

## Mechanism Analysis and Homogenization of Star-shaped Beam Profiles in Laser-Driven Proton Beamlines

**Authors:** Wu, Dr. Feiyu, Wang, Kedong, Dr. Kai Wang, Dr. Minjian Wu, I'm ready to translate your academic paper. Please provide the content wrapped in ... tags as specified., Li, Dr. Jie, Li, Dr. Yingjie, Zhang, Dr. Xu, Yan, Prof. Xue-qing, Zhu, Dr. Kun, Lin, Dr. Chen, Wang, Dr. Kedong

**Date:** 2025-09-08T17:46:43+00:00

### Abstract

Laser-driven proton accelerators have emerged as a promising technology with applications ranging from cancer therapy to advanced imaging. At Peking University's CLAPA facility (Compact Laser-Plasma Accelerator), supported by the Ministry of Science and Technology of China, researchers are developing proton beam optimization for medical applications. Within the CLAPA beam transport system, we have identified an unexpected star-shaped beam pattern that forms subsequent to the beam collection section. Once this distortion emerges, it persists unless specialized corrective measures are applied along the beam path. This phenomenon significantly degrades beam quality and limits system usability. Our study investigates both the formation mechanisms of these star-shaped beams and methodologies for their control. We present three effective solutions: beamline lattice reconfiguration, higher-order magnetic field beam shaping, and scatterer homogenization. This work provides essential guidance for enhancing beam control in similar laser-driven accelerator systems.

### Full Text

#### Preamble

#### Mechanism Analysis and Homogenization of Star-shaped Beam Profiles in Laser-Driven Proton Beamlines

Fei-yu Wu,<sup>1,2</sup> Ke-dong Wang,<sup>1,2,†</sup> Kai Wang,<sup>1,2</sup> Min-jian Wu,<sup>1,2</sup> Hao-yang Lan,<sup>1,2</sup> Jie Li,<sup>1,2</sup> Ying-jie Li,<sup>1,2</sup> Xu Zhang,<sup>1,2</sup> Xue-qing Yan,<sup>1,2,3,‡</sup> Kun Zhu,<sup>1,2,3</sup> and Chen Lin<sup>1,2,3</sup>

<sup>1</sup>State Key Laboratory of Nuclear Physics and Technology, and Key Laboratory of HEDP of the Ministry of Education, CAPT, Peking University, Beijing, 100871, China

<sup>2</sup>Beijing Laser Acceleration Innovation Center, Huairou, Beijing, 101400, China

<sup>3</sup>Institute of Guangdong Laser Plasma Technology, Baiyun, Guangzhou, 510540, China

Laser-driven proton accelerators have emerged as a promising technology with applications spanning cancer treatment to advanced imaging. At Peking University's CLAPA facility (Compact Laser-Plasma Accelerator), supported by China's Ministry of Science and Technology, researchers are developing medical-focused proton beam improvements. During beam transport in CLAPA's system, we identified an unexpected star-shaped beam pattern forming after the beam collection section. This distortion persists throughout the beamline without specialized treatment, significantly reducing beam quality and limiting usability. Our study examines both the formation mechanisms of these star-shaped beams and methods to control them. We present three effective solutions: beamline lattice reconfiguration, higher-order magnetic field beam shaping, and scatterer homogenization. This work provides essential guidance for improving beam control in similar laser accelerator systems.

**Keywords:** Laser-driven proton accelerators, Beam transport, Star-shaped beam, Beam homogenization

## Introduction

Laser-driven proton accelerators have attracted significant global research interest due to their wide-ranging applications in materials science [?, ?], imaging [?], and cancer therapy [?, ?, ?, ?]. Critical advances in laser technology since the 1980s—particularly the development of chirped pulse amplification (CPA) [?, ?] and Kerr-lens mode-locking techniques [?, ?]—have enabled modern laser systems to achieve extraordinary capabilities. These systems now produce peak intensities exceeding  $10^{19}$  W/cm<sup>2</sup> with ultrashort pulse durations ranging from picoseconds to tens of femtoseconds. Such breakthroughs have promoted progress in generating high-energy particle beams through laser-plasma interactions.

