibit significant deviations from this value. This discrepancy is mitigated through the introduction of a Gaussian-form three-body interaction in Eq. (8), which emulates effects of core deformation or core excitation [35,40,41]:

$$V_{3b}(\rho) = s_{3b} \exp\left[-\left(\frac{\rho}{r_{3b}}\right)^2\right]$$

When identical three-body potential parameters $s_{3b} = 3$, $r_{3b} = 14$ are applied to parameter sets 1, 4, and 5, an inverse correlation is observed: stronger neutron-core interaction potentials $V^c_{l=2}$ yield larger two-neutron separation energies S_{2n}. These results exhibit significant deviations from theoretical values. The deviation is reduced when the three-body potential strength is increased from the baseline of parameter set 5. This adjustment brings the calculated two-neutron separation energy S_{2n} into closer agreement with the theoretical value of $S_{2n} = 0.56^{+0.27}_{-0.20}$ MeV reported in Ref. [9]. The rationale for avoiding universally larger three-body potentials lies in Set 1's extremely shallow S_{2n} binding; further enhancement would induce a transition from bound states into the continuum. Consequently, each parameter set necessitates a customized three-body potential strength.

At this stage, the input parameters—including two-body and three-body potentials—have been calibrated through fixation of the r.m.s. matter radii and benchmarking against theoretical values of the two-neutron separation energy. These parameters are compiled in Table 1, with Sets 1, 2, and 3 selected as our primary configurations. Across these sets, only three parameters ($V^c_{l=2}$, s_{3b}, r_0) were varied while all others remained fixed.

The hyperangular momentum cutoff $K_{\max} = 20$ was employed throughout calculations to guarantee numerical convergence. The choice of $K_{\max}$ is briefly discussed in Ref. [42]; larger values yield better convergence behavior and enable higher accuracy of the numerical solution. However, the influence of $K_{\max}$ on the results is significantly smaller than that of the two-body potential. Moreover, our study of ^{22}C does not involve long-range interactions. Therefore, choosing $K_{\max} = 20$ is adequate for our calculations.

The neutron-neutron interaction is described using the Gogny-Pires-Tourreil (GPT) potential [43], with central, tensor, and spin-orbit terms included while omitting the spin-spin contribution. This potential provides good fits to the low-energy properties of nucleon-nucleon scattering.

Based on the parameter sets (Sets 1, 2, and 3) determined in Table 1, the orbital energy levels, spatial configurations, and density distributions were systematically calculated. Bound states prohibited by the Pauli principle, such as the $1s_{1/2}$ and $1p_{3/2}$ orbitals, are eliminated through supersymmetric transformations, restricting valence neutrons to the allowed $1d_{5/2}$ orbital. Furthermore, adjustment of the s-wave and p-wave potential strengths along with the spin-orbit coupling strength revealed that the $(1s_{1/2})$, $(1p_{3/2})$, and $(1p_{1/2})$ states depend mainly on s-wave and p-wave contributions, which are not listed explicitly.

Table 1 lists the key single-particle orbital energies (units: MeV), where the orbital energy for $1d_{5/2}$ ranges from -0.323 to -0.72 MeV, indicating that this orbital is weakly bound. The deeply bound $1s_{1/2}$ orbital (-18.306 to -19.889 MeV) has a compact wavefunction and is Pauli-blocked for valence neutrons. It should be noted that our research results do not rely on these energy values but rather on their single-particle wavefunctions. The different potential energies selected in this paper produce almost identical single-particle wavefunctions for each occupied orbital, and all potential energy sets are configured so that the energy of the $2s_{1/2}$ single-particle state approaches zero, consistent with the prescription in Ref. [12].

