Effects of Radius Measurement Uncertainty on RMF Model Parameters and Neutron Star Matter Properties

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Abstract

We investigate the impact of neutron star radius measurement precision on the constraints of relativistic mean-field (RMF) model parameters and the equation of state (EOS) of dense matter within a Bayesian framework. Using the canonical neutron star radius $R_{1.4} = 11.9$ km with uncertainties $\sigma_R = 1.0$, 0.5 and 0.2 km, we analyze six high-density density-dependent coupling parameters. It is found that reducing the observational uncertainty from 1.0 km to 0.5 km yields only minor improvements, whereas further reduction to 0.2 km significantly tightens constraints, particularly on the isoscalar couplings $\alpha''_S(2n_0)$ and $\alpha''_V(2n_0)$, with uncertainty reductions up to $\sim 90\%$. High-precision data favor a softer EOS, lower central pressures, and narrower credible intervals for the EOSs of symmetric nuclear matter, while the symmetry energy response depends strongly on the adopted couplings at subsaturation densities. Our results highlight the decisive role of future radius measurements with $\sigma_R \lesssim 0.2$ km, as expected from next-generation X-ray and gravitational-wave observatories, in refining the high-density behavior of the EOS and disentangling the density dependence of the symmetry energy from that of symmetric nuclear matter.

Full Text

Preamble

Effects of Radius Measurement Uncertainty on RMF Model Parameters and Neutron Star Matter Properties

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We investigate the impact of neutron star radius measurement precision on the constraints of relativistic mean-field (RMF) model parameters and the equation of state (EOS) of dense matter within a Bayesian framework. Using the canonical neutron star radius $R_{1\cdot4} = 11.9$ km with uncertainties σ_R = 1.0, 0.5 and 0.2 km, we analyze six high-density density-dependent coupling parameters. It is found that reducing the observational uncertainty from 1.0 km to 0.5 km yields only minor improvements, whereas further reduction to 0.2 km significantly tightens constraints, particularly on the isoscalar couplings α''_V($2n_0$), with uncertainty reductions up to 90%. High-precision data favor a softer EOS, lower central pressures, and narrower credible intervals for the EOSs of symmetric nuclear matter, while the symmetry energy response depends strongly on the adopted couplings at subsaturation densities. Our results highlight the decisive role of future radius measurements with σ_R 0.2 km, as expected from next-generation X-ray and gravitational-wave observatories, in refining the high-density behavior of the EOS and disentangling the density dependence of the symmetry energy from that of symmetric nuclear matter.

Keywords: Equation of state, Symmetry energy, Neutron star

Introduction

Understanding the equation of state (EOS) of dense strongly interacting matter remains a central goal in both nuclear physics and astrophysics [1–6]. Nevertheless, the properties of strongly interacting matter at large densities (n_V $2n_0$) are still unclear [7–10] despite that nuclear matter being well understood at densities around the saturation density $n_0 = 0.16$ fm^{-3} [11–16]. This is mainly attributed to the non-perturbative nature of Quantum Chromodynamics (QCD), while perturbative calculations become reliable only at extremely large densities n_V $40n_0$ [17, 18].

Fortunately, astrophysical observations of neutron stars (NSs) have achieved unprecedented precision. As natural laboratories that host matter under extreme densities and isospin asymmetries, NSs provide critical observational data to probe the EOS far beyond terrestrial conditions. For example, in order to reproduce the masses of pulsars PSR J1614-2230 (1.928 ± 0.017 M) [19, 20] and PSR J0348+0432 (2.01 ± 0.04 M) [21], the EOS for neutron star matter needs to be stiff. According to the measurements of the tidal deformability ($70 \leq \Lambda_{1\cdot4} \leq 580$) and radii (R = 11.9 ± 1.4 km) for 1.4 M neutron stars based on the binary neutron star merger event GRB 170817A-GW170817-AT 2017gfo [22], the EOSs of the corresponding neutron star matter need to be soft. Carrying out pulse-profile modelings using NICER and XMM-Newton data, the combined mass-radius measurements for PSR J0030+0451, PSR J0740+6620, and PSR J0437-4715 have become feasible [23–27], which corroborates the constraints on the EOSs of neutron star matter from other observations, i.e., the EOS is soft at low densities and becomes stiff at large densities. This indicates a unique signature for the speed of sound v of neutron star matter, i.e., there may exist a peak of v as a function of density which may correspond to a possible deconfinement phase transition [28–35].