The two primary proton acceleration mechanisms are target normal sheath acceleration (TNSA) [?] and radiation pressure acceleration (RPA) [?, ?]. When high-intensity lasers irradiate solid targets, energy is first transferred to electrons, creating a population of hot electrons. These electrons generate ultra-strong electric fields that can accelerate protons to high energies over distances as short as tens of micrometers. Laser-accelerated proton beams offer inherent advantages like compact source size and low emittance. However, challenges such as rapid electron expansion and plasma instabilities—including Coulomb explosion, Rayleigh-Taylor instability, Weibel instability, and nonlinear effects—result in undesirable beam properties like wide energy spreads and large divergence angles. These characteristics make specialized beam transport system

design essential for effective beam control and practical applications [?, ?, ?]. Major facilities addressing these challenges include the EU's ELIMED (Extreme Light Infrastructure-beamlines MEDical and multidisciplinary applications) [?, ?], Germany's ATHENA-h (Advanced Target High-Energy Network for Accelerators - high current) [?], and China's CLAPA (Compact Laser Plasma Accelerator) [?].

The CLAPA facility, developed by Peking University and funded by China's Ministry of Science and Technology, comprises two experimental systems: CLAPA I (Fig. 1 [Figure 1: see original paper]) and CLAPA II (Fig. 6 [Figure 6: see original paper]). The CLAPA I beamline employs simplified components for proof-of-principle experiments, while the CLAPA II beamline integrates a more sophisticated configuration for beam collection, transport, and shaping [?]. During beam transport in CLAPA, the large energy spread inherent to laser-accelerated proton beams results in distinct focal positions along the  $x$  and  $y$  directions when passing through quadrupole magnets. This energy-dependent focusing causes the initially circular beam profile to evolve into a star-shaped distribution due to the superposition of different energy slices. Without mitigation, this distorted profile persists throughout the beamline, severely degrading beam quality and necessitating urgent investigation into both the formation mechanisms and suppression strategies for star-shaped beams.

In this study, we systematically investigate the origins and mitigation of star-shaped beam profiles using two CLAPA beamlines. Section II presents beamline experiments on CLAPA I and the observed star-shaped beam phenomenon. Section III elucidates star-shaped beam formation mechanisms through theoretical and simulation analyses, and Section IV demonstrates three strategies for suppressing this distorted distribution—magnetic lattice reconfiguration, higher-order multipole shaping, and scatter homogenization—achieving complete star-shaped beam homogenization.

[Figure 1: see original paper]

## II. Experiment Settings

Figure 1 illustrates the experimental setup of the CLAPA I beam focusing system. An  $f/3.5$  off-axis parabolic mirror focuses the 30-fs laser pulse (800 nm wavelength) into a 5-m diameter focal spot (FWHM), containing approximately 30% of the total laser energy. This corresponds to a peak intensity of  $6 \times 10^{19} \text{ W/cm}^2$  at the target with 1.3 J on-target energy. The laser pulse is incident at  $30^\circ$  relative to the target normal on a 2.5- $\mu\text{m}$ -thick aluminum target. The initially accelerated TNSA proton beam exhibits an exponential energy spectrum with a cutoff energy of 6.5 MeV [?] and a divergence of hundreds of mrad [?]. The proton beam propagates along the target rear normal direction and is subsequently collected and focused by a triplet of electrostatic quadrupole lenses (Q1, Q2, Q3), where Q1 and Q3 focus in the  $x$ -direction while Q2 focuses in the  $y$ -direction. This beam collection system can collect 50 mrad

proton beams with various central energies.

After the proton beam is focused and collected by the triplet, a scintillator detector is positioned at 2.5 m downstream from the beamline exit for beam profile diagnostics. To improve the signal-to-noise ratio, this scintillator detector consists of a 15- $\mu\text{m}$ -thick aluminum foil used for blocking scattered light from the primary laser and the plastic scintillator [?, ?, ?, ?, ?]. The scintillator detector measures the superposition of beam distributions across all energy components, because it cannot discriminate proton beams of different energies. As shown in Fig. 2 [Figure 2: see original paper], the scintillator detector reveals a star-shaped beam profile after the proton beam is collected and focused by the triplet. This spatial distortion adversely impacts subsequent beam applications and therefore requires mitigation. A systematic investigation of the formation mechanisms of star-shaped beams and their suppression strategies is presented in the following sections.

[Figure 2: see original paper]

### III. Formation Mechanisms of Star-shaped Beams

Experimental data from Fig. 2 in the previous section demonstrate the star-shaped beam profiles observed in CLAPA I. As is known, proton beams generated via the TNSA mechanism exhibit broad energy spectra. In beamline design, a triplet is typically employed for laser-accelerated beam collection and focusing. Within quadrupole fields, the particle trajectory depends on the normalized quadrupole strength  $K_1$ , which is defined as  $K_1 = \frac{e}{P_0} \frac{\partial B_y}{\partial x}$ , where  $e$  is the elementary charge,  $P_0$  is the particle momentum at the beam central energy, and  $\frac{\partial B_y}{\partial x}$  is the field gradient of the quadrupole. For particles of different energies under identical magnetic gradient settings, variations in  $K_1$  lead to different trajectories.