Since there is no centrifugal potential barrier for neutrons in the $2s_{1/2}$ orbital in the average potential field, the extremely weak neutron binding energy leads to significant tunneling effects. This tunneling implies radial expansion of the $s_{1/2}$ orbital wavefunction. This energy level structure is consistent with typical characteristics of neutron dripline nuclei. Additionally, due to wavefunction expansion, dynamic coupling effects with other strongly bound orbitals are weakened beyond the static effects of the average potential field, indicating the presence of a pure $(s_{1/2})^2$ configuration and an unperturbed core. If two neutrons occupy the $2s_{1/2}$ orbital, their interaction becomes the only source of additional binding energy. Since the s-wave potential energy may be further weakened, the ground-state energy of ^{22}C is considered to be the minimum value in this analysis.

Table 2 presents the energies and r.m.s. radii of the ground state and excited state, calculated with three different three-body interaction parameter sets. Quantities with asterisks correspond to the excited state. All energies are in units of MeV, while all lengths are in fm.

Using the two-body and three-body interaction models from Table 1, we solved the Faddeev equations to obtain the energy values and r.m.s. matter radii of the ground state and excited state (relative to the n+^{20}C threshold) of ^{22}C. The calculation results are listed in Table 2. All energy values are given relative to the ^{20}C + n + n threshold, and the values are close to each other.

The $Z = 6$, $N = 14$ closed-shell cores are referred to as closed cores. The neutron part of the ^{20}C ground state contains not only the $1d_{5/2}$ closed-shell configuration but also other configurations such as $2s_{1/2}$. In the ground state,

the two-neutron separation energy $S_{2n} = -E = 0.265$–0.656 MeV. Since the parameters are set based on experimental values from Ref. [33], S_{2n} falls within the experimental range and is consistent with other theoretical values: $S_{2n} = 0.423 \pm 1.140$ MeV from Ref. [44] and $S_{2n} = -0.140 \pm 0.460$ MeV from Ref. [23], within the error range, confirming the reliability of the theoretical model.

In addition, the ^{22}C ground-state correlation core is introduced, featuring two valence neutrons occupying the halo s-orbital that must satisfy orthogonality with the ^{20}C core s-orbital. The excited state has spin-parity $J^{\pi} = 0^{+}$. It is noteworthy that the excited-state energy E^{*} is positive, indicating that the employed two-body and three-body potentials cannot bind the neutrons, consistent with the experimental observation that no bound state exists in the ^{21}C nucleus [45].

To analyze the three-body configuration of ^{21}C, the average distance parameters of valence neutrons were calculated (Table 3). In the ground state, the average distance between two neutrons is $r_{nn} \approx 6.49$ fm, and the distance from the nucleus to the center of mass of the neutron pair is $r_{c,nn} \approx 3.40$ fm. In the excited state, these values extend to $r_{nn} \approx 8.90$ fm and $r_{c,nn} \approx 4.65$ fm, corresponding to the size of a stable nucleus with mass number $A \approx 60$. Therefore, the excited state of ^{22}C can be described as a giant halo state. The key finding is that the ratio $r_{nn}/r_{c,nn} \approx 1.91$ remains constant. The identical proportions obtained in both states indicate that these two states have similar geometric structures, with the main difference being only a spatial scaling factor.

Table 3 presents the average distance r_{nn} between two valence neutrons in the ground and excited states of ^{22}C, and the average distance $r_{c,nn}$ from the core to the center of mass of the valence neutron pair. The superscript $*$ denotes the excited state. All lengths are in fm.

Additionally, the correlation density distribution between the ground and excited states of ^{22}C was analyzed. These values were calculated using the Jacobi coordinate system, with ^{20}C as the spectator particle. The formula for calculating the spatial correlation density distribution is as follows [41]:

$$P(r_{nn}, r_{C,(nn)}) \equiv x^2 y^2 \int |\Psi_{JM}(x,y)|^2 d\Omega_x d\Omega_y$$

The spatial distribution of two valence neutrons for the ground and excited states is shown in Figures 2 and 3, respectively. The spatial distribution function peaks at $(r_{nn}, r_{c,nn}) \approx (6.5, 3.5)$ fm (Figure 2 [Figure 2: see original paper]) in the ground state, corresponding to the maximum probability density. This configuration is consistent with a compact three-body arrangement. In contrast, the main peak of maximum probability density for the excited state occurs at $(9.5, 4.5)$ fm, with a secondary peak at $(12, 5.5)$ fm, which originates from contributions of the $(1d)^2$ orbital (Figure 3 [Figure 3: see original paper]). This double-peak structure reflects orbital rearrangement effects in the excited state:

some neutrons transition from s-wave to d-wave, causing the spatial configuration to differentiate into compact and extended modes.