Among various observational probes, the radius of neutron stars, particularly for canonical stars with mass 1.4M , has emerged as a key quantity for constraining the pressure and composition of dense matter [6, 36–39]. Recent advances in X-ray observations, especially from NICER, along with gravitational-wave measurements from LIGO and Virgo, have significantly improved the precision of such radius estimates [40–42]. High-precision NS radius measurements have been identified by the astrophysics community as a primary scientific objective for next-generation X-ray pulse-profile observatories, such as the enhanced X-ray timing and polarimetry mission (eXTP) [43] and the spectroscopic time-resolving observatory for broadband energy X-rays (STROBE-X) [44]. Concurrently, these measurements are a major science driver for third-generation gravitational-wave detectors, including the Einstein Telescope [45] and Cosmic Explorer [46].

As Bayesian statistical inference has proven to be a powerful tool for quantifying uncertainties in model parameters and systematically incorporating observational data into theoretical models [47–49], in this work we apply a Bayesian framework to the RMF model using neutron star radius data for a 1.4 M star with varying levels of observational precision. By analyzing the impact of reduced uncertainties from 1.0 km to 0.2 km in the measured radius, we aim to assess how improved observational accuracy enhances the constraints on model parameters, density-dependent coupling constants, and the resultant EOS of NS matter. Our results reveal that high-precision radius measurements strongly constrain the isoscalar sector, including scalar and vector coupling constants, and lead to tighter bounds on the internal pressure and sound speed of NS matter. Meanwhile, the constraints on symmetry energy from high-precision data depend strongly on the adopted couplings at subsaturation densities.

The paper is organized as follows. The theoretical frameworks of the RMF model and Bayesian inference approach are presented in Sec. II. The obtained constraints on the coupling constants and EOSs are illustrated in Sec. III. We draw our conclusion in Sec. IV.

II. Theoretical Framework

A. Relativistic Mean-Field Framework with Density-Dependent Couplings

The Lagrangian density for the relativistic mean-field (RMF) model [50], applicable to finite nuclei and uniform nuclear matter, is expressed as

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu\partial_\mu - \gamma^\mu(g_\omega\omega^\mu + g_\rho\rho^\mu\tau_3) - M - g_\sigma\sigma]\psi - \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho^\mu\cdot\rho_\mu$$

where denotes the nucleon field, M the bare nucleon mass, τ_3 the third isospin component, and (σ, ω^ , ^) the meson fields. The field tensors are ω_{ } = _ ω_ - _ ω *and* { } = _ _ - _ _ . For uniform matter, meson fields reduce to time components due to time-reversal symmetry, and only the third component of isospin for meson is considered to maintain charge conservation.

In uniform nuclear matter, derivative terms of mean fields vanish, and Eq. (1) can be simplified into a point-coupling form, i.e.,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - \alpha_S(\bar{\psi}\psi)^2 - \alpha_V(\bar{\psi}\gamma^\mu\psi)^2 - \alpha_{TV}(\bar{\psi}\gamma^\mu\tau_3\psi)^2$$

where $\alpha S = g\sigma^{2/\mathrm{m\_}\sigma}2$, $\alpha V = g\omega^{2/\mathrm{m\_}\omega}2$, and α_{TV} = g_ $^{2/\mathrm{m\_}}$ 2 correspond to the couplings in isoscalar-scalar, isoscalar-vector, and isovector-vector channels, respectively.

To incorporate in-medium effects, we adopt density-dependent couplings α_S(n_V), α *V(n_V), and* α{TV}(n_V) instead of nonlinear self-interactions U(σ, ω). The Dirac equation derived from Eq. (2) is then

$$[-i\vec{\alpha} \cdot \nabla + \beta(M - \alpha_S n_S)]\psi = (\epsilon - \Sigma_R - \alpha_V n_V - \tau_3\alpha_{TV}n_{TV})\psi$$

where n_S = $\bar{\psi}\psi$, n_V = $\bar{\psi}\gamma^0\psi$, and n_{TV} = $\bar{\psi}\gamma^0\tau$_3ψ are scalar, vector, and isovector densities. The rearrangement term Σ_R arises from density-dependent couplings, i.e., Σ_R = - α*S/ n_V n_S^2* - αV/ n_V n_V^2 - α{TV}/ n_V n{TV}^2.

The nucleon chemical potential _i is fixed by

$$\mu_i = \sqrt{\nu_i^2 + M^{*2}} + \Sigma_R + \alpha_V n_V + \tau_{3,i}\alpha_{TV}n_{TV}$$

where Fermi momentum _i is related to density n_i = _i$^{3/(3\pi}$2), and M* = M - α_S n_S is the nucleon effective mass.