The micron-scale source size of laser-accelerated beams permits treatment as a point source. At the triplet entrance, the transverse beam envelope satisfies the equation  $x_0^2 + y_0^2 = R^2$ , with divergence components  $x' = \frac{x_0}{D}$  and  $y' = \frac{y_0}{D}$ , where  $D$  is the target-to-triplet distance. Using beam dynamics theory under the thin-lens approximation, the triplet transfer matrix is derived. Post-transport, the transverse beam profile equation at distance  $f$  downstream becomes  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = R^2$ , where  $x_1$  and  $y_1$  denote particle coordinates at the triplet exit. Parameters  $a$  and  $b$  are formulated as functions  $a(E, H_1, H_2, H_3, d, L, f)$  and  $b(E, H_1, H_2, H_3, d, L, f)$ , where  $E$  is the proton kinetic energy;  $H_1, H_2, H_3$  are the magnetic field gradients of the three quadrupole magnets (T/m);  $d$  is the drift section length between the triplet;  $L$  is the effective length of each quadrupole magnet; and  $f$  is the distance from triplet exit to research point. The functional forms of  $a$  and  $b$  consist of complex mixed polynomials of proton energy and lattice parameters and are defined in the Appendix.

Under this lattice, the value of  $b$  is significantly larger than  $a$  at 95 MeV. As the

energy rises, the value of  $a$  first decreases and then increases, while the value of  $b$  gradually diminishes. This phenomenon leads to the shift of the major axis of the ellipse from the  $y$ -axis to the  $x$ -axis. Protons possessing energies higher than the central energy are focused into an ellipse whose major axis is oriented along the  $x$ -direction. Conversely, protons with lower energies form an ellipse with its major axis aligned along the  $y$ -direction. The superposition of these distributions from particles with different energies produces a star-shaped beam pattern. As depicted in Fig. 5(b) [Figure 5: see original paper], when the energy spread is introduced in simulation and the beam is transported through the triplet, a star-shaped beam is obtained at the exit.

[Figure 3: see original paper]

Figure 3 plots the energy-dependent evolution of semi-axes  $a$  and  $b$  for elliptical beam profiles after the triplet in Tracewin-simulated lattices, with black and red solid lines denoting  $a(E)$  and  $b(E)$  respectively.

Based on the above analysis, we find that different energy proton beams accelerated by lasers form a circular spot at the triplet entrance, which transforms into a strange shape (star-shape) after passing through the triplet. The shape of the terminal beam depends on both proton beam energy and the lattice configuration. The star-shaped beam pattern results from the superposition of diverse elliptical distributions formed by particles with varying energies as they transport through the triplet. We simulated the formation of this beam distortion using the Tracewin software developed by CEA, France [?]. In simulation, we configured the initial proton beam with a central energy of 100 MeV, an RMS energy spread of 5 MeV, and a beam divergence of  $\pm 50$  mrad. The beam distribution at the entrance of the triplet is shown in Fig. 5(a) [Figure 5: see original paper]. The lattice in simulation was set to focus protons with 100 MeV central energy into a small circular spot.

As shown in Fig. 4 [Figure 4: see original paper], we conducted simulations of the beam transport for monoenergetic proton beams with central energies of 95 MeV, 100 MeV, and 105 MeV, as well as for a 100 MeV central-energy beam with 10% energy spread. The simulation results demonstrate that when collecting laser-accelerated beams using a triplet to attain a small beam spot (with an approximate size of 5 mm), this star-shaped beam spot can be easily achieved. This is attributable to the fact that beam particles with varying central energies exhibit distinct focal positions in both the  $x$  and  $y$  directions. Moreover, the rates at which these focal positions change with energy also diverge between the  $x$  and  $y$  directions. As depicted in Fig. 4, when aiming to focus a polychromatic beam into a compact spot, the transverse ( $x$  or  $y$ ) focal positions of different energy components need to be set at the same longitudinal position, which inevitably leads to underfocusing or overfocusing in the orthogonal direction. The resulting superposition of these off-central energy beam envelopes leads to the formation of the star-shaped profile. Defining the beam transport direction as the  $z$ -axis, the  $x$ -direction and  $y$ -direction focus points shift toward increasing  $z$  with higher energies, while the  $y$ -direction focus point moves more slowly.