Orbital occupancy analysis reveals the s-wave dominant characteristics of ^{22}C. In the ground state, $(2s_{1/2})^2$ accounts for 97.77%, while the d-wave component accounts for only 2.23%. In the excited state, $(2s_{1/2})^2$ decreases to 72.90%, while $(1d_{5/2})^2$ increases to 27.10%. This change coincides with the secondary peak position of the density distribution, indicating s-d orbital mixing in the excited state. The reason may be that the d-wave component acquires significant weight, and core polarization along with the extended halo structure enhances coupling between the core and valence neutrons, inducing orbital mixing. Despite the increase in d-wave components, s-waves still dominate (>70%), so the excited state is essentially the same as the ground state—an s-wave halo nucleus [12]—but exhibits more complex multi-orbit coupling effects.

The Efimov effect is a universal quantum phenomenon in three-body systems, where an effective long-range attraction emerges from short-range two-body interactions near resonance, leading to a series of weakly bound states exhibiting discrete scale invariance [46]. Given the significantly larger r.m.s. radii and more diffuse spatial distribution of the excited state, it can be characterized as an Efimov state in the halo nucleus [47,48]. Since ^{22}C has been confirmed as a double-neutron halo nucleus, Ref. [49] points out that the Efimov effect is most likely to be observed in double-neutron halo nuclei. Our study also reveals an interesting geometric similarity between ground and excited state configurations—in Efimov physics, two consecutive Efimov states can be related through discrete spatial scaling factors [50]. The identical ratio relationship between r_{nn} and $r_{c,nn}$ in the ground and excited states of ^{22}C indicates that the nuclear configurations of the two states have highly similar geometric shapes, meaning that discrete scaling symmetry exists. The features we discovered in ^{22}C are highly consistent with the theoretical explanation of Efimov states [51].

IV. Conclusion

In this study, we systematically investigated the structural properties of the neutron-rich nucleus ^{22}C using a three-body Faddeev approach, constrained by the experimentally determined matter radius. Our calculations employed a Woods-Saxon potential for the neutron-core interaction and introduced a Gaussian-form three-body potential to account for core deformation effects. By fine-tuning the potential parameters, the experimental matter radius ($r_m = 3.296 \pm 0.123$ fm) was successfully reproduced, while the derived two-neutron separation energy S_{2n} showed agreement with both theoretical and experimental values, confirming the weakly bound nature of ^{22}C.

The ground state of ^{22}C exhibits a dominant $(2s_{1/2})^2$ configuration with a weakly bound $1d_{5/2}$ orbital, yielding $S_{2n} = 0.265$–0.656 MeV, consistent with previous results. An excited state displays an extended s-d hybrid structure (s-wave >70%) and a double-peak spatial distribution. The constant ratio $r_{nn}/r_{c,nn} \approx$

1.91 between states suggests discrete scaling symmetry, supporting ^{22}C as an Efimov candidate in Borromean nuclei.

Our results highlight the interplay between neutron-core interaction and three-body forces in shaping the properties of ^{22}C. The consistency between our predictions and experimental data validates the three-body approach as a powerful tool for studying neutron-rich nuclei. Future work could explore the dynamical effects of core excitation and the role of higher-order interactions in refining the description of ^{22}C and similar dripline nuclei. These insights contribute to a deeper understanding of exotic nuclear structures and their connections to universal few-body phenomena.

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