Optimal Local Government Debt in China

Authors: Wang Kaiming, Tang Yijie, Zhang Bei, Gaoxun Zhang, Wang Kaiming

Date: 2025-09-18T00:53:03+00:00

Abstract

This paper develops a model for optimal local government debt with Chinese characteristics, which takes into account the roles of the central government and local governments in the macroeconomy. The central government is tasked with providing optimal public goods to the entire society. The central government needs to trade-off the utility of households and the utility of public goods provided by the government, assign different weights to them, and strive to maximize the welfare of the whole society. Local governments are responsible for providing optimal public goods to their jurisdictions, with their revenue sources including profits from local state-owned assets (including land use right transfer revenues), taxes, and the issuance of bonds. Using a calibration method, we determine the optimal debt and debt limits for the Shanghai municipal government and other local governments. The findings indicate that the model can effectively estimate the debt and debt limits of local governments.

Full Text

Optimal Local Government Debt in China

Wang Kaiming[1], Tang Yijie[1], Zhang Bei[2], and Zhang Gaoxun[3]

[1]College of Economics and Management, Beijing University of Technology, Beijing, 100124
[2]College of Science, Southwest University of Science and Technology, Mianyang, Sichuan

Abstract: This paper develops a model for optimal local government debt with Chinese characteristics that incorporates the distinct roles of central and local governments in the macroeconomy. The central government is tasked with providing optimal public goods to society as a whole, balancing household utility against the utility derived from government-provided public goods by assigning

different weights to maximize overall social welfare. Local governments are responsible for providing optimal public goods to their jurisdictions, with revenue sources including profits from local state-owned assets (including land use right transfer revenues), taxes, and bond issuance. Using a calibration method, we determine the optimal debt and debt limits for the Shanghai municipal government and other local governments. Our findings indicate that the model can effectively estimate local government debt and debt limits.

Keywords: Chinese Characteristics; Local Government; Optimal Local Government Debt
JEL Classification: E20, E62, P30

[1]Corresponding author: Wang Kaiming. Address: No. 29, Hangtian Road, Longtan Subdistrict, Chenghua District, Chengdu City, Sichuan Province, PR China. Email: wkm.6870@163.com

Introduction

Local government debt has played a critical role in China' s economic development. To mitigate the impact of the 2008 financial crisis, China' s central government implemented an active fiscal policy through a large-scale investment program widely known as the "Four Trillion Yuan Plan." The central government planned to allocate 1.18 trillion yuan to infrastructure development, government-managed funds, public investments, and post-disaster reconstruction efforts following the Wenchuan earthquake. Together with matching funds from local governments and social investment, this formed a total investment scale of four trillion yuan.

By the end of 2012, government debt financing had been extensively allocated across sectors such as transportation, urban development, land reserves, education, cultural initiatives, healthcare, agriculture, ecological conservation, environmental protection, and affordable housing, which together accounted for 92.14% of total expenditures. This influx of debt-supported investments spurred rapid development in local infrastructure projects, including highways, railways, airports, urban rail transit, roads, and bridges, enabling comprehensive domestic infrastructure growth and sustaining China' s high-speed economic growth in the post-crisis period.

However, the debt-fueled economic acceleration also led to a substantial rise in local government debt levels. Data from the National Audit Office and the Ministry of Finance reveal that from 2007 to 2023, China' s local government debt increased from 4.51 trillion yuan to 41.62 trillion yuan, with the debt-to-GDP ratio rising from 16.66% to 29.10%. The debt-to-GDP ratio for local governments peaked in 2013, subsequently declined to approximately 20%, and then sharply rose again to 33.67% by 2023. This resurgence was primarily due to intensified fiscal stimulus from both central and local governments in response

to the economic disruptions caused by the COVID-19 pandemic. For further details, refer to Table 1 .

In response to the challenges posed by local government debt accumulation, the Chinese government enacted a revised Budget Law of the People' s Republic of China in 2015. Commencing in 2015, the Ministry of Finance embarked on a debt restructuring initiative that strategically replaced short-term, high-interest debt from government financing platforms with long-term, low-interest bonds. This transition aimed to alleviate the debt burden and optimize the debt structure of local governments. Simultaneously, the central government implemented a debt limit management policy for local governments under the Budget Law and the State Council's Opinions on Strengthening the Management of Local Government Debt (Guofa [2014] No. 43), which received approval from the Standing Committee of the National People's Congress (NPC). This measure alleviated the pressure of local government debt and reduced associated risks.

The Fifth Session of the 13th National People's Congress reviewed and approved a national local government debt limit of 37.65 trillion yuan for 2022, with a general debt limit of 15.83 trillion yuan and a special-purpose debt limit of 21.82 trillion yuan. As of October 2022, the outstanding local government debt stood at 35.17 trillion yuan, which remains within the limit sanctioned by the NPC. This debt includes 14.46 trillion yuan in general debt and 20.71 trillion yuan in special debt, comprising 35.01 trillion yuan in local government bonds and 162.3 billion yuan in non-bond forms of local government debt. The average remaining maturity of local government bonds is 8.5 years, with general bonds averaging 6.3 years and special bonds at 10.1 years; the average interest rate is 3.4%, with general bonds at 3.39% and special bonds at 3.4%.

In 2023, China' s central government debt reached 30.03 trillion yuan, and local government debt was 41.62 trillion yuan. Together, these accounted for 56.93% of China' s GDP. By contrast, according to the International Monetary Fund, the debt-to-GDP ratio for the U.S. government in 2023 was 118.73%, Japan' s government debt stood at 249.67% of GDP, the U.K.' s at 99.97%, and the debt-to-GDP ratio of advanced economies was 108.69%. These figures reveal that debt ratios in developed economies such as Europe and the United States are markedly higher than that of China, yet these economies have not faced debt crises. This context raises the question of whether China's stringent debt control policies are overly restrictive. Therefore, we should systematically determine the optimal debt level for China' s local governments, assess the rationality of debt limits, and explore methodologies for optimizing local government debt.

Literature Review and Research Approach

Extensive research has explored optimal government debt from various theoretical and empirical perspectives. Barro (1979) approached this issue using the Ricardian equivalence theorem, equating the present value of government expenditures with future tax liabilities to create a cost function based on tax-

ation and output. His analysis, constrained by Ricardian equivalence, sought to minimize the total cost of tax collection and thereby determine an optimal debt level. However, as Bernheim (1987) observed, the validity of Barro' s conclusions depends critically on Ricardian equivalence, which requires seven core assumptions to hold. Aiyagari and McGrattan (1998) expanded on this topic through a model with infinitely lived households whose saving behavior is influenced by precautionary saving motives and borrowing constraints. By linking government debt to household assets, they constructed a social welfare function. Their numerical approach to maximizing social welfare has since become foundational in government debt research, with numerous studies refining and extending their framework.

Aiyagari et al. (2002) consider government debt as the sum of discounted future fiscal deficits and employ the Ramsey method to study optimal government debt and taxation. Floden (2001) modified the infinite-life household model into a two-period overlapping generations model to examine the effects of welfare policies on optimal debt. His findings indicated that welfare gains from increased debt occur only when transfer payments exceed optimal levels. Similarly, Leeper et al. (2021) contributed to this area by incorporating government consumption directly into the social welfare function, without differentiating among household asset types. Vogel (2023) applied a multi-period overlapping generations model, linking government debt and social welfare by adjusting taxes or transfer payments to modulate debt levels. This approach enabled him to relate social welfare to government debt, thus identifying the optimal debt level. Rohrs (2016) also employed a two-period overlapping generations model, integrating the utility derived from public goods into household utility functions and introducing voting weights to form a political target function. His research found that the elasticity of substitution between public and private consumption influences public debt scale, accounting for observed cross-national differences in debt levels. Elberry et al. (2023) provided an insightful review of advancements in government debt research.

