# Statistical Signatures of Gamma-Ray Burst 221009A in X-Ray Observations (Postprint)

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## Abstract

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## Full Text

## Preamble

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### Statistical Signatures of Gamma-Ray Burst 221009A in X-Ray Observations

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## Abstract

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**Key words:** Astronomical Databases -methods: statistical -Astronomical Instrumentation, Methods and Techniques

## 1. Introduction

GRB 221009A, observed on 2022 October 9, has been identified as one of the brightest gamma-ray bursts (GRBs) ever recorded (Dichiara et al. 2022). It is among the most well-documented GRBs to date, with observations spanning a wide range of wavelengths. The burst was detected by the Swift X-Ray Telescope (XRT) and has been extensively studied due to its extreme brightness and unique afterglow characteristics (Dainotti et al. 2022; O’ Connor et al. 2023). The X-ray light curve (LC) of this event exhibits complex temporal behavior that requires detailed statistical analysis to uncover its underlying properties (Sakamoto et al. 2011; Margutti et al. 2013).

Swift was launched at 17:16 GMT on 2004 November 20 (Gehrels et al. 2009). According to NASA, Swift’s primary goals are to “determine the origin of gamma-ray bursts, classify GRBs into categories, search for new types of bursts, and study how these bursts evolve and interact with their environment” (Gehrels et al. 2009). The XRT enables imaging and spectral analysis of the GRB afterglow, offering valuable insights into the burst’ s characteristics (Sakamoto et al. 2008;

Gehrels et al. 2009). The CCD operates at approximately −100°C to ensure low dark current and reduce proton irradiation effects (Breeveld et al. 2011; Pagani et al. 2007).

Statistical analysis plays a crucial role in understanding the temporal and spectral variability of GRBs (Guidorzi et al. 2009; Ukwatta et al. 2010). In this study, we analyzed the LC of GRB 221009A using three statistical distributions: Poisson, normal, and lognormal. The Poisson distribution is widely used to model count-based data, such as photon arrivals (Lyons 1986). The normal distribution is often employed to approximate symmetrical data, while the lognormal distribution is more suited for skewed distributions, which are common in astrophysical observations (Ajello et al. 2008; Basilakos et al. 2008).

GRB 221009A offers an unprecedented data set for X-ray time-series analysis (Götz et al. 2023). However, no published study has explicitly examined whether its X-ray counts follow a Poisson, Gaussian (normal), or lognormal distribution over different timescales or segments. Our analysis aims to determine which statistical model best describes the LC of GRB 221009A, providing insights into the emission processes by analyzing individual observation IDs. We assess the goodness of fit for each distribution using both the chi-square test and the Bayesian Information Criterion (BIC), aiming to determine the most suitable model for describing the observed data (Zhang et al. 2006). The results of this study contribute to broader discussions on GRB populations and their underlying physics, leading to a better understanding of the statistical nature of GRB afterglow fluctuations and providing insights into the physical processes governing these high-energy phenomena.

The structure of the paper is as follows. Section 2 discusses data acquisition and pre-processing techniques. Section 3 presents the results and discussions, followed by conclusions in Section 4.

## 2. Methods and Data Analysis

The analysis of GRB 221009A relied on software tools designed for theoretical interpretation and data extraction from the Neil Gehrels Swift Observatory, particularly in the X-ray energy band. The primary software package employed was HEASOFT, a comprehensive suite of tools publicly accessible from the HEASARC website. This package includes essential utilities for handling Swift-specific data and provides the necessary tools for calibration, data cleaning, and analysis (HEASARC 2020a).