Experimental observations further revealed a tilted deformation superimposed on the star-shaped beam profile. This phenomenon was attributed to mechanical misalignment that occurred during the installation of the triplet. As demonstrated in Fig. 5(c) [Figure 5: see original paper], simulation results show that a triplet rotation error of 0.5 degrees around the  $z$ -axis gives rise to a rotational deviation of nearly 45 degrees around the  $z$ -axis in the exit-focused star-shaped beam spot. The rotational misalignment of quadrupole fields transforms the standard elliptical beam profiles (for monoenergetic components) into tilted elliptical profiles. Therefore, in numerous laser acceleration beam collection systems, the observed star-shaped beam spots are tilted. This simulated behavior correlates well with the experimentally observed beam spot morphology in Fig. 1.

We further investigated whether star-shaped beam profiles would emerge in the CLAPA II horizontal beamline configuration. As depicted in Fig. 6 [Figure 6: see original paper], within the completed CLAPA-II horizontal beam transport system, the collection section comprises three superconducting solenoids, followed by two dipole magnets for beam deflection and four quadrupole magnets to minimize dispersion-induced envelope distortions and instabilities. Subsequent quadrupole and octupole magnets implement beam homogenization and shaping. Since solenoid magnetic fields exhibit azimuthal symmetry about the central axis, standard solenoid configurations do not induce star-shaped beam formation. We simulated the beam evolution from solenoid injection through pre-homogenization stages for a 100 MeV central-energy proton beam with 5 MeV RMS energy spread. As demonstrated in Fig. 7 [Figure 7: see original paper], during the beam collection process employing the solenoid system, no star-shaped distortion occurred. After the beam passes through the quadrupoles between the collection section and the shaping section, star-shaped distortion of the beam is observed, consistent with our theoretical framework. This arises from the coupling between focal lengths in the transverse directions and beam energy inherent to quadrupole fields. Given the critical impact of beam profile on downstream applications, the following section discusses optimized strategies tailored for both the CLAPA I and CLAPA II beamline architectures.

[Figure 6: see original paper]

#### IV. Suppression of Star-shaped Beams and Beam Homogenization

The existence of star-shaped beam profiles exerts a detrimental influence on proton beam applications at the treatment terminal. As a result, it is essential to suppress them within the beamline. To effectively characterize star-shaped distortion and evaluate homogenization efficacy, we introduce the star-shaped Level (SL) parameter, defined as the normalized deviation from an ideal uniform circular profile:

$$\text{SL} = \left( \frac{1}{8} - \frac{n}{N} \right)^2$$

where  $n$  is the particle count within the angular sector spanning  $22.5^\circ - 67.5^\circ$  between half and full beam spot radius, and  $N$  is the total particle count in the corresponding annular region. For an ideal uniform circular beam distribution, the value of  $n/N$  approximates  $1/8$ . The SL quantifies the deviation of a given beam distribution from ideal uniformity, with  $\text{SL} > 1$  indicating star-shaped beam formation. Lower SL values (best approaching 0) correspond to improved beam homogenization. Under this definition, the beam distribution shown in Fig. 5(b) exhibits an SL value of 1189.6, while that in Fig. 7 exhibits an SL value of 4268.4, both indicating severe star-shaped distortion.

We investigate three suppression methods: (1) magnetic lattice reconfiguration, (2) higher-order multipole field homogenization, and (3) scattering foil homogenization. All approaches successfully suppress star-shaped beams and homogenize beam distributions, achieving reductions of the SL parameter below unity ( $\text{SL} < 1$ ).

### A. Magnetic Lattice Reconfiguration of the Triplet

For the quadrupole group used to collect and focus the beam, reconfiguring the magnetic lattice can release the beam spot distortion within the large energy dispersion range. Adjusting the magnetic field gradients induces controlled overfocusing in the  $x$ -direction and underfocusing in the  $y$ -direction across all energy components. This intentional mismatch prevents the formation of elongated or flattened beam structures for individual energy slices. The resulting superposition of these modified beam envelopes significantly enhances transverse homogeneity.

