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Full Text

Preamble

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fiDrizzle-MU: A Fast Iterative Drizzle with Multiplicative Updates

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Abstract

In this paper, we introduce a new algorithm, fiDrizzle-MU, to coadd multiple exposures with multiplicative updates in each iteration instead of the difference correction terms used in the preceding version. We find that multiplicative update mechanisms demonstrate superior performance in decorrelating adjacent pixels compared to additive approaches, reducing noise complexity in the final stacked images. After applying fiDrizzle-MU to JWST-NIRCam F277W band data, we obtain a comprehensive reconstruction of a potential gravitational lensing candidate substantially blurred by the JWST pipeline' s resampling process.

Key words: methods: analytical -techniques: image processing -gravitational lensing: strong

1. Introduction

The rapid deployment of astronomical telescopes has been driving the production of vast amounts of observational data, posing significant challenges to data processing methodologies and capabilities within the astronomical community. One critical aspect is the combination of multiple exposures of under-sampled images to attain certain degrees of image quality (e.g., increasing signal-to-noise ratios (SNRs), upgrading image resolution, and diminishing noise). In general, astronomical telescopes aim to achieve a large field of view during observations. However, due to various factors including cost-effectiveness, the need to control readout noise, and technical limitations, the number of detectors in imaging arrays is typically insufficient to meet the Nyquist sampling criterion (Nyquist 1928; Shannon 1949), resulting in under-sampling. Many telescopes are under-sampled (Fruchter & Hook 2002), at least in some bands. For instance, the pixel size of the Hubble Space Telescope (HST) Wide Field Camera is 0.046 , while the angular resolution of HST is 0.05 . According to the Nyquist sampling theorem, this represents severe under-sampling. Additionally, the James Webb Space Telescope Near Infrared Camera (JWST-NIRCam) is under-sampled for short wavelengths below 2 m and long wavelengths below 4 m (Makidon et al. 2007; Wang et al. 2025). In under-sampled images, components with different frequencies are blurred together, a phenomenon referred to as aliasing. Recovering additional details from under-sampled images often requires a dither strategy (Lauer 1999; Hook & Fruchter 2000), which introduces small but random shifts between exposures, thereby reducing aliasing and improving spatial resolution.

For the purpose of coadding multi-exposure images (i.e., the dithered frames), the community has developed several valuable and enlightening methods, such as interlacing, shift-and-add (Bates & Cady 1980; Farsiu et al. 2004), and Drizzle (Fruchter & Hook 2002). In this regard, Drizzle, which draws upon advantageous aspects of both interlacing and shift-and-add, has become the de facto standard for combining images photographed by HST and JWST. Despite its effectiveness in restoring high-resolution images of resolved objects, Drizzle struggles with the reconstruction of unresolved structures and high-frequency details (Fruchter 2011). These high-frequency components correspond to sharp intensity transitions and small-scale features, which are crucial for characterizing compact sources and preserving morphological fidelity. However, they are particularly vulnerable to information loss due to under-sampling, noise, and the blurring effects of instrument optics. Meanwhile, the Drizzle algorithm distributes the flux of a single input pixel across several output pixels, which breaks the independence between output pixels and leads to the emergence of noise correlation.

Mathematically, CCD sampling, dithering, and resampling spread output pixel flux into adjacent pixels—analogous to convolving with a diffusion kernel, which can be effectively modeled by the so-called "Nota Spike"profile (see the Appendix in Wang et al. 2022 for reference). The upper left panel of Figure 1 [Figure 1: see original paper] depicts the convolution kernel according to this profile with an up-sampling factor of 10. Drizzle involves two parameters, p and s, which represent the linear scaling ratio between the shrunk input pixels and the original input pixels, and that between the output pixels and the original input pixels, respectively. As anticipated from an intuitive perspective, a lower p/s ratio reduces pixel correlations in the output Drizzled image. The trade-off, however, is amplification of high-frequency artifacts, as the noise from every single input pixel is condensed into fewer output pixels, resulting in a lack of smoothing.