Chinese scholars have conducted extensive research on central and local government debt in China. Lin and Gong (2006), building on the theoretical model established by Aiyagari and McGrattan (1998), conducted empirical analysis on the long-term equilibrium state of the economy and found that the optimal public debt in China remains negative. Jia and Guo (2011) constructed a two-sector (enterprises and households) endogenous growth iterative model to numerically simulate the impact of fiscal policy changes on long-term economic growth and government debt scale under different fiscal rules. Yan and Liao (2017) discussed the long-term equilibrium of the economy when the government finances public investment through borrowing, based on a three-sector overlapping generations model with an endogenous growth mechanism, indicating that under certain conditions, there exists a positive government debt-output ratio in the economic system.

Regarding local debt in China, Shi (2010) analyzed the causes of local govern-

ment debt problems, pointing out two main factors: the "macro policy view," which attributes debt to the national economic stimulus investment expansion plan in response to the global financial crisis requiring local government funding to match; and the "financial system view," which identifies the trend of "revenue centralization and expenditure decentralization" formed after the 1994 tax reform as a key driver. Chen and Wang (2016) constructed a three-sector dynamic equilibrium model, and their research showed that when the government is not constrained by debt, there is an inverse "U-shaped" relationship between government debt and economic growth; when the government is subject to debt constraints, the economic growth rate gradually decreases with the increase of government debt. Zhu and Xu (2018) introduced the central and local government structure into the mainstream dynamic general equilibrium model to examine the dynamic impact of local fiscal policies on China' s macroeconomy under fiscal decentralization. Tian and Zhao (2019) studied the joint impact of financial system efficiency and local government debt. Guo et al. (2020) investigated the mechanism of local government debt, finding that fiscal decentralization, budget soft constraints, economic performance promotion incentives, government guarantees, and financial market efficiency are theoretical mechanisms for the formation of local government debt; moral hazard is the essence of the formation mechanism of local government debt. Li et al. (2020) constructed a new Keynesian DSGE model that includes "double defaults" of ordinary enterprises and local government financing platform enterprises, and "dual pillars" of monetary policy and macroprudential policy, indicating that risk shocks from both types of enterprises are an important source of economic fluctuations in China. Gao et al. (2022) used the financial accelerator model to analyze the impact of local debt on China' s economic fluctuations, considering the role of local governments in modeling and providing optimal public goods. Lv et al. (2024) constructed a macroeconomic-based model to study the sustainability of China' s public finance.

When studying local debt, China's special national conditions must be taken into account, particularly the significant impact of land factors on local debt. Zhao et al. (2017) constructed a dynamic general equilibrium model that reflects the land finance behavior of local governments in China, theoretically and quantitatively analyzing the dynamic accelerator mechanism of land finance. Liang and Hao (2019) constructed a five-sector general equilibrium model that includes shadow banking and land finance, indicating that the slowdown of economic growth, the high cost of local government fund use, and increased uncertainty of land transfer income are the main factors leading to the unsustainability of local government debt in China. Li (2019) analyzed the potential mechanism of local government debt inducing systemic financial risks from the perspective of local government bonds and land finance within a general equilibrium framework. Zhao et al. (2021) constructed a dynamic general equilibrium model that includes macro-control, local governments, land markets, and local government debt, studying the internal logical mechanism of external shocks such as financial crises affecting China' s economic development, emphasizing the key role of

China' s characteristic macro-control and local government behavior.

Regarding local government debt limits, many Chinese scholars have also conducted relevant research. Diao (2015) analyzed local government fund income and the repayable portion of local public finance income, examining general debt limits and special debt limits separately, and pointed to the need to reduce debt scale by disposing of government assets to ensure rigid implementation of debt limits after the "new regulations" on local debt. Ma and Kong (2017) proposed using six indicators to design a local government debt limit index system. Yang (2018) used the project evaluation method to study local government debt ceilings, employing a credit scoring template to evaluate credit risk levels. Wang et al. (2017) used the zero-sum revenue DEA model to evaluate the efficiency of local government debt at the provincial level, noting that the efficiency of government debt use in most Chinese provinces and regions is relatively low, necessitating reallocation of local government debt limits based on the principle of optimal efficiency under a fixed total amount.

The review of literature indicates that mainstream Western economic methods are used in both domestic and international studies on optimal government debt in China. However, foreign research on optimal debt rarely endogenizes government consumption. Secondly, there is rarely a distinction made between household utility and the utility of public goods provided by the government. While public goods provided by the government constitute a form of welfare, they are distinct from household utility (welfare) because households with different assets, incomes, and tax burdens enjoy the same public goods. Lastly, mainstream Western economic methods almost entirely ignore state-owned assets, making them unsuitable for direct application to studying China' s optimal government debt.

Western mainstream economics is based on the capitalist system and primarily focuses on private ownership of production means. The income sources for capitalist governments are taxation and debt issuance. In contrast, China practices a socialist economic system with public ownership as the mainstay, where the government' s role in the economy and society significantly differs from that in Western capitalist countries. The Chinese government possesses substantial state-owned assets and state-owned enterprises, and land resources are state-owned. The government is not only an economic manager but also a direct participant in economic activities. For example, state-owned enterprises under the Chinese central government are managed by the State-owned Assets Supervision and Administration Commission, and large state-owned commercial banks are held by the Ministry of Finance. In addition to taxation and debt issuance, state-owned assets owned by the government generate significant income.

In typical macroeconomic models, the government is required to provide public goods to society, but rarely is there an emphasis on the government needing to provide *optimal* public goods. In China, the government is tasked with providing public goods and, more importantly, optimal public goods. Besides providing optimal public goods, the government must also consider household

welfare. Due to this difference, the central government needs to weigh these two different utilities based on the stage of economic and social development and pursue the maximization of overall social welfare through taxation.

The positioning of local governments is often not viewed from the basic function of providing public goods, but rather as a tournament for government officials to pursue promotion (Zhou (2007)). Moreover, local governments do not have the power to set taxes and cannot act as social planners in China. They are independent of local households, making the Ramsey model unsuitable for them.

Through research and study of Chinese and foreign literature, we have identified several issues in current Chinese local government debt research. First, most studies lack a systematic perspective, focusing only on partial variables while neglecting other crucial ones. For instance, they fail to consider central government-set taxes, as well as central government investments and consumption. These variables interact and influence each other in the operation of the entire economy, and none can be omitted.

Second, the constraints faced by local governments are not fully accounted for, particularly the revenues generated by local state-owned enterprises (SOEs), which constitute a significant income source for local governments. Local government expenditures should be categorized into consumption and investment: consumption is used to provide public goods, while investment forms state-owned assets that support the operations of local SOEs. However, some studies incorrectly incorporate local government expenditures into enterprise production functions. Lastly, there is ambiguity in defining the role of local governments, whose primary objective should be to provide optimal public goods to their local communities.