In Fig. 8 [Figure 8: see original paper], simulations adopt the same initial beam parameters as in Fig. 5. Under this reconfigured lattice, central-energy, high-energy, and low-energy protons all exhibit overfocusing in the  $x$ -direction and underfocusing in the  $y$ -direction, with comparable beam sizes in both planes. This configuration results in a nearly uniform distribution when energy-slice superposition is performed. The calculated SL of the homogenized profile is reduced to 0.067, demonstrating significant improvement over the original SL value of 1189.6 in Fig. 5(b). Black and red solid lines in Fig. 9 [Figure 9: see original paper] correspond to the beam profile in Fig. 5(a), while blue and green solid lines in Fig. 9 represent the homogenized profile achieved via lattice reconfiguration.

To quantify transverse homogeneity, the normalized full width at half maximum (NFWHM) is employed as a metric, where larger NFWHM values indicate enhanced uniformity. Here the NFWHM is defined as the ratio of the beam profile width at half-maximum height to the total width. A significant improvement in the beam distribution is evident compared to Fig. 5. Quantitative analysis

shows the original profile (Fig. 5) exhibits NFWHM values of 5% ( $x$ -direction) and 6% ( $y$ -direction), whereas the optimized profile achieves 70% ( $x$ ) and 80% ( $y$ ), demonstrating effective homogenization with transverse uniformity exceeding 80% in both planes.

[Figure 8: see original paper]

## B. Octupole-based Homogenization

When a proton beam traverses ideal nonlinear magnets, the transverse equations of motion can be expressed as a power series in  $x$  and  $y$  [?]:

$$\begin{aligned} x'' + K_4(s)x + \sum_{n=3}^{\infty} \frac{K_{2n}}{(n-1)!} \operatorname{Re}\{(x+iy)^{n-1}\} &= 0 \\ y'' - K_4(s)y + \sum_{n=3}^{\infty} \frac{K_{2n}}{(n-1)!} \operatorname{Re}\{i(x+iy)^{n-1}\} &= 0 \end{aligned}$$

where  $K_4$  is the quadrupole field strength and  $K_{2n}$  represents the  $2n$ -order nonlinear field strength. Derivatives are taken with respect to the beam propagation coordinates. Obtaining analytical solutions for such coupled equations is exceptionally challenging. To address this, we expand the higher-order terms as [?]:

$$\begin{aligned} x'' + K_4(s)x + K_{6xy} + K_8 \frac{x^3 - 3xy^2}{3!} + \dots &= 0 \\ y'' - K_4(s)y - K_{6xy} + K_8 \frac{y^3 - 3x^2y}{3!} + \dots &= 0 \end{aligned}$$

From Eq. (3), transverse coupling can be decoupled when horizontal motion dominates ( $|\max(y)/\max(x)| \ll 1$ ) or vertical motion dominates ( $|\max(x)/\max(y)| \ll 1$ ). Therefore, to decouple transverse motion, nonlinear magnets are positioned at locations where the beam envelope in one transverse dimension significantly exceeds that in the other, thereby enabling homogenization. This is achieved using two quadrupole doublets in CLAPA II. For beam homogenization, octupole magnets are implemented in CLAPA II. The initial beam is assumed to follow a Gaussian distribution [?]:

$$\rho_0 = \frac{N}{\sqrt{2\pi}\sigma_0} \times \exp\left(-\frac{x_0^2}{2\sigma_0^2}\right)$$

After traversing the octupole magnet, the beam profile evolves into the following form [?]:

$$\rho_t = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{X^2}{2\sigma_t^2}\right) \left[1 - \tilde{R}_8 \frac{X^2}{2} \tan \phi_{K8}\right]$$

where subscripts 0 denote initial beam parameters and  $t$  represent post-homogenization parameters. The term  $K_8$  corresponds to the octupole field strength. This equation demonstrates that central beam homogenization can be achieved by appropriately adjusting  $K_8$ , effectively transforming the core region of the beam into a uniform distribution.

In the CLAPA II beamline, two quadrupole doublets are employed to generate flat beam profiles in the  $x$ - and  $y$ -directions, respectively, each followed by an octupole magnet for transverse homogenization, with a final triplet for beam refocusing. As shown in Fig. 10 [Figure 10: see original paper] and Fig. 11 [Figure 11: see original paper], Fig. 10(a) illustrates the beam envelope during higher-order homogenization of the star-shaped beam from Fig. 7; Figs. 10(b) and 10(c) show horizontally and vertically flattened profiles after the first and second doublets, respectively. The original star-shaped beam (Fig. 7) exhibits  $SL = 4268.4$  with  $NFWHM = 16\%$  ( $x$ ) and  $24\%$  ( $y$ ). After octupole-based homogenization (Fig. 10(d)), the central region achieves  $SL = 0.5$  and  $NFWHM = 97\%$  ( $x$ ) and  $99\%$  ( $y$ ). Fig. 11 demonstrates transverse beam profiles before and after homogenization, confirming the elimination of star-shaped distortion and achieving excellent homogenization.