The energy density E_{NM} and pressure P_{NM} of nuclear matter are:

$$E_{NM} = E_k + \alpha_S n_S^2 + \alpha_V n_V^2 + \alpha_{TV} n_{TV}^2$$

$$P_{NM} = \mu_n n_n + \mu_p n_p - E_{NM}$$

where E_k is the kinetic energy. For neutron stars, leptons (l = e,) contribute additional energy and pressure:

$$E = E_{NM} + \sum E_l, \quad P = P_{NM} + \sum P_l$$

under charge neutrality (n_p = n_e + n_) and β-equilibrium (_n = _p + _l) conditions. The energy per nucleon in symmetric nuclear matter is defined through the energy density E_{NM} indicated in Eq. (6) and the nucleon rest mass M as _0(n_V) = E_{NM}|_{n_p=n_n}/n_V - M.

The symmetry energy is expressed as

$$\epsilon_{sym}(n_V) = \alpha_{TV} n_V$$

where n_p = n_n = n_V/2 = $^{3/3\pi}$2 and E* = ^2 + M^{*2}.

We partition the density domain into three regions: n_{on} ≤ n_V ≤ n_0, n_0 ≤ n_V ≤ 2n_0, and n_V > 2n_0 with n_{on} = 0.1 fm^{-3} and n_0 being the saturation density. In each region, couplings follow an exponential form:

$$\alpha_\xi(n_V) = \sqrt{\alpha_\xi(n_I)^2 + \alpha'_\xi(n_I)\alpha''_\xi(n_I)(n_V - n_I) + \frac{1}{2}\alpha''_\xi(n_I)(n_V - n_I)^2} + \alpha_\xi(n_I)$$

where = S, V, TV and n_I {n_{on}, n_0, 2n_0}. Continuity of α_ and its first derivative α'_ is enforced at intersection densities n_I, while second derivatives α''_ (n_I) are free parameters to be determined by observational data.

For n_V ≤ n_0, parameters are fixed to replicate the relativistic density functionals DD-ME2 [51], TW99 [52], and PKDD [53] at n_V = n_{on}, where the values of the coupling constants, their first and second derivatives at n_V = n_{on} are listed in Table 2 . The predictions of those functionals for nuclear saturation properties are summarized in Table 1 , where K and J_0 respectively denote the incompressibility and skewness parameters for symmetric nuclear matter, and L and K_{sym} stand for the slope and curvature coefficients of the symmetry energy. In such cases, the variations of saturation properties of nuclear matter can be examined, where the relativistic density functional DD-ME2 predicts a stiff EOS for symmetric nuclear matter with large K and J_0, PKDD predicts a stiff EOS for asymmetric nuclear matter with large L, and TW99 predicts EOSs with moderate stiffness.

Finally, there are six high-density parameters that remain unconstrained: α''_S(n_0), α''*V(n_0),* α''{TV}(n_0), α''*S(2n_0),* α''V(2n_0), and α''*{TV}(2n_0). According to our previous investigation [35], these parameters modulate the EOS at supranuclear densities (n_V > n_0) and exhibit linear correlations with coupling strengths constrained by various pulsar observations, showing reduced sensitivity at higher densities. For n_V ≤ n*{on}, neutron star matter becomes nonuniform and we adopt unified EOSs fixed with single-nucleus approximation employing the relativistic density functionals DD-ME2, TW99, and PKDD [54].

B. Bayesian Inference Approach

The foundational principle of Bayesian analysis is mathematically formalized through Bayes' theorem, which governs probability updating upon data acquisition:

$$P(M|D) = \frac{P(D|M)P(M)}{\int P(D|M)P(M)dM}$$

In this formulation, P(M|D) denotes the posterior probability of model M conditional on dataset D, representing updated belief after data assimilation. P(D|M) represents the likelihood function, quantifying the probability of observing data D under model M. P(M) corresponds to the prior probability, encapsulating pre-existing knowledge about M before data consideration. The denominator constitutes a normalization constant ensuring posterior probabilities integrate to unity across the model space, enabling rigorous model comparison.

The six density-dependent coupling parameters p_i=1,…,6 (α' '_S(n_0), α'' *V(n_0),* α' '{TV}(n_0), α"_S(2n_0), α'' *V(2n_0),* α' '{TV}(2n_0)) are sampled uniformly within prior bounds (Table 3). Note that the initial parameter ranges (in units of 10^{-16} MeV^{-8}) are set to be [-10, 10] for α"_S(n_0), α'' *V(n_0),* α' '{TV}(n_0), α"_S(2n_0), α' ' *V(2n_0), and [-10, 50] for* α' '{TV}(2n_0), with subsequent refinement via preliminary low-precision posterior analysis.