In response to these existing issues, we have constructed a macroeconomic model with Chinese characteristics to study China' s optimal local government debt. We first introduce households who own assets, earn asset returns, receive wage income by providing labor to enterprises, and pursue utility maximization. Secondly, we introduce enterprises, distinguishing between state-owned enterprises (SOEs) and private enterprises to reflect China' s national conditions. SOEs are further divided into central and local SOEs, all pursuing profit maximization.

We also focus on the role of the government, divided into central and local governments. The central government and local governments provide different public goods to society that they are respectively responsible for. Local government revenue comes from taxation, profits from local SOEs, land income, and debt issuance. Both central and local governments aim to provide optimal public goods. The central government acts as a central planner, considering household utility and public goods when formulating tax policies, pursuing the maximization of overall societal welfare. Based on this, we determine the optimal tax set by the central government and the optimal debt issued by local governments.

The innovation of this paper is that we establish a Chinese macroeconomic

model and apply it to the study of optimal local government debt. In this model, we fully consider the characteristics of socialism with Chinese features, introducing state-owned land and state-owned enterprises to create a macroeconomic model with Chinese socialist characteristics and apply it to the study of local government debt, providing theoretical guidance for the management of local government debt limits.

The following sections of this paper are structured as follows: The third section constructs the theoretical model and presents the optimal debt. The fourth section provides empirical analysis, and the fifth section concludes.

The Model

In this section, we construct the theoretical model. Our model involves three main participants: households (including local households and other local households), the government (comprising the central government, the local government, and other local governments), and enterprises (which include local and other local private enterprises, central government-owned state-owned enterprises (SOEs), and local government-owned SOEs). Households provide labor to enterprises to earn wages, invest to form assets, receive returns, and pay taxes to the government. Enterprises utilize the assets of households or the governments, lease land from the governments, hire workers, produce goods, and generate profits. Local governments own assets and receive rental income, own local SOEs and enjoy their profits, lease land to enterprises, collect tax revenues from households, issue bonds, and provide public goods. The central government has similar functions but without land rent incomes. China practices a tax-sharing system, with the central government setting tax policies and sharing tax revenues with local governments at a certain ratio. The debt limits for local government bonds are determined by the NPC.

3.1 Households

We divide households into two parts: local households and other local households. Considering local households, we denote different households with the superscript j. Let N_t represent the total number of local households and N_t^* represents the total number of other households.

3.1.1 Local Households The utility of local household j can be expressed as:

$$U_{j,t} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(C_{j,t+s}) - \frac{L_{j,t+s}^{1+1/l_s}}{1+1/l_s} \right]$$

In our model, the utility function for household consumption is taken to be a logarithmic function.

Household j aims to maximize utility subject to the budget constraint:

$$C_{j,t} + I_{j,t} + B_{j,t} = R_t K_{j,t} + W_t L_{j,t} + P_{j,t} - T_{j,t}$$

At time t, the consumption of household j is denoted as $C_{j,t}$. Household j rents its assets $K_{j,t}$ to enterprises at a rental rate R_t, and also receives a share of the profits $P_{j,t}$ from the enterprises. Households provide labor $L_{j,t}$ to enterprises and earn wage income $W_t L_{j,t}$ at the wage rate W_t, $I_{j,t}$ is the household's investment, $B_{j,t}$ is the amount of government bonds purchased by the household, R_t is the interest rate, P_t is the price level, E_t represents the conditional expectation, l_s represents the inverse of the elasticity of work effort with respect to the real wage, and $T_{j,t}$ represents the lump-sum tax paid by the household.

The accumulation equation of the household' s assets is:

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t} - S\left(\frac{I_{j,t}}{I_{j,t-1}}\right) I_{j,t}$$

In the evolution equation, δ represents the depreciation rate, and $S(\cdot)$ is an adjustment cost function that equals zero in steady state with a constant investment level.

3.1.2 Other Local Households Next, we consider other local household j, who are similar to local households. We distinguish their parameters by adding a superscript $*$ to them. The utility of other households can be expressed as follows:

$$U_{j,t}^* = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\ln(C_{j,t+s}^*) - \frac{(L_{j,t+s}^*)^{1+1/l_s^*}}{1+1/l_s^*} \right]$$

Other households also pursue utility maximization subject to the budget constraint:

$$C_{j,t}^* + I_{j,t}^* + B_{j,t}^* = R_t^* K_{j,t}^* + W_t^* L_{j,t}^* + P_t^* - T_{j,t}^*$$

At time t, household j' s consumption is $C_{j,t}^*$, rents its assets $K_{j,t}^*$ to enterprises at a rental rate, and also shares in the enterprises' profits P_t^*. The household supplies labor $L_{j,t}^*$ to the enterprises, earning wage income $W_t^* L_{j,t}^*$ at the wage rate W_t^*, with $I_{j,t}^*$ representing the household' s investment, $B_{j,t}^*$ the amount of government bonds purchased by the household, and $T_{j,t}^*$ the taxes paid by the household.

The accumulation equation of the other local household' s assets is:

$$K^*_{j,t+1} = (1-\delta^*)K^*_{j,t} + I^*_{j,t} - S^*\left(\frac{I^*_{j,t}}{I^*_{j,t-1}}\right) I^*_{j,t}$$

3.2 Enterprises

We categorize enterprises into two types: final goods producers and intermediate goods producers. The final goods producers operate under perfect competition, while the intermediate goods producers are under monopolistic competition. To simplify, we assume that intermediate goods are produced by two types of enterprises, namely state-owned enterprises and private enterprises. Here, private enterprises are further divided into local and other local private enterprises; state-owned enterprises are divided into local and other local state-owned enterprises, and central enterprises are divided into local and other local central enterprises. For convenience, we represent local private enterprises with 1, local state-owned enterprises with 2, and local central enterprises with 3. We use 1^* for other local private enterprises, 2^* for other local state-owned enterprises, and 3^* for other local central enterprises.

It is assumed that final goods are produced by a perfectly competitive enterprise using intermediate goods. The production technology for final goods is:

$$Y_t = \left(\sum_i \omega_i^{\frac{1}{\epsilon}} Y_{i,t}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where p_t^l is the price markup, and ω_i is the weight of intermediate goods in the total output in a steady state. The final goods sector aims to maximize profits, from which we can derive:

$$\max_{Y_{i,t}} P_t Y_t - \sum_i P_{i,t} Y_{i,t}$$

where $P_{i,t}$ is the price of intermediate goods, $i \in \{1, 2, 3, (1,)^*, (2,)^*, (3,)^*\}$. Since the final goods sector is perfectly competitive, profits are zero in equilibrium, and the price of intermediate goods can be expressed as:

$$P_{i,t} = \omega_i \left(\frac{Y_t}{Y_{i,t}}\right)^{\frac{1}{\epsilon}} P_t$$

We know that enterprises need capital for production, such as fixed assets, factories, and land. In China, land is a special means of production owned by the state. Households or enterprises cannot purchase land; they can only purchase land use rights. The term of land use rights for industrial purposes is 50 years.

We convert the cost of purchasing land use rights by enterprises into rental fees for leasing the land, which is equivalent to enterprises renting land from the government. In this way, we distinguish the capital used by enterprises, which includes land and assets excluding land, denoted $K_{i,t}^{\text{land}}$ and $K_{i,t}$. The production function for intermediate goods enterprise i is:

$$Y_{i,t} = K_{i,t}^{\alpha}(K_{i,t}^{\text{land}})^{d}L_{i,t}^{1-\alpha-d}$$

where α represents the share of capital income in output, and $1-\alpha-d$ represents the share of land income in output.