iDrizzle (Fruchter 2011) was proposed as a refined variant of Drizzle. The framework of this algorithm was designed in an iterative fashion, where a low-pass filter would be applied after oversampling in each iteration, effectively suppressing high-frequency artifacts. As the updates continue, the pixfrac parameter decreases gradually, which mitigates noise correlations due to Drizzling. However, the oversampling—low-pass filtering—interpolating process decelerates convergence and slows computational speed. The application of the tapering function in the Fourier domain turns white noise into reddish noise, which is another cause of noise correlation. Wang & Li (2017) developed another iterative Drizzle-based method, fiDrizzle, which uses a difference-correction term (hereafter fiDrizzle-DC). fiDrizzle-DC proves more computationally efficient compared with iDrizzle and performs better than iDrizzle given the same number of iterations. Without the use of a tapering function and operations in the Fourier domain, fiDrizzle-DC suffers less from noise correlation compared with iDrizzle. However, due to its difference-correction term, fiDrizzle-DC is relatively sensitive to ringing effects. Additionally, the difference-based updates appear insufficiently efficient in signal deblending and pixel decorrelation.

In this work, we modify fiDrizzle by applying a multiplicative update in each iteration. The multiplicative updates enable the program to converge to the optimal reconstruction with fewer iterations. Apart from this, we impose positivity constraints on all approximations, which suppress the ringing effect to a significant degree. With these measures, fiDrizzle-MU can restore high-resolution images with extensive details from under-sampled dithered frames, resolve sources with minimal spatial separation, invert the Drizzling kernel to concentrate aliased signals from peripheral electronic pixels, and alleviate noise correlation and artificial ringing effects. The paper is organized as follows: We describe the fiDrizzle algorithm in detail in Section 2 and illustrate its reconstruction power and convergence efficiency in Section 3. The complete characterization of a potential gravitational lensing system that had previously been only partially resolved in existing JWST data—enabled by the astrometric precision of our algorithm—represents an interesting outcome of this investigation, and the de-pixelation power of the algorithm is discussed quantitatively, which constitutes the primary focus of this work. In Section 4, we evaluate the significant advantage of fiDrizzle-MU over previous algorithms in terms of computational consumption. We present a comprehensive discussion of the results and provide conclusive remarks in Section 5.

2. Method

As described in Wang & Li (2017), fiDrizzle-DC achieves a noteworthy balance between image fidelity, noise control, and convergence speed. As a reasonable generalization, in circumstances where all pixel values in the input images are positive, we can perform a multiplicative update to refresh each iterative result. The workflow of this generalization, i.e., the fiDrizzle-MU algorithm, is described below.

Given N dithered exposures of the same imaging field, $\{\mathrm{I}_1, \mathrm{I}_2, \cdots, \mathrm{I}\}$, as input, we first Drizzle them onto a finer grid to produce a high-resolution image F_0. The subscript denotes the iteration order. Next, we map the result of the 0th iteration, F_0, back to the same N frames of the input images to simulate those dithered exposures, producing a set of approximations to the observation, G . We then divide the original images by their corresponding approximations to yield a set of ratio images sharing the same frames as the dithered ones. As the term literally implies, R denotes a ratio. Returning to the first step, we now take this new image set as input to Drizzle into a new image R_1 on the finer grid. A pixel-wise normalization is conducted on R_1 by the effective overlapping layers L . Continuing as before, for $i \geq 1$, the direct product of Drizzle is R , which serves as a multiplicative correction to update the $(i-1)$th reconstruction, while the ith reconstruction, F , is obtained by multiplying R by $\mathrm{F}_{\ 1}$.

A reminder is warranted that the up-sampling operation employed in the above steps operates with a pixfrac value of 1, which in practice corresponds to the shift-and-add technique. The down-sampling process from fine-grid to coarse-grid pixels is implemented through an area-weighted flux conservation scheme, where the value of each coarse pixel is computed as the integration of overlapping contributions from all intersected fine-grid pixels. The convergence rate can be algorithmically modulated by elevating the multiplicative update to the γ-powered step-size parameter, where γ defines the exponential scaling factor for iterative refinement. In this work, we intentionally set $\gamma = 1$ as the baseline value—a conservative yet theoretically justified choice to ensure algorithm stability across all test conditions. For clarity, we denote S to signify the up-sampling operation from the kth coarse frame to the fine one and S to signify the corresponding backward down-sampling operation (see the variables in Table 1). The complete algorithm can subsequently be expressed mathematically as follows:

[Mathematical expression would appear here]

By applying multiplicative updates, fiDrizzle-MU converges to the optimal value faster than fiDrizzle-DC. In general, the number of steps required for convergence depends on the number of dithered frames and the PSR parameter. Intuitively, employing a larger number of dithered frames in conjunction with smaller PSR parameters necessitates more iterative steps to achieve optimal reconstruction. Additionally, stronger noise backgrounds extend the iterations needed for convergence.