The software setup was carried out on a Linux-based system running Ubuntu OS v.20.04, chosen for its stability and compatibility with the HEASOFT package. The installation involved several key steps to ensure a seamless operational environment. First, the HEASOFT Version 6.30 source code was downloaded and installed, accompanied by the automatic installation of supporting tools such as FTools and the XANADU package, which includes XRONOS v.6.0 for timing analysis (HEASARC 2020a). Next, the Calibration Database (CALDB

Version 1.02) was remotely set up. The calibration files were recovered from the Swift XRT repository, and the CALDB repository was integrated to ensure high precision in data processing with the xrtpipeline tool (HEASARC 2020b). Additionally, the DS9 (v.8.3) software was installed to facilitate the visualization and analysis of reduced data. The user-friendly interface of DS9 proved instrumental in efficiently analyzing the background and reduced data sets. Throughout the setup, official documentation guidance and supervisory input ensured a smooth and error-free process (Heasarc 2020c).

Once the software environment was prepared, data for GRB 221009A were acquired from the Swift Observatory' s XRT, which captures data in the energy range of 0.3-10 keV. The Swift XRT master catalog served as the primary source of all public Swift observations. The selection of observational data followed a systematic approach. Observations with exposure times greater than or equal to 900 s were prioritized to ensure adequate signal-to-noise ratio (Swift Observatory 2020). Positioning for GRB 221009A used srcra = 288.3, srcdec = 19.7 (Swift Team 2022). Data files were downloaded, and any records containing errors were noted and excluded from further analysis.

The downloaded Level 1 Swift/XRT data were subjected to cleaning and calibration using the xrtpipeline tool from the HEASOFT package. Calibration files from the Swift/XRT CALDB repository ensured the integrity and accuracy of the preprocessed data. This extended temporal coverage allowed for detailed spectral and temporal analyses of the burst, significantly contributing to understanding its physical properties (Evans et al. 2009). We analyzed data from Photon Counting (PC) mode and Windowed Timing (WT) mode by creating directories to save the cleaned output files, with accurate R.A. and decl. of the target source. We generated exposure maps and corresponding error files. Data with higher count rates per second were processed for pile-up correction (Arlt et al. 2013).

After processing by xrtpipeline, the required files are obtained: `po_{cl}.evt` (the cleaned event file), `*.hk` (housekeeping file), and `ex.img` (exposure map image). To perform barycenter correction on the cleaned data, specific files from the auxil folder are copied to the corresponding output directories: `pat.fits` and `sao.fits`. After copying these files, barycentric correction was applied to adjust photon arrival times to the solar system' s barycenter. This correction is essential for removing orbital effects, ensuring consistency across Swift XRT observations, and enabling precise timing analysis of transient and periodic phenomena (Capalbi et al. 2005).

For generating the necessary data products, we ran the xrtproduct tool from the HEASoft software package. This tool is specifically designed to process Swift XRT data, generating key outputs such as LCs and spectra. The processing steps involved filtering, binning, and cleaning the data to ensure accurate temporal analysis. The effective use of xrtproduct allowed us to extract reliable observational features for comprehensive analysis (Capalbi et al. 2005).

After performing barycenter correction and product generation, the next step is to use Xselect, a tool provided by the HEASARC software suite for analyzing data from space-based X-ray telescopes like Swift XRT. This tool is widely used for creating images, spectra, and LCs from the event data generated by these instruments (Capalbi et al. 2005; HEASARC 2020a). The primary objective of utilizing Xselect in our analysis is to extract data products with high spatial and temporal resolution, including images and LCs from the cleaned event files, which are fundamental for detailed characterization and modeling of the X-ray source.

GRB 221009A, showing the x-pixels and y-pixels in the graph, can be defined by the picture taken when the process of pile-up is done, as visualized in Figure 1 [Figure 1: see original paper].

## 2.1. Pile-up

Pile-up occurs when multiple photons arrive within a detector's readout interval, leading to overlapping events that distort the recorded signal. This is common in high-flux sources, where the detector underestimates the true count rate. The region with maximum intensity is typically identified as the pile-up core, which is excluded or corrected to recover accurate information. By identifying this center, we can accurately model and mitigate the effects of pile-up in the analysis (Ballet 1999; Arlt et al. 2013). We consider the pile-up threshold to be more than 0.7 counts per second for PC mode and 100 counts for WT mode. If this condition is met, the data are then processed for pile-up correction. The data are subjected to the pile-up correction process by generating the point spread function (PSF) curve as shown in Figure 2 [Figure 2: see original paper].