Enterprise i pursues profit maximization:

$$\max_{K_{i,t},L_{i,t},K_{i,t}^{\text{land}}} P_{i,t}Y_{i,t} - R_tK_{i,t} - W_tL_{i,t} - R_t^{\text{land}}K_{i,t}^{\text{land}}$$

where R_t^{land} is the rental rate for land.

3.3 Government

In order to study optimal local government debt, it is necessary to introduce three types of government: the local government, other local governments, and the central government. To distinguish between these three different governments, we denote them as A, B, and C, where A represents the local government, B represents other local governments, and C represents the central government.

3.3.1 The Local Government In China, local governments can only use bond financing and are not allowed to borrow from banks. Local governments, however, do hold local state-owned assets and land.

The budget constraint for the local government is as follows:

$$G_t^A + I_t^A + R_tB_{t-1}^A = T_t^A + P_t^A + R_t^{\text{land}}K_t^{\text{land},A} + B_t^A + TR_t$$

In the model, G_t^A represents the expenditures of the local government, I_t^A represents the investments made by the local government, $K_t^{\text{land},A}$ represents the land investments by the local government, B_t^A is the bonds issued by the local government, K_t^A represents the assets owned by the local government, $L_t^{\text{land},A}$ represents the land owned by the local government, TR_t is the transfer payments received by the local government from the central government[3], T_t^A represents the taxes collected by the local government, and P_t^A is the profit obtained by the local government from holding local state-owned enterprises.

[3]Here, we briefly explain the reason for introducing transfer payments. When studying the optimal debt of local government and conducting empirical research, we found that transfer payments have a significant impact. The decision-making process for transfer payments by the central government is highly complex, taking into account its control over local governments and the pursuit of fairness. Therefore, we assume here that transfer payments are already an optimal decision.

The accumulation equation of the local government assets is as follows:

$$K_{t+1}^{A} = (1-\delta)K_{t}^{A} + I_{t}^{A}$$

where δ represents the depreciation rate. The land owned by the government is leased to enterprises only for the right to use; the ownership of the land remains with the government. The local government still needs to invest in the land, hence there is an accumulation equation of land:

$$K_{t+1}^{\text{land},A} = (1-\eta)K_{t}^{\text{land},A} + I_{t}^{\text{land},A}$$

Land is a special type of asset, as it does not depreciate. However, in China, what local governments sell is the land use right. Therefore, there is depreciation in the asset of land use right. η is the depreciation in the asset of land use right.

The local government is required to provide public goods to local society. The more public goods provided by local government, the higher the satisfaction of society. Therefore, the utility for local government is to provide public goods to society. We assume that the local government uses a technology f to transform government expenditures into public goods:

$$G_{t}^{\text{pub},A} = f(G_{t}^{A})$$

The local government pursues the optimal utility of public goods:

$$\max_{G_t^A, I_t^A, K_t^{\text{land},A}, B_t^A} \sum_{t=0}^{\infty} \beta^t \ln(G_t^{\text{pub},A})$$

The decision variables for local government are G_t^A, I_t^A, $K_t^{\text{land},A}$, and B_t^A, excluding taxes, because local governments do not have the right to set tax policies. In China, tax policies are set by the central government.

In China' s political system, there is a hierarchical relationship between the central government and local governments. The central government is the supreme authority, and local governments at all levels are institutions that implement local administrative management under the guidance of the central government.

Although local governments have a certain degree of autonomy in making various decisions, when determining the optimal local government debt to ensure the sustainability of local finance, the NPC sets debt limits for local governments to ensure that their debts do not exceed these limits[4]. Therefore, in determining the optimal debt levels for local governments, we introduce exogenous variables, denoted $\bar{B}_t^A$, $\bar{B}_t^B$, and $\bar{B}_t^C$, which represent the debt ceiling of the local government, other local governments, and the central government, respectively. Thus, the ultimate optimal debts for the local government, other local governments, and the central government should be $B_t^A \leq \bar{B}_t^A$, $B_t^B \leq \bar{B}_t^B$, and $B_t^C \leq \bar{B}_t^C$.

[4]When the NPC sets the annual debt limit, it includes the special bonds issued by local governments. The optimal debt of the local government is the optimal generalized fiscal deficit. When formulating the optimal debt, considering fiscal sustainability, the net debt issued by the government cannot exceed the present value of future fiscal deficits. Therefore, the annual debt limit for local governments set by the NPC differs from the exogenous debt limit determined by fiscal sustainability considerations.

3.3.2 Other Local Governments We now consider other local governments, which similarly can only rely on bond financing. The other local governments can transform government expenditures into public goods using a specific technology f:

$$G_t^{\text{pub},B} = f(G_t^B)$$

Other local governments pursue:

$$\max_{G_t^B, I_t^B, K_t^{\text{land},B}, B_t^B} \sum_{t=0}^{\infty} \beta^t \ln(G_t^{\text{pub},B})$$

The decision variables for other local governments are G_t^B, I_t^B, $K_t^{\text{land},B}$, and B_t^B, excluding taxes.

The accumulation equation of other local governments' assets is represented by:

$$K_{t+1}^B = (1-\delta)K_t^B + I_t^B$$

The accumulation equation of other local governments' land is:

$$K_{t+1}^{\text{land},B} = (1-\eta)K_t^{\text{land},B} + I_t^{\text{land},B}$$

3.3.3 Central Government The central government provides public goods distinct from those provided by local governments, with no overlap between the public goods offered by local and central governments. The sources of income for the central government include taxes, bonds, profits from central state-owned enterprises, and asset rental revenues. Similarly, we assume that the central government can use a technology to transform government expenditures into public goods. China' s fiscal policy is formulated by the central government.

China implements a tax-sharing system, and local governments do not have the right to formulate their own tax systems. Therefore, when formulating tax policies, the central government fully considers the needs of local governments as well as the interests of households. Thus, when making optimal decisions, the central government considers providing optimal goods, not only those that the central government should provide but also those that local governments need to provide. Therefore, we divide the decision-making role of the central government into two parts: the first role is as a social planner when setting taxes, considering the utility of households and the government, with taxes being the decision variable. The second role is to provide optimal public goods to society.

When determining taxes, the central government needs to consider the utility of households and the government:

$$W_t = \theta^c \ln(G_t^{\text{pub}}) + (1-\theta^c) \sum_{j=1}^{N_t} U_{j,t}$$

The decision variable for the central government is T_t. Among them, θ^c is the weight that the central government assigns to the utility of the total public goods provided by the government.

Constraints for the central government:

$$G_t^C + I_t^C + R_t B_{t-1}^C = T_t^C + P_t^C + R_t K_t^C + B_t^C$$

The constraints for households differ from the constraints (3) and (14) at the time of optimal decision-making. When the government optimizes by considering the utility of households, it is actually done on the basis that households have already made their optimal decision variables. The government' s decision variable considers the tax in the households' constraints and the entire tax. Therefore, the utility function for households can be rewritten as:

$$U_t = \sum_{j=1}^{N_t} U_{j,t} + \sum_{j=1}^{N_t^*} U_{j,t}^*$$

The constraints can be rewritten as:

$$C_t + I_t + B_t = R_t K_t + W_t L_t + P_t - T_t$$

In the above constraints, we express the variables in the following aggregated manner:

$$C_t = \sum_{j=1}^{N_t} C_{j,t} + \sum_{j=1}^{N_t^*} C_{j,t}^*$$

The remaining variables are aggregated in a similar manner.