With the intention of attenuating high-frequency artifacts induced by iterative processing such as ringing effects, we simply apply a non-strict positivity constraint to the reconstructions, which serves as a standard regularization term frequently employed in image combination and signal processing applications:

F = max(F , 0) (2)

3.1. Dither, Sampling and Pixel Correlation

We begin our exploration by examining the procedure for producing a high-resolution image with Drizzle from the true brightness distribution of a light source, denoted by L . This process involves a series of operations such as PSF convolution, dithering, CCD sampling, and resampling (all conducted without considering noise in this analysis). Before reaching the detector, which is typically a CCD array, L is convolved by the telescope’ s optics, transforming it into a light distribution blurred by the telescope’ s PSF: L = L K , where K represents the PSF and L denotes the PSF-blurred light distribution.

Multiple exposures with sub-pixel dithering are widely used to attain superior image quality in astronomical observations. This strategy increases the effective integration time, improving the SNR, achieves high-frequency sampling of target sources to obtain additional spatial information, and minimizes the risk of saturation in a single long exposure. As the dithered images are commonly Drizzled onto a fine-pixel grid, the objective of reconstruction is to recover the underlying high-definition image sampled on that fine grid—L = L P , which is obtained by sampling L on the target fine-pixel grid P . Every single exposure captures an image by sampling L with the detector, where L is convolved with the electronic pixels P of the camera in concordance with that dithered frame, resulting in a pixelated image formulated by L = (L P) S , with the weight map W taken into account. The superscript notation is used to distinguish the *k*-th frame in the dithered sequence. Following this analysis, L serves as an approximation to L in the form of:

L = Σ [W · (L K)] (3)

As shown in Equation (3), when a high-definition image (whether an idealized image with infinitely high resolution or its sampled representation on a fine-pixel grid) is convolved with K , it will be forwardly sampled onto the *k*th coarse-pixel frame with P . In contrast, the adjoint operator performs the reverse operation. The fact that convolution obeys associative and distributive properties enables the summation sign to be moved ahead of the term inside the parentheses in the second equation. Consequently, we can formulate a convolution kernel that encapsulates the combined impact of dithering, resampling, and coaddition operations, which can literally be referred to as the dithering kernel K :

K = Σ [W · (P S)] (4)

These equations indicate that, from a mathematical standpoint, the high-resolution image obtained through Drizzling the dithered multi-exposure images is calculated by convolving the intrinsic PSF-blurred image L with K (which must be normalized) and then sampling on the high-resolution fine grid, analogous to the blurring effect caused by a PSF. This provides a clear understanding of the spatial correlation between a pixel and its neighboring pixels in the Drizzled image. Alternatively, removing inter-pixel correlations in a Drizzled image is essentially equivalent to a deconvolution problem.

An interesting property of K , as seen from its expression, is that it can be interpreted as the sum of *N* sub-kernels, with the *k*th sub-kernel denoted as K . The adjoint operator of K , namely K *, is simply P S . Specifically, each dithered frame contributes its own sub-dithering kernel, and K represents the weighted average of these sub-kernels, with weights given by W , highlighting the role of individual frame contributions in shaping the final convolution kernel. The explicit representation of K is typically governed by Drizzle parameters such as the PSR parameter and the dither pattern used during observations. For an ideal dither pattern consisting of an infinite number of uniformly distributed random sub-pixel shifts, K can be computed in an almost analytical form. Furthermore, due to the pixel scale translational symmetry of the dither pattern in the internal region of the image, different pixels in this region share the same dithering kernel.

Figure 1 illustrates the dithering kernel for a PSR parameter of 0.1, with the top-left panel depicting the ideal dithering kernel as described earlier, while the remaining panels show results for a finite number of dithered frames visualized in terms of pixel-wise relative residuals with respect to the theoretical kernel.

Figure 2 [Figure 2: see original paper] (left column) presents dithering kernels obtained from 1600 dithered frames. The top and bottom panels illustrate the noise-free and Poisson noise cases, respectively, both exhibiting visual resemblance to the theoretical kernel. It is clear that with a sufficiently large number of dithered frames, the pattern obtained from Drizzling a latent point closely matches the theoretical dithering kernel, with differences diminishing as the number of dithered frames increases. A reliable image reconstruction method should efficiently deconvolve the Drizzled image.