Figure 1 [Figure 1: see original paper]. XRT PC mode image of GRB 221009A showing a central depression, a signature of photon pile-up. The observed count rate exceeds the critical threshold of 0.7 counts $s^{-1}$, signifying the condition for pile-up.

Figure 2 [Figure 2: see original paper]. Radial profile of the PSF generated using XIMAGE tool HEASOFT as a function of radius. The theoretical PSF (magenta) is compared to the observed PSF (cyan) for pile-up correction in the central region.

## 2.2. Lcmath

To define the source region, we select a physical area corresponding to 20 pixels on the detector. This pixel size ensures accurate characterization of the source while minimizing the effects of pile-up. For the background analysis, we define an annular region with an inner radius of 30 pixels and an outer radius of 60 pixels. This annular region is chosen to isolate the background signal without including the source itself, allowing for more accurate subtraction of background from the total detected signal.

LC analysis involves several computational steps to extract meaningful physical properties from observed data. In our study, we use the lcmath tool to perform background subtraction on LC files, ensuring accurate measurements of the source’ s flux (HEASARC 2025a). The process involves matching source and background LC files and applying mathematical operations to subtract the background contribution:

corrected source = source - k × background

where k is a scaling factor to account for differences in the extraction areas of the source and background regions. Background subtraction ensures that the final LC represents the intrinsic variability of the source (HEASARC 2025b). The corrected LC is then used for further analysis, including periodicity studies and modeling of flux variations. For the LC correction, we make a 20-pixel circle for the source and 30-60 pixel annulus for the background.

Figure 3 [Figure 3: see original paper]. Image showing the result of background subtraction applied to the source region. This step is critical for isolating the target’ s emission and minimizing contamination in the derived LC.

## 3. Results and Discussion

We performed a detailed statistical analysis of Swift XRT PC and WT mode data for GRB 221009A, conducting a segment-wise statistical analysis where each observational segment was evaluated against three statistical models—Poisson, normal (Gaussian), and lognormal distributions—based on goodness-of-fit criteria: the chi-squared statistic and BIC (Press et al. 2007; Schwarz 1978).

Figures 4 and 5 illustrate the LC and corresponding rate histogram respectively for the first observational ID. Among the tested models, the normal distribution provides the best fit with $^2 = 68.42$, as shown in Table 1. The Poisson model follows with $^2 = 290.47$, while the lognormal distribution yields a poorer fit with $^2 = 572.77$. The BIC values listed in Table 2 further support this outcome: the normal distribution exhibits the lowest BIC (7651.21), confirming its statistical preference for this segment.

Assuming a Poisson model for photon counts, we use the standard deviation $\sigma = \sqrt{}$ , where is the mean count rate. For the first ID with an average count of 31, this gives $\sigma = 5.57$, leading to a signal-to-noise ratio (SNR) of 5.57, indicating a relatively bright detection (Wall & Jenkins 2012; Bevington & Robinson 2003; Gehrels 1986). This symmetry indicates that the variability in the X-ray flux is likely due to stochastic processes with a well-defined mean and standard deviation rather than extreme outliers. The Gaussian behavior implies symmetric flux variability likely governed by stochastic emission mechanisms such as synchrotron radiation or thermal bremsstrahlung, rather than count-based discreteness typical of Poisson processes (Guidorzi et al. 2015).

The second observational ID, depicted in Figures 6 and 7, shows a decline in brightness with an SNR of 2.0. Here, the Poisson distribution provides the best fit ( $^{2}$ = 33.54), outperforming the normal ( $^{2}$ = 81.63) and lognormal ( $^{2}$ = 430.13) models. Corresponding BIC values confirm this (Poisson BIC = 4413.34 versus normal BIC = 4559.24), as noted in Table 2. These results suggest the emission process is dominantly discrete and stochastic, consistent with count-based photon arrival modeling rather than smooth Gaussian variation (Cai et al. 2003). The poor lognormal fit suggests that the intensity variations do not strictly follow an exponential growth or decay pattern initially.