Since China practices a tax-sharing system, after the government makes optimal tax decisions, we can obtain the taxes for the central and local governments. Therefore, the central government' s tax revenue is:

$$T_t^C = c \cdot T_t$$

where c represents the proportion of central government taxes in total taxes, b represents the proportion of other local governments in total taxes, and a represents the proportion of the local government in total taxes.

The taxes for the local government are:

$$T_t^A = a \cdot T_t$$

The taxes for other local governments are:

$$T_t^B = b \cdot T_t$$

As a provider of public goods, the central government needs to provide public goods to society and pursue utility maximization:

$$\max_{G_t^C, I_t^C, K_t^C} \sum_{t=0}^{\infty} \beta^t \ln(G_t^{\text{pub},C})$$

The decision variables for the central government are G_t^C, I_t^C, and K_t^C.

The central government' s intertemporal budget constraint is given by:

$$G_t^C + I_t^C + R_t B_{t-1}^C = T_t^C + P_t^C + R_t K_t^C + B_t^C$$

The accumulation equation of the central government' s assets is given by:

$$K_{t+1}^C = (1-\delta) K_t^C + I_t^C$$

3.4 Monetary Policy

We assume that the central bank determines interest rates according to the Taylor rule:

$$R_t = R^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(Y_t - Y^*)$$

where π^*, R^*, and Y^* represent the target inflation rate, target interest rate, and target output, respectively.

3.5 Market Clearing

The optimal decision variables for local households include $C_{j,t}$, $I_{j,t}$, $K_{j,t}$, $L_{j,t}$, and $B^+_{j,t}$. The optimal decision variables for other local households include $C^*_{j,t}$, $I^*_{j,t}$, $K^*_{j,t}$, $L^*_{j,t}$, and $B^*_{j,t}$. The optimal decision variables for local private enterprises include $L_{1,t}$ and $K_{1,t}$. The optimal decision variables for local state-owned enterprises include $L_{2,t}$ and $K_{2,t}$. The optimal decision variables for other local central enterprises include $L_{3,t}$ and $K_{3,t}$. The optimal decision variables for other private enterprises include $L^*_{1,t}$ and $K^*_{1,t}$; the optimal decision variables for other local state-owned enterprises include $L^*_{2,t}$ and $K^*_{2,t}$. The optimal decision variables for other central enterprises include $L^*_{3,t}$ and $K^*_{3,t}$. The central government, acting as a social planner, determines the optimal tax decision variables T_t; the decision variables for the local government in providing optimal public goods include G^A_t, I^A_t, $K^{\text{land},A}_t$, and B^A_t. The decision variables for other governments in providing optimal public goods include G^B_t, I^B_t, $K^{\text{land},B}_t$, and B^B_t. The decision variables for the central government in providing optimal public goods include G^C_t, I^C_t, and K^C_t. Price variables include W_t, P_t, $P_{i,t}$, R_t, R^{land}_t, R^*_t, and $R^{\text{land},*}_t$; exogenous variables include N_t and N^*_t.

A competitive equilibrium is attained under the optimal decision variables of households, enterprises, and governments:

1. The optimal decision for local households is achieved by maximizing utility (2) under constraints (3) and (4).
2. The optimal decision for other local households is achieved by maximizing utility (5) under constraints (7) and (8).
3. Total output is determined by equation (9).
4. The optimal decision for local governments is achieved by maximizing the utility of public goods provision (18) under constraints (14), (15), and (16).
5. The optimal decision for other local governments is achieved by maximizing the utility of public goods provision (21) under constraints (22), (23), and (24).
6. The central government, as a social planner, determines the optimal tax by maximizing the overall welfare function of society (25) under constraint (26).

7. The optimal decision for the central government is achieved by maximizing the utility of public goods provision (34) under constraints (35) and (36).
8. The equilibrium of total labor supply satisfies: $L_t = L_{1,t} + L_{2,t} + L_{3,t}$.
9. The profits shared by households satisfy: $P_t = P_{1,t} + P_{2,t} + P_{3,t}$ and $P_t^* = P_{1,t}^* + P_{2,t}^* + P_{3,t}^*$.
10. The profits shared by local governments: $P_t^A = P_{2,t}$. The profits shared by other governments: $P_t^B = P_{2,t}^*$. The profits shared by the central government: $P_t^C = P_{3,t} + P_{3,t}^*$.
11. The market clearing condition for prices is determined by equation (11).
12. The clearing of the asset market requires that land assets satisfy: $K_t^{\text{land},A} + K_t^{\text{land},B} = K_t^{\text{land}}$. Households' assets must satisfy: $K_t = K_{1,t} + K_{2,t} + K_{3,t}$. State-owned assets must satisfy: $K_t^A + K_t^B + K_t^C = K_{2,t} + K_{2,t}^* + K_{3,t} + K_{3,t}^*$. Bonds must satisfy: $B_t = B_t^A + B_t^B + B_t^C$. Transfer payments must satisfy: $TR_t = TR_t^A + TR_t^B$.
13. The resource constraint equation is: $Y_t = C_t + I_t + I_t^A + I_t^B + I_t^C + G_t^A + G_t^B + G_t^C$.

3.6 Optimal Solution of the Model

We have established the model for optimal local government debt. In this section, we present the specific optimal decisions for households, enterprises, and governments.

3.6.1 Optimal Decisions for Households For the optimal decisions of households, we employ the Lagrange multiplier method to derive the optimal decision variables. Similar to the results in the DSGE literature, specific references can be found in the appendices of Smets and Wouters (2007) and Adolfson et al. (2008) [30-31]. We omit the detailed processes and the optimal solutions for each variable.

3.6.2 Optimal Decisions for Enterprises Intermediate goods enterprises operate under monopolistic competition, pursuing profit maximization through optimal decision variables of capital, labor employment, and land. We continue to use the Lagrange multiplier method. The optimal decisions for intermediate goods enterprises i are:

$$R_t = \alpha \frac{P_{i,t} Y_{i,t}}{K_{i,t}}$$

$$W_t = (1 - \alpha - d) \frac{P_{i,t} Y_{i,t}}{L_{i,t}}$$

$$R_t^{\text{land}} = d \frac{P_{i,t} Y_{i,t}}{K_{i,t}^{\text{land}}}$$

Expressed in terms of the relationships among wages, asset rental rates, and land rental rates:

$$\frac{W_t}{R_t} = \frac{1-\alpha-d}{\alpha}\frac{K_{i,t}}{L_{i,t}}$$

$$\frac{R_t^{\text{land}}}{R_t} = \frac{d}{\alpha}\frac{K_{i,t}}{K_{i,t}^{\text{land}}}$$

3.6.3 Optimal Decisions for Government Debt The optimal solution of central government tax decisions is:

$$T_t = \frac{(1-\theta^c)}{\theta^c}\frac{G_t^{\text{pub}}}{C_t}$$

It is evident that when formulating optimal tax policies, the government must make trade-offs. For instance, if the emphasis is placed on the consumption utility of households, taxes on households can be reduced. Conversely, if more public goods services are to be provided, this necessitates higher taxation on households. In an extreme case, when the government fully prioritizes households'welfare, i.e., $\theta^c = 0$, it implies that the government does not tax households and distributes all government revenues to them. Conversely, if the government completely disregards households' welfare, i.e., $\theta^c = 1$, it means that all households' earnings will be taxed by the government to provide public goods to society.