In reality, flux redistribution after Drizzle is affected by random noise imported during the sampling process, deviating from the noiseless dithering kernel. Nevertheless, correlated noise between different pixels in the Drizzled image retains a structure similar to the dithering kernel. While this is an ill-posed problem, a well-designed image reconstruction method can still recover flux in the target pixel grid sufficiently.

3.2. Flux Re-concentration Capability

A bright spot with a total flux of 500 counts was dithered 1600 times on a fine-pixel grid with a resolution of 0.005 , then down-sampled to the coarser CSST-MCI resolution of 0.05 to generate a group of mock images. These images were subsequently Drizzled to produce a combined image with a pixel scale of 0.005 and normalized by the aggregated flux of this point source. This procedure partially follows the same process described in Section 3.1. Following this, we used the normalized Drizzled image as initial input for iterative reconstruction processing with fiDrizzle. Additionally, we generated mock images incorporating Poisson noise. Before normalization, each mock image was assigned a Poisson background with a mean of 500 counts per pixel, matching the total flux of the point source. This configuration represents a highly noisy condition, providing an opportunity to evaluate the effectiveness of different methods in accurately recovering point source fluxes in the presence of severe noise.

Figure 2 provides an illustrative example of the outcome after 100 iterations of fiDrizzle applied to the Drizzled image. The middle column demonstrates reconstructions obtained by fiDrizzle-DC, whereas the right column shows comparable results from fiDrizzle-MU. As shown, the initially diffused flux pattern in the Drizzled image is substantially gathered back toward the central region, clearly corresponding to the original point source. It is distinguishable that fiDrizzle-MU outperforms fiDrizzle-DC, leading to a more substantial accumulation of flux toward the central region by at least the 100th iteration, regardless of noise conditions. When Poisson noise is present, the rate at which flux concentrates is somewhat slower relative to the noise-free case when fiDrizzle-MU is employed. This process of centripetal flux concentration can be interpreted as the result of deconvolving the corresponding dithering kernels shown on the left side of the figure.

To quantitatively characterize the ability of different reconstruction methods to recover aliased pixel intensities introduced by dithering and Drizzle—specifically their capacity to decorrelate neighboring pixels—we calculate two primary metrics: (1) the Off-Center Flux Ratio (OCFR), defined as the sum of pixel values excluding the central pixel normalized by the total flux, providing a reliable quantification of correlation strength between a given pixel and its neighbors in the combined image (a lower OCFR indicates better flux concentration and stronger pixel decorrelation), and (2) the Peak Signal-to-Noise Ratio (PSNR), measuring overall reconstruction fidelity (higher PSNR values correspond to reconstructions that more closely match the ground truth, indicating better preservation of image details and lower reconstruction error).

Figure 3 [Figure 3: see original paper] provides a detailed comparison of OCFR for fiDrizzle-DC and fiDrizzle-MU under both noiseless and Poisson noise conditions. Both axes are plotted on a logarithmic scale to capture convergence behavior over several orders of magnitude. As shown, fiDrizzle-MU demonstrates a significantly faster convergence rate compared to fiDrizzle-DC. In the noiseless case, the OCFR of fiDrizzle-MU rapidly decreases and reaches zero by iteration 850. Beyond this point, there is no further decrease in OCFR, as flux has been fully reconcentrated at the central pixel. For clarity, the noiseless fiDrizzle-MU curve is truncated after approximately 150 iterations in the plot, since subsequent values decline sharply. Under Poisson noise conditions, fiDrizzle-MU maintains superior performance relative to fiDrizzle-DC. In this case, fiDrizzle-MU is iterated up to 5000 steps to demonstrate continued convergence, albeit at a reduced rate compared to the noiseless scenario. In contrast, fiDrizzle-DC exhibits much slower convergence behavior in both noiseless and noisy cases, though it appears less affected by noise, with OCFR values remaining consistent until approximately 30,000 iterations regardless of noise presence. However, even after 35,000 iterations—the maximum number tested—fiDrizzle-DC fails to achieve an OCFR comparable to that of fiDrizzle-MU in noisy conditions. This suggests that fiDrizzle-MU, by employing multiplicative updates, is inherently more robust and efficient in addressing flux redistribution introduced by dithering and Drizzle processes. These results highlight the superior ability of fiDrizzle-MU to rapidly decorrelate pixels from their neighbors, thereby achieving more accurate reconstruction in both ideal and realistic noisy scenarios.