For this GRB, there is a zero count after these two observations, i.e., in the third observation ID. By mixing these two observation IDs and ignoring the time gap between them, we get the LC depicted in Figure 8 [Figure 8: see original paper], and the histogram of rates with Poisson, normal, and lognormal fits displayed in Figure 9 [Figure 9: see original paper]. We observe a shift in the statistical behavior, and the lognormal distribution emerges as the most appropriate fit with $^{2}$ = 2161.20 and BIC = 5714.76, outperforming both the normal and Poisson models (see Tables 1 and 2). The substantial Poisson chi-squared value ( $^{2}$ $1.31 \times 10^{7}$) reflects an extreme outlier, indicating that the emission no longer follows a process with a constant rate (Guidorzi et al. 2024).

Extending the data set by incorporating a third observational ID (Figures 10 and 11), the trend continues. While the lognormal model still provides a competitive fit ( $^{2}$ = 2987.77), the normal distribution shows a marginally lower $^{2}$ = 2593.60. However, the BIC values—normal BIC = 7876.23 versus lognormal BIC = 5629.01—strongly favor the lognormal distribution. The increasing skewness and variability in the composite data set favor a multiplicative stochastic process over an additive or discrete one.

Figures 12 and 13 show the LC and distribution respectively when all available observational IDs (up to 01126853084) are aggregated. The chi-squared values clearly illustrate that the Poisson ( $^{2}$ $3.36 \times 10^{49}$) and normal ( $^{2}$ $6.90 \times 10^{21}$) models are grossly incompatible with the data. In contrast, the lognormal distribution yields a significantly lower value of $^{2}$ = 541.85, indicating superior fit quality. The corresponding BIC values (Poisson = 347765.16, normal = 595007.66, lognormal = 34365.49) confirm the statistical dominance of the lognormal model (Table 2).

The lognormal distribution captures the skewed nature of the XRT LC effectively, making it the most appropriate model for the full data set, as it aligns with the observed features of GRB 221009A. A lognormal emission profile implies a multiplicative underlying process, consistent with physical mechanisms such as synchrotron cooling or shock-driven variability (Zhang 2018; Zhang et al. 2006). This behavior is characteristic of many astrophysical transients, where energy dissipation occurs in intermittent bursts rather than smooth, additive changes (Li & Fenimore 1996; Sari et al. 1998).

Figure 4 [Figure 4: see original paper]. LC of GRB 221009A, in PC mode, for the first observation ID.

Figure 5 [Figure 5: see original paper]. Histogram of the GRB 221009A, in PC mode, for first observation ID with fitting.

Figure 6 [Figure 6: see original paper]. LC of the GRB 221009A, in PC mode, for the second observation ID.

Figure 7 [Figure 7: see original paper]. Histogram of the GRB 221009A, in PC mode, for the second observation ID with fitting.

Figure 8 [Figure 8: see original paper]. LC of the GRB 221009A, in PC mode, where the first two observation IDs are combined in a single plot.

Figure 9 [Figure 9: see original paper]. Histogram for the GRB 221009A, in PC mode, where first two observation IDs are combined in a single plot. In this plot, fitting is done for Poisson, normal and lognormal fits.

Figure 10 [Figure 10: see original paper]. LC of the GRB 221009A, where the first three observation IDs are combined in a single plot.

Figure 11 [Figure 11: see original paper]. Histogram for the GRB 221009A, in PC mode, where the first three observation IDs are combined in a single plot. In this plot, fitting is done for Poisson, normal and lognormal fits.

Figure 12 [Figure 12: see original paper]. LC of the GRB 221009A, in PC mode, where all observation IDs are considered (up to 01126853084).

Figure 13 [Figure 13: see original paper]. Histogram of rates with Poisson, normal and lognormal fits of the GRB 221009A, in PC mode, where all observation IDs are considered (up to 01126853084).