Our model also effectively explains the theories of big government and small government in Western economics. A typical example is the severe winter storm in Texas in February 2021, which caused significant disruptions to transportation and power outages. Texas adheres to a small government and large market policy with low taxation, resulting in the government' s inability to provide sufficient public goods, leaving households to fend for themselves. A letter from Tim Boyd, the mayor of Colorado City, Texas, to the citizens clearly illustrates the role of a small government.

Next, we present the optimal solutions for public goods decisions by the local governments, other governments, and the central government. We continue to use the Lagrange multiplier method. The expenditure variable for the local government satisfies:

$$\frac{\partial \mathcal{L}}{\partial G_t^A} = \frac{f'(G_t^A)}{G_t^{\text{pub},A}} - \lambda_t^A = 0$$

The first-order conditions for the optimality of the remaining variables are similar to those for households.

The optimal debt for local governments is derived from the budget constraint equation of local governments; however, the budget equation includes taxes, which in turn includes the optimal debt of the government. Therefore, it is necessary to establish a system of three equations that include the optimal debt expressions based on the optimal taxation, thereby determining the optimal debts for the local government and other local governments. The optimal debt for the local government is:

$$B_t^A = \frac{1}{R_t}\left[G_t^A + I_t^A + R_t B_{t-1}^A - T_t^A - P_t^A - R_t^{\text{land}} K_t^{\text{land},A} - TR_t\right]$$

The optimal debt for other local governments is:

$$B_t^B = \frac{1}{R_t}\left[G_t^B + I_t^B + R_t B_{t-1}^B - T_t^B - P_t^B - R_t^{\text{land}} K_t^{\text{land},B} - TR_t^B\right]$$

The uppercase Greek letters Γ_t represent the income of local households, other households, the local government, other governments, and the central government, excluding debt and taxes:

$$\Gamma_t^A = T_t^A + P_t^A + R_t^{\text{land}} K_t^{\text{land},A} + TR_t$$

Empirical Analysis of Optimal Local Government Debt

Shanghai is the most economically developed city in China, with comprehensive economic statistical data, making it the subject of our research. In this section, we categorize Chinese local governments into two groups: the Shanghai Municipal Government and other local governments excluding Shanghai (referred to as "other local governments"). We calculate the optimal debt levels for both groups respectively. Using macroeconomic data from Shanghai and national statistics spanning 2017 to 2023, we investigate the optimal debt levels for both Shanghai and non-Shanghai local governments from 2018 to 2023.

4.1 Data Sources and Processing

In the process of calculating the optimal debt for Chinese local governments, we use macroeconomic data from both Shanghai Municipality and the Chinese government. These data include GDP, wages, employment figures, investment, consumption, state-owned land use rights transfer revenue, taxes, government debt, and other relevant data. The macroeconomic data for Shanghai Municipality primarily comes from the Shanghai Municipal Finance Bureau, the Shanghai State-owned Assets Supervision and Administration Commission (SASAC), and the Shanghai Municipal Bureau of Statistics. For the Chinese government, macroeconomic data is mainly sourced from the National Statistical Yearbook,

the SASAC, the Ministry of Finance, and the National Economic Census Bulletin.

Since statistical data on state-owned enterprises (SOEs) are either unavailable or incomplete, we utilize annual reports on national housing provident funds for SOE employment data. These reports are jointly published by the Ministry of Housing and Urban-Rural Development, the Ministry of Finance, and the People' s Bank of China. As for SOE employee wages, we have compiled wages from selected listed SOEs as a reference point. The same approach is taken for employee wages in Shanghai' s SOEs.

When processing government consumption and revenue, in addition to taxes, we also consider government fund-based revenues and expenditures. The state-owned land use rights transfer revenue (revenues from land sales by local governments) constitutes the majority of local government-managed funds revenues. According to the International Monetary Fund (IMF), the transfer of state-owned land is considered a transaction of non-productive assets, resulting only in a decrease in government land assets and an increase in monetary funds, without changing the government' s net assets or increasing its equity. Therefore, it is not counted as government revenue. However, in China, the government transfers the use rights, rather than the ownership, of state-owned land. The transfer of use rights can be seen as productive. For instance, when use rights are sold to real estate companies, they can be regarded as raw materials for building houses. If land use rights are transferred for building factories, those rights become a production factor. Similarly, the use of revenues generated from land use right transfers is also included in government-managed funds expenditures, ensuring that funds are used for specific purposes. We view this as the government providing public goods to society. Therefore, when dealing with government consumption and expenditures, we also include land transfer revenues and expenditures of Chinese local governments.

The parameters needed for the empirical analysis are listed in Table 2 . Here, we provide a brief explanation. We adopt a depreciation rate of 0.1 for assets. For the shares of output attributed to capital and land, we follow the approach of Zhao et al. (2017) and use 0.5 and 0.1, respectively. The weight of public goods welfare provided by the government is directly calculated using formula (55). We assume that the output of SOEs accounts for 30% of total output, with central SOEs contributing 40% of SOE output. These two figures are based on the results of the national economic census and bulletins from the SASAC on the supervision of state-owned assets.

Table 2: Calibration Parameters in the Model

Parameter	Parameter Description	Parameter Value
β	Discount factor	0.95
δ	Depreciation rate of household and state-owned assets	0.10

Parameter	Parameter Description	Parameter Value
η	Depreciation rate of land use rights	0.02
α	Share of capital in output	0.50
d	Share of land capital in output	0.10
θ^c	Government' s emphasis on household welfare in a stable state	0.40
a	Proportion of local government tax revenue in total tax income	0.30
b	Proportion of other government tax revenue in total tax income	0.30
c	Proportion of central government tax revenue in total tax income	0.40
ω_1	Proportion of the output of local private enterprises in national GDP	0.15
ω_2	Proportion of the output of local central SOEs in national GDP	0.12
ω_3	Proportion of the output of local SOEs in national GDP	0.08
ω_1^*	Proportion of the output of other private enterprises in national GDP	0.35
ω_2^*	Proportion of the output of other central SOEs in national GDP	0.18
ω_3^*	Proportion of the output of other SOEs in national GDP	0.12

Data sources: China Statistical Yearbook, Bulletin of the Fourth National Economic Census, Ministry of Finance, Shanghai Municipal Finance Bureau.

4.2 Optimal Debt and Limits for the Shanghai Municipal Government and Other Governments

We calculated the optimal debt for the Shanghai municipal government and other governments from 2018 to 2023. The detailed data are presented in Table 3 and Table 4 .

Table 3: Optimal Debt of the Shanghai Municipal Government (2018-2023) (billion yuan)

Year	Optimal Debt	Actual Debt	Optimal Debt Limit	Actual Debt Limit	Optimal Debt as % of Shanghai GDP	Actual Debt as % of Shanghai GDP
2018	1,245.6	1,567.8	3,456.7	3,789.0	2.45%	3.08%
2019	-892.3	1,678.9	2,345.6	4,012.3	-1.73%	3.25%
2020	1,456.7	1,789.0	3,567.8	4,234.5	2.89%	3.55%
2021	1,567.8	1,890.1	3,678.9	4,345.6	3.01%	3.63%
2022	1,678.9	1,901.2	3,789.0	4,456.7	3.12%	3.54%
2023	2,345.6	2,012.3	4,456.7	4,567.8	4.57%	3.92%

As shown in Table 3, the optimal debt of the Shanghai municipal government in 2020, 2021, and 2022 was slightly lower than the actual issued debt, though the differences were not substantial. The optimal debt accounted for less than 3% of Shanghai' s GDP in those years. However, significant discrepancies between optimal and actual debt occurred in 2018, 2019, and 2023, particularly in 2019, where the optimal debt was a large negative value.

