Study on Plastic Zone of Layered Surrounding Rock Based on Modified Hoek-Brown Criterion (Postprint)

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Abstract

To investigate the distribution characteristics and evolution patterns of plastic zones in layered surrounding rock, the boundary of the surrounding rock plastic zone considering rock mass anisotropy was derived for both hydrostatic and non-hydrostatic pressure fields based on the modified Hoek-Brown criterion for anisotropic rock masses. The influence patterns of rock strata dip angle, degree of rock mass anisotropy, and lateral pressure coefficient on plastic zone morphology were examined. The results demonstrate that rock mass anisotropy exerts a significant influence on the morphology of surrounding rock plastic zones, with the plastic zone expansion range primarily concentrated in the direction at a 30° angle to the normal of the rock strata; as the degree of rock mass anisotropy increases, the plastic zone morphology exhibits a transition trend from approximately circular to butterfly-shaped; when the lateral pressure coefficient is relatively large, the superposition effect of rock mass anisotropy and lateral pressure renders the plastic zone expansion at the tunnel shoulder more pronounced.

Full Text

Study on the Plastic Zone in Layered Surrounding Rock Based on the Modified Hoek–Brown Criterion

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Abstract

To investigate the distribution characteristics and evolution of the plastic zone in layered rock masses, this study derives the plastic zone boundaries under hydrostatic and non-hydrostatic stress fields based on the modified Hoek–Brown criterion for anisotropic rock. The influences of bedding inclination, degree of anisotropy, and lateral pressure coefficient on plastic zone morphology are systematically analyzed. The results indicate that rock mass anisotropy has a significant influence on the morphology of the plastic zone. The expansion of the plastic zone predominantly occurs within 30° of the normal direction to the bedding planes. As the degree of anisotropy increases, the shape of the plastic zone evolves from an approximately circular form to a distinctive butterfly pattern. Furthermore, under high lateral pressure coefficients, the combined effect of rock mass anisotropy and lateral stress significantly amplifies plastic zone development around the tunnel shoulders.

Keywords: modified Hoek–Brown criterion; layered rock; rock anisotropy; plastic zone

Introduction

Tunnel excavation induces significant changes in the stress state of surrounding rock, transforming the rock mass from its original triaxial stress condition to a biaxial stress state and triggering stress redistribution. When the induced secondary stress exceeds the yield limit of the rock mass, an elastoplastic zone forms. The rock within the plastic zone undergoes plastic flow and potential failure, making the determination of plastic zone extent crucial for stability control and tunnel support design [1].

Under hydrostatic pressure, elastoplastic analysis can yield analytical solutions for stress and deformation in surrounding rock, thereby defining the plastic zone boundary. Under non-hydrostatic pressure, the irregular shape of the plastic zone boundary makes analytical solutions difficult to obtain. Existing research primarily employs the approximate Kastner method, which bypasses rigorous plastic zone analysis by directly substituting the Kirsch stress solution for a circular tunnel under non-uniform pressure into the strength criterion to obtain an approximate plastic zone boundary equation [2-3]. The selection of an appropriate strength criterion is critical for accurate elastoplastic analysis of surrounding rock, with commonly used criteria including the Mohr-Coulomb criterion [4] and the Hoek-Brown criterion.

Layered rock masses, commonly encountered in tunnel and underground engineering projects, exhibit pronounced anisotropic mechanical properties that complicate deformation and strength characteristics, with significant variations in failure modes and deformation patterns at different locations [6]. Current research on plastic zones in surrounding rock primarily focuses on isotropic rock masses, while studies on anisotropic rock masses such as layered rock are mainly conducted through numerical simulation methods [7], with theoretical investigations remaining scarce. Regarding anisotropic strength criteria for layered rock masses, H. Saroglou [8] modified the Hoek-Brown criterion by introducing an anisotropy coefficient k_β for the parameter m_i, recognizing that both k_β and the uniaxial compressive strength $\sigma_{c\beta}$ vary with the angle between the maximum principal stress and the rock bedding planes.

This paper investigates the distribution characteristics of plastic zone boundaries in layered rock masses under hydrostatic and non-hydrostatic pressure fields based on the modified Hoek-Brown criterion, and examines the influence of bedding inclination, rock mass properties, and lateral pressure coefficient on plastic zone extent.

2. Plastic Zone Boundary Derivation

2.1 Basic Assumptions

To obtain concise analytical solutions and practical analysis methods for tunnel engineering, the following fundamental assumptions are adopted:

(1) The tunnel cross-section is circular with an axial length far exceeding its cross-sectional dimensions, allowing the tunnel excavation problem to be simplified as a plane strain issue.

(2) The surrounding rock is a homogeneous, continuous, ideal elastoplastic material, with plastic zone stresses satisfying the modified Hoek-Brown criterion. For simplification, the tangential stress σ_θ direction is approximated as the maximum principal stress direction, where β represents the angle between σ_θ and the rock bedding plane, calculated according to Equation (2). Due to symmetry, the plastic zone exhibits central symmetry, requiring analysis only within the range $\theta = 0 \sim \pi$.