[Figure 10: see original paper]

### C. Scattering Foil Optimization

We further propose a scattering-foil homogenization method, which offers greater operational simplicity and spatial efficiency compared to other methods. When traversing scattering foils, proton beams undergo multiple Coulomb scattering (MCS) [?], inducing both deceleration (energy loss via Coulomb interactions with atomic electrons) and angular spreading (nuclear scattering with atomic nuclei). Material selection and geometric design of the scattering foil critically determine homogenization efficacy: low- $Z$  materials preferentially decelerate protons, while high- $Z$  materials enhance scattering.

As illustrated in Fig. 12(a) [Figure 12: see original paper], a compound scattering foil comprising lead (high- $Z$ , dark green) and LEXAN polymer (low- $Z$ , maroon-colored) achieves optimal homogenization. The proton beam first enters the LEXAN polymer, which preserves the incident-to-exit energy spectrum via controlled deceleration. Subsequent traversal through the lead scattering foil induces angular spreading while minimizing energy loss. Due to the higher central energy of the laser-accelerated proton beam, the lead foil is designed with a radially graded thickness profile (thick central region, thin peripheral zones) to optimize energy-dependent scattering efficiency.

The homogenization process of star-shaped beams through the scattering foil, which is installed 545 mm downstream of the second octupole, was simulated using GEANT4 [?]. A square collimator with an aperture size of 10 mm  $\times$  10 mm was positioned downstream of the scattering foil to constrain the scattered beam envelope. At 250 mm downstream of the scattering foil, a nearly uniform

beam distribution was achieved. As shown in Fig. 12(b), the homogenized beam profile exhibits near-perfect uniformity, with blue and green solid lines in Fig. 13 [Figure 13: see original paper] demonstrating approximate flat-top transverse distributions in the  $x$ - and  $y$ -directions, respectively. The NFWHM values for both transverse planes exceed 98%, confirming exceptional homogenization efficacy.

[Figure 12: see original paper]

## V. Conclusion

This study presents a comprehensive investigation of star-shaped beam formation mechanisms in the CLAPA I and CLAPA II beamlines through theoretical, experimental, and simulation analyses. The star-shaped Level (SL) parameter quantifies beam distortion severity, while the normalized full width at half maximum (NFWHM) metric evaluates transverse homogeneity. Unmitigated star-shaped profiles propagate through the beamline, severely degrading beam quality and limiting practical applications. To address this, three suppression strategies were proposed and validated: lattice reconfiguration, octupole-based homogenization, and scattering foil optimization. As shown in Fig. 14 [Figure 14: see original paper], all methods demonstrated significant homogenization efficacy (NFWHM maximum 99%) while suppressing star-shaped distortion ( $SL < 1$ ).

Quadrupole lattice reconfiguration offers the most expedient homogenization but is limited to electrostatic quadrupole-based star-shaped beams; octupole magnet homogenization applies broadly but incurs higher costs and spatial footprint; scattering foil optimization is operationally straightforward and more effective for high-energy proton beams. Selection among these methods should align with specific application requirements. This study constructs a theoretical framework aimed at suppressing star-shaped beams and optimizing beam transport during the design and implementation of laser-driven beam transport systems.

[Figure 14: see original paper]

## Appendix

For a triplet composed of quadrupole magnets, its transfer matrix under the thin-lens approximation is given by:

$$M = \begin{pmatrix} L + f + (L + d)\frac{H_3}{L}(L + f) + (L + d)\frac{H_3}{L}(L + f) & \cdots \\ \vdots & \ddots \end{pmatrix}$$

For a beam with an initial circular envelope defined by  $x_0^2 + y_0^2 = R^2$ , the envelope transforms into  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = R^2$  after traversing the triplet, where:

$$a = L + f + (L + d)\frac{H_3}{L}(L + f) + (L + d)\frac{H_3}{L}(L + f)$$

$$b = L + f - (L + d)\frac{H_3}{L}(L + f) + (L + d)\frac{H_2}{L}$$

The functional forms consist of complex mixed polynomials of proton energy and lattice parameters as defined above.

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