Sampled parameters are transformed into density-dependent couplings α_S(n_V), α *V(n_V), and* α{TV}(n_V) via equation (11). These couplings parameterize the RMF model to generate NS EOS under β-equilibrium. Neutron-star structure is determined by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [55, 56]:

$$\frac{dP}{dr} = -\frac{G(E+P)(M+4\pi r^{3P})}{r(r-2GM)}$$

$$\frac{dM}{dr} = 4\pi E r^2$$

where G = 6.707×10^{-45} MeV^{-2} denotes the gravitational constant. Theoretical radii R_{th} derived from TOV solutions are confronted with observational constraints R_{obs} $\pm$ σ_{obs},j via the radius likelihood function:

$$P_R[D|M(p_i)] = \frac{1}{\sqrt{2\pi}\sigma_{obs,j}} \exp\left[-\frac{(R_{th}-R_{obs})^2}{2\sigma^2_{obs,j}}\right]$$

In this work we adopt the observational data from LIGO/Virgo collaboration but with three uncertainties [57], i.e., R_{obs} = 11.9 km, and σ_{obs},j=1,2,3 = 1.0, 0.5 and 0.2 km.

The composite likelihood integrates three components:

$$P[D|M(\{p_i\})] = P_{filter} \times P_{mass,max} \times P_R$$

where P_{filter} enforces thermodynamic stability (dP/dE $\geq$ 0) and causality (c_s < c), and P_{mass},max requires EOS solutions to satisfy M_{max} $\geq$ 1.97M . Posterior distributions are sampled via Markov Chain Monte Carlo (MCMC) using the Metropolis-Hastings algorithm. Parameter probability density functions (PDFs) are derived through marginalization:

$$P(p_i|D) \propto \int P(D|M) \prod_{j\neq i} dp_j$$

Initial 40,000 samples (burn-in phase) are discarded to mitigate non-equilibrium sampling effects [47–49, 58–60]. Posterior statistics are computed from 10^6 subsequent samples, consistent with established convergence diagnostics.

III. Results and Discussion

Figures 1, 2 and 3 present the one-dimensional (1D) PDFs for six RMF model parameters and the associated two-dimensional (2D) PDFs illustrating their correlations. The neutron star observational constraints employed in the calculations are the radius of the canonical star $R_{1\cdot 4}$ = 11.9 km with 1σ uncertainties of 1.0 km, 0.5 km, and 0.2 km. For the density region below saturation density, three different nuclear energy density functionals were utilized: DD-ME2 (Fig. 1 [Figure 1: see original paper]), TW99 (Fig. 2 [Figure 2: see original paper]), and PKDD (Fig. 3 [Figure 3: see original paper]). The 68% and 90% credible intervals for these six parameters are presented in Table 4 .

We observe that the results for σ_R = 0.5 km and σ_R = 1.0 km exhibit only minor differences, evident in both the 1D PDFs and the 2D PDFs. In contrast, significant changes emerge when comparing results for σ_R = 0.2 km to those for σ_R = 1.0 km for the three parameters defined at saturation density (listed in Table 4):

1. **Parameter α’’_S(n_0):** The peak value of the posterior distribution shifts from predominantly negative to predominantly positive. Specifically, the peak locations shift from -0.56 $\times 10^{-16}$ MeV^{-8} to 0.12 $\times 10^{-16}$ MeV^{-8} (DD-ME2), from -0.56 $\times 10^{-16}$ MeV^{-8} to 0.44 $\times 10^{-16}$ MeV^{-8} (TW99), and from -0.68 $\times 10^{-16}$ MeV^{-8} to 0.20 $\times 10^{-16}$ MeV^{-8} (PKDD). The constraints on α’’_S(n_0) (reflected in the width of the credible intervals) show no substantial improvement for DD-ME2 and TW99. However, for

PKDD, the constraint tightens by approximately 40% at the 68% credible level.

2. **Parameter α''_V(n_0):** Consistent with α''_S(n_0), its peak value also shifts from negative to positive: from -0.36 $\times$ 10^{-16} MeV^{-8} to 0.04 $\times$ 10^{-16} MeV^{-8} (DD-ME2), from -0.24 $\times$ 10^{-16} MeV^{-8} to 0.24 $\times$ 10^{-16} MeV^{-8} (TW99), and from -0.32 $\times$ 10^{-16} MeV^{-8} to 0.12 $\times$ 10^{-16} MeV^{-8} (PKDD). Unlike α''_S(n_0), the constraints on α''_V(n_0) improve markedly for all scenarios adopting different functionals at subsaturation densities, with reductions in uncertainty of 54.5% (DD-ME2), 46.2% (TW99), and 75% (PKDD).