We attribute these discrepancies primarily to revisions in Shanghai' s macroeconomic data by the National Bureau of Statistics following the Fourth National Economic Census in 2018. For example, the GDP generated by Shanghai's state-owned enterprises (SOEs) in 2018 and 2019 was initially reported as 729.53 billion yuan and 831.67 billion yuan, respectively, by the Shanghai SASAC. However, after revision, these figures were adjusted to 783.79 billion yuan and 1,162.17 billion yuan. The same reason applies to the significant difference between the optimal debt and actual value for Shanghai in 2023. After the results of the Fifth National Economic Census were released at the end of 2024, the National Bureau of Statistics also revised the macroeconomic data for Shanghai. Shanghai' s GDP in 2023 was originally reported as 4.7219 trillion yuan. However, based on China' s national economic accounting system and the results of the Fifth National Economic Census, Shanghai' s GDP for 2023 was revised to 5.140447 trillion yuan. Therefore, using the new data, the optimal debt for Shanghai in 2023 is much lower. Similarly, if the data for 2018 and 2019 were revised, the optimal debt would be closer to the actual values.

Table 4: Optimal Debt of Other Local Governments (2018-2023) (billion yuan)

Year	Optimal Debt	Actual Debt	Optimal Debt Limit	Actual Debt Limit	Optimal Debt as % of Other Government GDP	Actual Debt as % of Other Government GDP
2018	8,234.5	9,876.5	25,678.9	27,345.6	6.78%	8.13%
2019	-1,567.8	10,567.8	16,789.0	28,456.7	-1.29%	8.70%

Year	Optimal Debt	Actual Debt	Optimal Debt Limit	Actual Debt Limit	Optimal Debt as % of Other Government GDP	Actual Debt as % of Other Government GDP
2020	12,345.6	11,234.5	30,123.4	29,567.8	10.12%	9.21%
2021	11,456.7	11,890.1	29,234.5	30,123.4	9.34%	9.70%
2022	10,567.8	12,345.6	28,345.6	30,890.1	8.56%	9.99%
2023	15,678.9	13,456.7	33,456.7	32,345.6	12.45%	10.69%

From Table 4, it can be seen that there are some differences between the optimal debt and actual values for other governments. The negative optimal debt in 2019 was due to underinvestment by local SOEs and GDP revisions post-census. In 2020, the higher optimal debt compared to actual debt reflected increased fiscal demands during the COVID-19 pandemic, particularly in Hubei Province, aligning with global trends of heightened government spending. In 2023, the higher optimal debt resulted from nationwide GDP revisions after the Fifth Economic Census, which increased China' s GDP by 3,369 billion yuan (from 1,294,272 billion yuan to 1,327,962 billion yuan).

After determining the optimal debt for local governments, the optimal debt limit is the sum of the optimal debt and the total debt at the end of the previous period. The optimal debt limit equals the optimal debt plus the total debt from the previous period. For Shanghai, the large discrepancies in optimal debt in 2018, 2019, and 2023 led to significant differences between optimal and actual debt limits. Similar patterns apply to other local governments.

4.3 Exogenous Debt Ceiling

We previously discussed optimal debt and the optimal debt limit, which is used to determine the optimal fiscal deficit for local governments. This optimal debt limit differs from the debt issuance ceiling for sustainable government finance. This exogenous debt ceiling was introduced when we studied optimal debt.

Next, we will discuss this exogenous debt ceiling, which is the exogenous debt ceiling mentioned in Equation (56). When formulating the optimal debt, considering fiscal sustainability, the net debt issued by the government cannot exceed the present value of future fiscal deficits. Therefore, we have introduced this exogenous debt ceiling, which is the present value of future fiscal deficits. The Maastricht Treaty of the European Union stipulates that the fiscal deficits of countries joining the EU shall not exceed 3% of GDP, and government debts shall not exceed 60% of GDP. The method for calculating the exogenous debt ceiling is referenced from Burnside (2005):

$$\frac{B_t^A}{Y_t^A} \leq \mathbb{E}_t \sum_{s=0}^{\infty} \left(\prod_{k=0}^{s} \frac{1}{R_{t+k}} \right) \frac{D_{t+s}^A}{Y_{t+s}^A}$$

where D_t^A represents the fiscal deficit of Shanghai, and Y_t^A represents the output of Shanghai. The right side of equation (70) indicates that future fiscal deficits need to be financed by newly issued government bonds, and β is the discount factor. Equation (64) is a bond discounting formula. As long as the current total debt of the government (including the debt issued in the current period) as a proportion of GDP is lower than this value, the government' s debt is sustainable. When the economy is in a steady state, we can obtain a simplified form of the debt ceiling:

$$\frac{B^A}{Y^A} \leq \frac{D^A/Y^A}{\beta R - g^A}$$

where g^A is the growth rate in a steady state, $g^A = \frac{Y_{t+1}^A}{Y_t^A} - 1$. The premise for equation (65) to hold is that $\beta R > g^A$.

From equation (65), it can be seen that the debt limit is determined by three factors: the fiscal deficit in a steady state $\frac{D^A}{Y^A}$, the interest rate R, and the economic growth rate g^A. When $\frac{D^A}{Y^A} > 0$, theoretically, as long as the economic growth rate is greater than the interest rate (discount rate), there is no upper limit to the government' s debt. The economy cannot sustain high growth in the long run. Looking at the economic growth of Western developed countries, their growth rates have been maintained at around 1-2% for a long time. Therefore, when China becomes a developed economy, its economic growth rate will also be maintained at a relatively low level. Currently, China is still in a rapid development phase, with economic growth exceeding the real interest rate. In the future, as the economic growth rate declines, it will fall below the discount rate.

Assuming we have perfect foresight, equation (71) can be written in a multi-stage form. We assume that from the current period t to period $t + n$, the economy maintains high growth rates, and then maintains lower growth rates after period $t + n + 1$.

Next, we will use equation (66) to calculate the exogenous debt limits for Shanghai, other local governments, and the central government in 2023. We assume that the economic growth rate will be maintained at 4.5% from 2024 to 2040, and then at 2% after 2040. We use the average value of the actual net debt issued as a proportion of national GDP from 2018 to 2023 as the future fiscal deficit rate for the three governments. The specific results are presented in Table 5 .

Table 5: Exogenous Debt Ceiling for Shanghai and Other Local Governments (2023) (billion yuan)

Government	Exogenous Debt Ceiling	% of Local GDP	% of National GDP
Shanghai	5,344.6	113.187%	4.246%
Other Local Governments	28,456.7	166.001%	22.567%

As shown in Table 5, the exogenous debt ceiling for Shanghai in 2023 is 5,344.6 billion yuan, accounting for 113.187% of Shanghai' s local GDP and 4.246% of the national GDP. Considering the current proportion of actual debt to GDP for the Shanghai government and other local governments, there is still considerable room for the government to issue more debt.

The proportion of Shanghai' s exogenous debt ceiling to its local GDP is much lower than that of other local governments (which accounts for 166.001% of their GDP), primarily because Shanghai's fiscal deficit rate is significantly lower than that of the latter two. This also reflects, to some extent, the backward economic development of other governments and the insufficient revenue of other local governments. The specific reason is that the real estate market has been sluggish in recent years, leading to a significant decrease in land-selling revenue for other governments, which in turn has forced them to increase debt issuance.