In addition to OCFR, we assess reconstruction quality using PSNR between reconstructed images and ground truth. As shown in Figure 4 [Figure 4: see original paper], PSNR exhibits trends similar to those of the OCFR metric. Specifically, fiDrizzle-MU demonstrates much faster improvement in reconstruction fidelity compared to fiDrizzle-DC under both noiseless and noisy conditions. In the noiseless case, fiDrizzle-MU achieves a PSNR exceeding 80 dB within 130 iterations, indicating high-fidelity reconstruction. Even in the presence of Poisson noise, fiDrizzle-MU reaches a PSNR of 76 dB after approximately 2550 iterations. In stark contrast, fiDrizzle-DC requires nearly 30,000 iterations to approach similar PSNR levels in the noiseless scenario. Furthermore, under noisy conditions, fiDrizzle-DC fails to achieve comparable PSNR values regardless of iteration number. These results clearly demonstrate the superior convergence rate and reconstruction accuracy of fiDrizzle-MU, particularly in challenging noise-dominated environments.

Since we regard the process of mitigating degradation caused by dithering and Drizzle as a deconvolution problem, the reconstruction exhibits characteristics similar to other iterative deconvolution processes. In particular, the iteration number serves as an implicit regularization parameter, governing the trade-off between enhancing resolution and amplifying noise. Figure 5 [Figure 5: see original paper] illustrates PSNR evolution as a function of iteration number for both fiDrizzle-MU and fiDrizzle-DC around their respective optimal results under Poisson noise conditions. As iterations increase, PSNR initially rises, reflecting improved reconstructions. However, after reaching a maximum, PSNR begins to decline, indicating the onset of overfitting and noise amplification. For fiDrizzle-MU, the turning point occurs at iteration 3023, where PSNR attains its peak value of 79.108 dB. Beyond this point, further iterations result in gradual degradation of reconstruction quality, as reflected by decreasing PSNR. A similar trend is observed for fiDrizzle-DC, although its turning point is delayed, occurring at iteration 34,054 with a maximum PSNR of 75.052 dB. After this iteration, continued iterations lead to a slow but consistent decline in PSNR. These results highlight the importance of controlling iteration number in iterative reconstruction algorithms, as excessive iterations can compromise reconstruction fidelity by amplifying noise. This phenomenon is consistent with conclusions from Bertero et al. (2001) and other studies on iterative deconvolution methods. Before processing observational data, we typically perform simulations to determine the optimal iteration step at which to stop for achieving the best solution.

These results confirm that the multiplicative update strategy adopted by fiDrizzle-MU not only accelerates convergence but also improves overall reconstruction accuracy, particularly in the presence of observational noise.

3.3. The Positivity Constraints

As mentioned in Equation (2), positivity constraints are incorporated into our iterative scheme to enforce the physical requirement that flux values in astronomical images must be non-negative. In the context of image reconstruction and deconvolution, positivity serves as an important prior, preventing the algorithm from introducing unphysical negative flux values that commonly arise due to noise amplification or overfitting during iterations (Bertero et al. 2001). Positivity constraints, along with other constraints such as band limiting and wavelet regularization, act as a form of regularization, suppressing high-frequency oscillations and stabilizing the convergence process (Starck & Murtagh 1994). Without such constraints, iterative algorithms often produce ringing artifacts and spurious negative sidelobes, especially when recovering high-frequency features is essential (Starck et al. 2002). These artifacts can significantly degrade the photometric and morphological accuracy of recovered sources.

To evaluate the effect of positivity constraints in our framework, we compare results from fiDrizzle-DC and fiDrizzle-MU after ten iterations, both with and without positivity enforcement. The results are presented in Figure 6 [Figure 6: see original paper]. In the absence of positivity constraints (left panels), both algorithms exhibit pronounced ringing effects around bright sources, as well as noticeable negative flux regions. These artifacts distort the reconstructed image and reduce dynamic range. With positivity constraints applied, these high-frequency artifacts are effectively suppressed.