(3) The rock mass is subjected to vertical and horizontal pressures with a lateral pressure coefficient λ, while rock mass self-weight is neglected.

(4) No support is installed, meaning the support pressure at the tunnel periphery is zero.

2.2 Strength Criterion and Mechanical Model

H. Saroglou [8] proposed the modified Hoek-Brown criterion for layered rock masses, expressed as:

$$\sigma_1 = \sigma_3 + \sigma_{c\beta}\left(m_i \frac{\sigma_3}{\sigma_{c\beta}}\right)^{k_\beta}$$

where σ_1 and σ_3 are the maximum and minimum principal stresses, respectively; m_i is the strength parameter; and $\sigma_{c\beta}$ and k_β are correction coefficients for uniaxial compressive strength and m_i, respectively. Both coefficients depend on the angle β between the maximum principal stress direction and the rock bedding plane, as illustrated in [Figure 1: see original paper].

The stress field in the surrounding rock is shown in [Figure 2: see original paper]. P_0 represents the vertical in-situ stress, λ is the lateral pressure coefficient, λP_0 is the horizontal in-situ stress, and R_0 is the tunnel radius. θ denotes the polar angle, r the polar radius, σ_r the radial stress, σ_θ the tangential stress, and $\tau_{r\theta}$ the shear stress at any point in the rock mass; α represents the bedding inclination angle.

2.3 Plastic Zone Boundary for $\lambda = 1$

According to elastic theory, when the lateral pressure coefficient $\lambda = 1$, the stress components in the tunnel surrounding rock depend only on r. The equilibrium and geometric equations become:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r}$$

Considering axisymmetry and substituting the strength criterion into the equilibrium equation yields:

$$\frac{d\sigma_r}{dr} + \frac{1}{r}\left[0.5\sigma_{c\beta}\left(m_i \frac{\sigma_r}{\sigma_{c\beta}}\right)^{k_\beta}\right] = 0$$

Solving this differential equation gives the expression for σ_r:

$$\sigma_r = \frac{\sigma_{c\beta}}{m_i}\left[\ln\left(\frac{r}{R_0}\right)\right]^{1/k_\beta}$$

where C is a constant. With no support ($\sigma_r = 0$ at $r = R_0$), the constant C is determined as:

$$C = -\frac{\sigma_{c\beta}}{m_i}\left[\ln(R_0)\right]^{1/k_\beta}$$

Substituting this into the previous expression yields:

$$\sigma_r = \frac{\sigma_{c\beta}}{m_i}\left\{\left[\ln\left(\frac{r}{R_0}\right)\right]^{1/k_\beta} - \left[\ln(R_0)\right]^{1/k_\beta}\right\}$$

From which σ_θ can be obtained:

$$\sigma_\theta = \sigma_r + \sigma_{c\beta}\left(m_i \frac{\sigma_r}{\sigma_{c\beta}}\right)^{k_\beta}$$

At the elastoplastic interface ($r = R_p$), the following conditions must be satisfied:

$$\sigma_r^{(e)} = \sigma_r^{(p)}, \quad \sigma_\theta^{(e)} = \sigma_\theta^{(p)}$$

The elastic zone stresses under no support condition are:

$$\sigma_r = P_0\left(1 - \frac{R_0^2}{r^2}\right), \quad \sigma_\theta = P_0\left(1 + \frac{R_0^2}{r^2}\right)$$

From elastic theory, the relationship between principal stresses and stress components in plane strain is:

$$\sigma_{1,3} = \frac{\sigma_r + \sigma_\theta}{2} \pm \sqrt{\left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2 + \tau_{r\theta}^2}$$

Substituting the strength criterion yields the plastic zone radius R_p:

$$R_p = R_0 \exp\left[\frac{m_i}{\sigma_{c\beta}}\left(P_0 - \frac{\sigma_{c\beta}}{m_i}\right)\right]^{k_\beta}$$

2.4 Plastic Zone Boundary for $\lambda \neq 1$

When $\lambda \neq 1$, the plastic zone boundary becomes irregular, making analytical solutions difficult. The approximate method typically involves first determining the elastic stress distribution using elastic theory, then substituting these elastic solutions into the rock mass strength criterion to obtain the approximate plastic failure range.

Using superposition principle, the Kirsch stress formulas for a circular tunnel under non-uniform pressure can be obtained. The elastic stresses are:

$$\sigma_r = \frac{P_0}{2}\left[(1+\lambda)\left(1 - \frac{R_0^2}{r^2}\right) + (1-\lambda)\left(1 - 4\frac{R_0^2}{r^2} + 3\frac{R_0^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{P_0}{2}\left[(1+\lambda)\left(1 + \frac{R_0^2}{r^2}\right) - (1-\lambda)\left(1 + 3\frac{R_0^4}{r^4}\right)\cos 2\theta\right]$$

$$\tau_{r\theta} = \frac{P_0}{2}(1-\lambda)\left(1 + 2\frac{R_0^2}{r^2} - 3\frac{R_0^4}{r^4}\right)\sin 2\theta$$

Let $m = R_0/r$, $a_1 = 0.5(1+\lambda)P_0$, $a_2 = 0.5(1-\lambda)P_0\cos 2\theta$, and $a_3 = 0.5(1-\lambda)P_0\sin 2\theta$. The stress expressions become:

$$\sigma_r = a_1(1-m^2) + a_2(1-4m^2+3m^4)$$

$$\sigma_\theta = a_1(1+m^2) - a_2(1+3m^4)$$

$$\tau_{r\theta} = a_3(1+2m^2-3m^4)$$

Substituting into the strength criterion yields the plastic zone boundary equation:

$$0.5\sigma_{c\beta}\left(m_i\frac{\sigma_3}{\sigma_{c\beta}}\right)^{k_\beta} + \sigma_3 - \sigma_1 = 0$$

where σ_1 and σ_3 are determined from the stress components above.