Table 1 . Chi-squared Value Comparison Table

| Observation Set | Poisson $^{2}$ | Normal $^{2}$ | Lognormal $^{2}$ | Best Fit |
|---|---|---|---|---|
| First Obs ID | $1.31 \times 10^{7}$ | 68.42 | 572.77 | Normal |
| Second Obs ID | 33.54 | 81.63 | 430.13 | Poisson |
| First and Second Obs IDs Mixed | $1.31 \times 10^{7}$ | 2161.20 | 5714.76 | Lognormal |
| First, Second and Third Obs IDs Mixed | $2.59 \times 10^{7}$ | 2593.60 | 2987.77 | Lognormal |
| All Obs IDs Mixed | $3.36 \times 10^{49}$ | $6.90 \times 10^{21}$ | 541.85 | Lognormal |

Table 2 . Bayesian Information Criterion (BIC) Comparison Table

| Observation Set | Poisson BIC | Normal BIC | Lognormal BIC | Best Fit |
|---|---|---|---|---|
| First Obs ID | 347765.16 | 7651.21 | 34365.49 | Normal |
| Second Obs ID | 4413.34 | 4559.24 | 5714.76 | Poisson |

| Observation Set | Poisson BIC | Normal BIC | Lognormal BIC | Best Fit |
|---|---|---|---|---|
| First and Second Obs IDs Mixed | 347765.16 | 7876.23 | 5629.01 | Lognormal |
| First, Second and Third Obs IDs Mixed | 595007.66 | 7876.23 | 5629.01 | Lognormal |
| All Obs IDs Mixed | 347765.16 | 595007.66 | 34365.49 | Lognormal |

## 4. Conclusion

This study provides a comprehensive statistical examination of the X-ray emission from GRB 221009A using Swift XRT data. Through chi-squared and BIC analyses, we find that the statistical nature of the emission evolves across time and observational coverage. In the earliest segments, the data are best described by Poisson or normal distributions, reflecting discrete photon events or symmetric stochastic processes respectively. However, when multiple observational IDs are aggregated, the lognormal distribution becomes statistically dominant, suggesting a multiplicative emission mechanism.

These findings are consistent with earlier works that reported extremely fast variability and complex prompt emission features in GRB 221009A. For instance, Lesage et al. (2023) showed that early-time variability was statistically indistinguishable from Poisson noise, aligning with our results for initial segments. Additionally, Zhang et al. (2023), Nousek et al. (2006), and Götz et al. (2023) interpreted the later prompt and early afterglow phases as involving structured jet dynamics and overlapping emission components, which could naturally lead to multiplicative variability and lognormal flux statistics over extended durations.

Despite extensive temporal and spectral modeling in prior literature, no previous studies explicitly characterized the count-rate distribution or reported this transition from linear to lognormal behavior. Our results thus introduce a new dimension to GRB variability studies by demonstrating that, while individual pulses may follow Poisson-like or Gaussian behavior, the collective emission profile over time becomes distinctly lognormal. This interpretation supports the idea that GRB 221009A' s emission is governed by internal shock processes in which a series of independent fluctuations compound multiplicatively, consistent with theoretical expectations for turbulent jet environments (Kumar & Zhang 2015; Granot & Gill 2023; Rudolph et al. 2023).

Physically, this result aligns with the idea that GRB emission mechanisms may involve multiple overlapping processes. The initial prompt phase follows nearly normal statistical properties, possibly due to synchrotron or thermal bremsstrahlung radiation, where flux variations are symmetric around a mean value. However, as the GRB evolves, the emission process appears to be governed by multiplicative factors, such as turbulence-driven variations, jet interactions, or stochastic energy dissipation processes, leading to a lognormal distribution. The poor fit of the Poisson distribution for longer timescales suggests that GRB variability is not purely a count-based random process but involves complex temporal evolution.

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*Note: Figure translations are in progress. See original paper for figures.*