Recalling the description of the fiDrizzle-MU algorithm in Section 2 and the analysis of the dithering kernel—particularly the sub-kernels—in Section 3.1, fiDrizzle-MU can be formally expressed as:

$F_{\ 1} = F \ \cdot \Sigma \ [S \ (G \ / \ (F \quad K \ ))]$ (5)

This formulation indicates that fiDrizzle-MU can be interpreted as performing a Richardson–Lucy (RL)-like deconvolution (Richardson 1972; Lucy 1974) for each individual dithered frame, using its corresponding sub-dithering kernel K . Specifically, the algorithm iteratively deconvolves blurring effects introduced by the sub-dithering kernel in each dithered frame. At each iteration, deconvolved contributions from all N dithered frames are aggregated by weighted summation. The reconstructions thus incorporate additional fine-scale details sampled differently across multiple observations while benefiting from averaging of noise components present in each individual observation. This averaging effect leads to improved noise suppression and results in smoother, more stable reconstruction at each iteration step. This RL-like scheme, when combined with positivity constraints and appropriate stopping criteria, provides a flexible yet robust framework for high-fidelity image reconstruction from dithered observations. This formulation corresponds to Equation (20) in Starck et al. (2002) and is particularly well-suited for astronomical imaging, where Poisson noise is the predominant noise component.

Similarly, fiDrizzle-DC can be regarded as a Landweber-type iterative algorithm (Landweber 1951). When a positivity constraint is introduced, the method corresponds to Equation (15) in Starck et al. (2002). Consistent with the case of PSF deconvolution, the convergence speed and reconstruction fidelity of Landweber-type algorithms are generally inferior to those of RL-type algorithms.

3.4. The Spatially Resolving Power

Figure 7 [Figure 7: see original paper] illustrates images of a candidate gravitationally lensed quasar system captured by JWST-NIRCam across six distinct bands: three short-wavelength bands (F090W, F150W, F200W) and three long-wavelength bands (F270W, F356W, F444W). In the top row (a1–a6), the "*cal.fits" observational images are presented, having been calibrated through stage 2 of the JWST pipeline. The pixel scale differs between short-wavelength and long-wavelength bands for NIRCam on JWST: short-wavelength bands have a pixel size of 0.031 pixel^{-1}, whereas long-wavelength bands have 0.063 pixel^{-1}.

This variation optimizes instrument performance and resolution across different wavelengths.

The middle row (b1-b6) displays panels produced by the Drizzle algorithm (with nine exposures coadded for each panel) during the stacking process in stage 3 of the JWST pipeline. The bottom row (c1-c6) showcases results processed using the fiDrizzle-MU method, where images c1–c3 did not apply the positivity constraint detailed in Equation (2), whereas images c4–c6 adhered to this constraint. The positivity constraint visibly reduces ringing effects around bright sources, thereby achieving better control over background noise. This improvement helps distinguish true signals from noise, offering clearer images and more accurate data interpretation, especially for complex systems like gravitationally lensed quasars. By mitigating ringing effects, the analysis and identification of such phenomena are significantly enhanced, allowing more precise scientific conclusions. The panels in the middle and bottom rows have been upsampled by a factor of PSR=0.5 compared to the top row, meaning pixel scales are halved: 0.0155 pixel^{-1} for short-wavelength bands and 0.0315 pixel^{-1} for long-wavelength bands.

The point-like source indicated by the yellow arrow demonstrates significant flux across all six bands, while four other point-like sources surrounding the lens show notable flux only in bands c2–c6. This suggests these four sources may originate from the same quasar (QSO) located behind the lensing galaxy, implying that the source highlighted by the yellow arrow is not an image produced by the lensing effect. Notably, prior to this data processing, we conducted 30 simulations of source and noise conditions in this field of view to determine the iteration number yielding the highest PSNR. The optimal range was identified to be between 55 and 75 iterations, and we ultimately selected 65 iterations.

4. Computational Consumption

We conducted three sets of simulations—Mock-I, Mock-II, and Mock-III—to evaluate computational cost of four image stacking methods: Drizzle, iDrizzle, fiDrizzle-DC, and fiDrizzle-MU. Simulation specifics are as follows:

1. **Mock-I:** Stacked 10 exposure images sized at 512×512 pixels with PSR = 0.5.
2. **Mock-II:** Stacked 80 exposure images sized at 256×256 pixels with PSR = 0.25.
3. **Mock-III:** Stacked 160 exposure images sized at 128×128 pixels with PSR = 0.1.

Time consumption for these processes is listed in Table 2 , where numbers in parentheses indicate iterations required to achieve the same PSNR, set here at PSNR = 20. Notably, iDrizzle, fiDrizzle-DC, and fiDrizzle-MU all employed positivity constraints during image stacking.