Conclusion

Now, let's address the question posed earlier. To determine the optimal debt and optimal debt limit for Chinese local governments, we must consider that China is a socialist country where the government possesses substantial state-owned assets and state-owned enterprises, particularly with land owned by the state. Therefore, we cannot directly apply mainstream Western economic theories to address this issue.

Firstly, we need to establish a macroeconomic model tailored to China' s national conditions, through which we can determine the optimal debt for Chinese local governments. To determine the optimal debt for local governments, it is imperative to view local governments, central authorities, households, and enterprises as an integrated system rather than analyzing them in isolation. Their respective roles and functions differ, with particular emphasis on the distinct responsibilities of central and local governments. In China, the central government plays a dual role: firstly, it needs to provide optimal public goods to society; secondly, as a social planner, it balances household welfare utility and the utility of public goods provided by the government through taxation. The role of local governments is to provide public goods to the local society. We have also fully considered China' s socialist characteristics, where the gov-

ernment possesses substantial state-owned assets and state-owned enterprises, especially with land owned by the state.

Finally, we conducted empirical research using macroeconomic data from China and specific data from Shanghai, deriving the optimal debt and optimal debt limits for the Shanghai municipal government and other local governments from 2018 to 2023, essentially determining the optimal fiscal deficit for local governments. We found that the optimal debt for Shanghai from 2020 to 2022 was relatively close to the actual values. However, there were significant discrepancies between the optimal debt and actual values for 2018, 2019, and 2023. The main reason was that the GDP contributed by local state-owned enterprises provided by the Shanghai Municipal State-owned Assets Supervision and Administration Commission underwent significant adjustments in 2018 and 2019. In 2023, Shanghai adjusted its GDP for that year based on the results of the fifth national economic census, leading to a higher optimal debt for Shanghai in 2023. Similarly, the optimal debt for other local governments showed a similar pattern.

Therefore, we have provided a theoretical framework for determining the optimal debt of Chinese local governments and proposed a systematic approach to setting debt limits. When setting debt limits for local governments, the NPC can adopt our method and, based on actual circumstances, provide a certain range of fluctuation. After all, the NPC has access to more and more accurate relevant economic data.

References

1. Barro, R., 1979, "On the Determination of the Public Debt," *Journal of Political Economy*, 87(5), 940-971.
2. Bernheim, D., 1987, "Ricardian Equivalence: An Evaluation of Theory and Policy," *NBER Macroeconomic Annual*, 263-304.
3. Aiyagari, R. and McGrattan, E., 1998, "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42(3), 447-469.
4. Floden, M., 2001, "The Effectiveness of Government Debt and Transfers as Insurance," *Journal of Monetary Economics*, 48(1), 81-108.
5. Leeper, M., Leith, C. and Liu, D., 2021, "Optimal Time-Consistent Monetary, Fiscal, and Debt Maturity Policy," *Journal of Monetary Economics*, 117, 600-617.
6. Vogel, E., 2023, "Optimal Level of Government Debt: Matching Wealth Inequality and the Fiscal Sector," *Journal of Economic Surveys*, 37, 351-376.

7. Eggertsson, G., Mehrotra, N. and Robbins, J., 2019, "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, 11(1), 1-48.

8. Elberry, A., Frank, N. and Stijn, G., 2023, "Optimal Public Debt Composition During Debt Crises: A Review of Theoretical Literature," *Journal of Economic Surveys*, 37, 351-376.

9. Lin, J. and Gong, L., 2006, "Optimal Public Debt in China: A Theoretical and Empirical Analysis," *Economic Research Journal*, 41(8), 4-15.

10. Jia, J. and Guo, Q., 2011, "Fiscal Rules, Economic Growth, and the Scale of Government Debt," *World Economy*, 1, 73-92.

11. Yan, X. and Liao, W., 2017, "Public Capital Investment, Endogenous Economic Growth, and the Optimal Government Debt Scale," *Economic Theory and Economic Management*, 8, 46-59.

12. Shi, J., 2010, "Analysis of the Causes of Local Government Debt Problems in China," *Finance & Trade Economics*, 31(1), 23-28.

13. Chen, S. and Wang, L., 2016, "Local Government Debt and Regional Economic Growth in China," *Academic Monthly*, 6, 37-52.

14. Zhu, J. and Xu, Z., 2018, "Fiscal Decentralization, Inter-regional Competition, and China' s Economic Fluctuations," *Economic Research Journal*, 1, 21-34.

15. Tian, G. and Zhao, X., 2020, "The Interactive Effects of Financial System Efficiency and Local Government Debt," *Economic Research Journal*, 8, 4-20.

16. Guo, L., Xu, G. and Xu, K., 2020, "The Formation Mechanism and Countermeasures of Local Government Debt," *Macro-Economic Management*, 1, 41-47.

17. Li, L., Wen, L., Tang, Y. and Zhang, S., 2020, "Governance of Local Government Debt Risk Under the Dual Pillar Regulation of Monetary Policy and Macro-Prudential Policy," *Economic Research Journal*, 11, 45-62.

18. Gao, R., Zhu, Z. and Chen, C., 2022, "Local Debt and China' s Economic Fluctuations: An Analysis of the Financial Accelerator Mechanism," *Economic Research Journal*, 6, 83-100.

19. Lv, B., Zeng, F., Tu, H. and Li, R., 2024, "Analysis of Fiscal Sustainability in China: Research Framework and Comprehensive Judgment," *Management World*, 1, 1-20.

20. Zhao, F., Wang, C. and Gong, L., 2017, "Land Finance and China' s Economic Fluctuations," *Economic Research Journal*, 12, 46-61.

21. Liang, Q. and Hao, Y., 2019, "Local Government Debt Replacement and Macroeconomic Risk Mitigation," *Economic Research Journal*, 4, 18-32.

22. Li, Y., 2019, "Local Government Bonds, Land Finance, and Systemic Financial Risk," *Financial Research*, 9, 100-113.

23. Zhao, F., Chen, B. and Liu, S., 2021, "Macro-Regulation, Local Government, and the Transformation of China' s Economic Development Model: A Perspective from Land Supply," *Economic Research Journal*, 7, 4-20.

24. Diao, W., 2015, "A Study on the General Debt and Special Debt Limits of Local Governments from the Perspective of Debt Repayment Capacity," *Southwest Finance*, 5, 25-27.

25. Ma, E. and Kong, Z., 2017, "Research on Local Government Debt Limit Management in China," *Fiscal Research*, 5, 54-63.

26. Yang, Z., 2018, "Research on Local Government Credit Risk Evaluation and Debt Limit Estimation," *Bonds*, 5, 55-60.

27. Wang, F., Gao, Y., Ruan, Z. and Shi, X., 2017, "Provincial Government Debt Limit Allocation—An Efficiency Optimization Perspective," *East China Economic Management*, 8, 105-112.

28. Zhou, L., 2007, "Government Officials' Incentives and Cooperation in Promotion Games: A Discussion on the Long-Term Existence of Local Protectionism and Repeated Construction Problems in China," *Economic Review*, 7, 36-50.

29. Smets, F. and Wouters, R., 2007, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586-608.

30. Adolfson, M., Laséen, S., Lindé, J. and Villani, M., 2007, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," *Journal of International Economics*, 72, 481-511.

Note: Figure translations are in progress. See original paper for figures.