3. Case Study

Using the rock parameters from reference [8] as examples, the analysis employs a tunnel radius $R_0 = 6$ m, in-situ stress $P_0 = 50$ MPa, and $m_i = 6$. The degree of rock mass anisotropy is characterized by $RC = \sigma_{ci}(90)/\sigma_{ci(\min)}$, where $\sigma_{ci}(90)$ is the uniaxial compressive strength perpendicular to bedding ($\beta = 90°$) and $\sigma_{ci(\min)}$ is the minimum uniaxial compressive strength at various bedding angles. A larger RC value indicates more pronounced anisotropy.

Table 1 Strength parameters for different rock types [8]

Rock Type	$\sigma_{ci}(90)$ (MPa)	$\sigma_{ci(\min)}$ (MPa)	RC
Gneiss A	120	80	1.5
Gneiss B	100	60	1.67
Schist	80	70	1.14
Marble	90	75	1.2

Under hydrostatic pressure ($\lambda = 1$), the plastic zone boundaries for different bedding inclinations α are shown in [Figure 3: see original paper]. The anisotropic characteristics of layered rock significantly influence the yield and failure patterns of surrounding rock, with plastic zone expansion concentrated within 30°

of the bedding plane normal direction. This occurs because when the angle β between the maximum principal stress σ_θ and bedding plane is 30°, the strength parameters $\sigma_{c\beta}$ and k_β reach minimum values, making the rock more susceptible to yielding under identical stress conditions.

[Figure 4: see original paper] illustrates the plastic zone morphology for different rock types at $\alpha = 0°$. When the degree of anisotropy RC is low, the plastic zone boundary approximates a circular shape. As anisotropy increases, the variation in plastic zone radius at different θ angles becomes more pronounced, exhibiting butterfly-shaped distribution characteristics. The plastic zone extents in schist and marble are significantly smaller than those in gneiss A and B, primarily due to their lower strength parameters $\sigma_{c\beta}$ and k_β.

Table 2 Comparison of analytical and approximate solutions for plastic zone radius in Gneiss A

θ (°)	Analytical R_p/R_0	Approximate R_p/R_0	Error (%)
0	1.21	1.18	-2.5
30	1.45	1.38	-4.8
60	1.32	1.28	-3.0
90	1.15	1.13	-1.7

Under non-hydrostatic stress conditions ($\lambda \neq 1$), the plastic zone boundary is calculated using the approximate method. To evaluate the error introduced by this approximation, analytical and approximate solutions for Gneiss A at $\lambda = 1$ are compared in Table 2, showing small differences that validate the approximate approach.

[Figure 5: see original paper] presents plastic zone morphology under various lateral pressure coefficients λ. When $\lambda < 1$, larger plastic zones develop at the tunnel sidewalls. As λ increases, butterfly-shaped characteristics become more pronounced. At bedding inclinations of $\alpha = 0°$ and $\alpha = 60°$, plastic zone expansion at the tunnel shoulders becomes particularly significant due to the superposition effect of rock anisotropy and high lateral pressure. The influence of lateral pressure coefficient on plastic zone morphology becomes more significant with increasing rock mass anisotropy.

4. Conclusions

Existing research [9] indicates that when $\lambda < 1$, plastic zones exhibit a transverse elliptical shape with larger sidewall extents, whereas high λ values produce butterfly-shaped plastic zones with significantly larger radii at the tunnel shoulders.

Based on the modified Hoek-Brown criterion for anisotropic rock, this study derives plastic zone boundaries for layered rock masses under hydrostatic and non-hydrostatic stress fields, analyzing the distribution characteristics and evolution patterns under various bedding inclinations, rock properties, and lateral pressure coefficients. The main conclusions are:

(1) Rock mass anisotropy significantly influences plastic zone distribution. Plastic zone expansion concentrates within 30° of the bedding plane normal direction because rock strength parameters decrease substantially when the angle between maximum principal stress and bedding plane is 30°, facilitating yield initiation.

(2) At low degrees of anisotropy, plastic zone boundaries approximate circular shapes. At high anisotropy levels, significant variations in plastic zone radius at different locations produce distinctive butterfly-shaped distributions.

(3) Rock mass anisotropy amplifies the effect of lateral pressure on plastic zone morphology. Under high lateral pressure coefficients, the combined effect of anisotropy and lateral stress causes more pronounced plastic zone development at tunnel shoulders.

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Note: Figure translations are in progress. See original paper for figures.