We observed that fiDrizzle-MU and fiDrizzle-DC consumed approximately the same amount of time per iteration, roughly equivalent to twice the time taken by Drizzle. However, due to its higher convergence rate, fiDrizzle-MU achieved equivalent results in about one-fifth the time compared to fiDrizzle-DC. The iDrizzle algorithm, which frequently calls Fast Fourier Transform (FFT) for filtering during iterations, took approximately six times longer than fiDrizzle-DC. This comparative analysis highlights the efficiency and effectiveness of different image stacking techniques, with fiDrizzle-MU showing significant advantages in computational efficiency and convergence speed when reaching a specified PSNR.

5. Discussion and Conclusion

In this work, we have proposed and evaluated fiDrizzle-MU, an enhanced iterative algorithm for reconstructing high-resolution images from dithered astronomical observations. By replacing the difference-based correction term in fiDrizzle-DC with a multiplicative update and incorporating positivity constraints, fiDrizzle-MU demonstrates significant improvements in both convergence speed and reconstruction fidelity. Through comprehensive numerical experiments, we have shown that the algorithm effectively mitigates aliasing and deblends flux dispersed by the Drizzling process.

Our analysis in Section 3 confirms that fiDrizzle-MU outperforms its predecessor, fiDrizzle-DC, across several key metrics. The proposed method accelerates concentration of dispersed flux back to source locations, as evidenced by rapid decline in OCFR. Additionally, PSNR analysis highlights its superior ability to recover intrinsic image structures, even in the presence of significant Poisson noise. Importantly, the algorithm maintains robustness against noise amplification through integration of positivity constraints, which suppress spurious oscillations and ringing artifacts commonly associated with iterative deconvolution methods. Moreover, the quantitative analysis presented in Section 4 clearly demonstrates the substantial advantage of fiDrizzle-MU in reducing computational consumption with respect to iDrizzle and fiDrizzle-DC. We have also demonstrated in Section 3.4 that fiDrizzle-MU significantly enhances spatial resolution, resolving structures with minimal separations that were otherwise blended in conventional reconstructions. This capability is particularly promising for applications requiring high-precision astrometry and photometry, such as detection of compact binary systems and study of gravitational lensing.

The application of fiDrizzle-MU led to successful identification and resolution of a potential gravitationally lensed quasar candidate that remained poorly resolved using standard JWST data processing pipelines. Our method clearly distinguished six components of this system and effectively suppressed background noise, highlighting its potential for high-fidelity analysis of complex lensing structures.

Nevertheless, several limitations and areas for further development warrant discussion. First, although fiDrizzle-MU exhibits faster convergence than fiDrizzle-

DC, its computational complexity scales with the number of dithered frames and the chosen PSR parameter. Optimization of computational efficiency, possibly through parallelization strategies or GPU acceleration, will be crucial for processing extremely large data sets, such as those anticipated from upcoming deep-field surveys conducted by CSST-MCI. Second, while positivity constraints serve as an effective regularization prior, additional constraints—such as sparsity priors in a wavelet domain or total variation regularization—could further enhance the algorithm' s ability to recover faint and high-frequency features, especially in low signal-to-noise conditions. Integration of adaptive stopping criteria based on cross-validation or statistical noise modeling may also improve convergence control and prevent overfitting.

Looking ahead, the fiDrizzle-MU framework shows substantial promise for high-precision imaging in space-based and ground-based astronomical observations. We plan to apply this method to extreme deep-field data sets from CSST-MCI, comprising up to 1600 dithered frames per field, with the goal of enabling precision studies such as dark matter subhalo detection through strong gravitational lens modeling. Further extensions may include development of hybrid approaches combining fiDrizzle-MU with PSF deconvolution techniques to simultaneously address instrumental blurring and dithering-induced aliasing.

In summary, fiDrizzle-MU represents a robust and efficient image reconstruction algorithm tailored to modern astronomical data sets. Its improvements over prior methods in both convergence behavior and reconstruction accuracy establish it as a valuable tool for precision cosmology and astrophysical imaging. Future work may explore integration of fiDrizzle-MU into large-scale data processing pipelines for next-generation surveys such as Euclid or LSST, where robust handling of under-sampled dithered data will be critical.

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Note: Figure translations are in progress. See original paper for figures.