3. **Parameter α''*{TV}(n_0):** For DD-ME2 and TW99 adopted at subsaturation densities, the value of α''*{TV}(n_0) decreases, while a substantial decrease is observed for PKDD. The constraints improve by approximately 50% for all three functionals. These results are influenced not only by the observational data and its precision but also by the intrinsic properties of the density functionals at saturation density, as detailed in Table 1.

Utilizing the higher-precision observational data σ_R = 0.2 km, we find that the PDFs for the parameters α''*S(2n_0) and α''*V(2n_0) narrow significantly, with uncertainty reductions of the order of 90%. In contrast, the changes for the symmetry energy parameter α''*{TV}(2n_0) are less pronounced. Notably, relative to their prior ranges (see Table 3), the PDFs for α''*{TV}(2n_0) show minimal change under the DD-ME2 and TW99 functionals adopted at n_V < n_0. Meanwhile, a substantial improvement in the constraint is observed when adopting the PKDD functional at n_V < n_0. For the 2D PDFs, the sign of the correlations between parameters remains consistent with our previous calculations [35]. Crucially, the extent of the joint credible regions is markedly reduced for σ_R = 0.2 km. This demonstrates that high-precision measurement of $R_{1\cdot4}$ not only imposes strong constraints on the 1D PDFs of the parameters but also significantly constrains their 2D PDFs.

The coupling constants of the RMF model are derived from the six parameters given in Eq. (11). Consequently, the constraints on these parameters presented in Figs. 1, 2 and 3 directly influence the density dependence of the three coupling constants α_S, α*V, and* α{TV}, as illustrated in Fig. 4 [Figure 4: see original paper]. The top, middle, and bottom panels of Fig. 4 display the results obtained using the DD-ME2, TW99, and PKDD functionals at n_V < n_0, respectively. Overall, all three coupling constants decrease with increasing nucleon density and exhibit asymptotic behavior at high densities, consistent with our earlier calculations [35]. Compared with the constraints obtained with σ_R = 1.0 km and σ_R = 0.5 km, the uncertainty ranges for α_S and α_V narrow significantly for σ_R = 0.2 km, with the constraint on α*V showing particularly remarkable improvement. Conversely, the range for* α{TV} exhibits a substantial broadening, especially in the region where the nucleon density n_V > 2n_0. This indicates that the current neutron star observational data provide weaker constraints on the symmetry-energy-related coupling constant. It is worth noting that the application of higher-precision data $\sigma R = 0.2$ *km leads to a divergent effect on* α{TV}: the constraint weakens (i.e., the uncertainty range broadens) if the DD-ME2 and TW99 functionals are adopted at n_V < n_0, whereas it strengthens (i.e., the range narrows) if the PKDD functional is adopted at n_V < n_0. This contrasting behavior highlights the role of the intrinsic saturation-density properties of each functional in governing the density dependence of their respective coupling constants.

Figure 5 [Figure 5: see original paper] displays the 90% credible intervals for the pressure inside neutron stars (P), the nuclear symmetry energy (_{sym}), and the energy per nucleon in symmetric nuclear matter (_0) as functions of nucleon density. Utilizing higher-precision neutron star radius observations σ_R = 0.2 km reveals the following:

1. **Pressure Reduction and Constraint:** The neutron star internal pressure decreases overall, and its uncertainty range contracts significantly. This indicates that higher-precision observational data favor a softer neutron star matter equation of state. Crucially, this pressure arises from two competing contributions: the symmetric matter term _0 and the symmetry energy term _{sym}.
2. **Sensitivity of Symmetric Matter EOS:** The energy per nucleon of symmetric nuclear matter _0 exhibits exceptional sensitivity to the observational precision. Its credible interval narrows substantially across the density range, and the values themselves trend lower.
3. **Divergent Behavior of Symmetry Energy:** For the symmetry energy _{sym}, adopting the DD-ME2 and TW99 functionals at n_V < n_0 shows a broadening uncertainty range accompanied by a reduction in the central values. In contrast, adopting the PKDD functional exhibits a narrowing range and an increase in the central values.

Net Effect: The combined result is a tendency towards a softer overall neutron star EOS with improved constraints. This effect is particularly pronounced when employing the PKDD density functional.

Complementing the results in Fig. 5, Fig. 6 [Figure 6: see original paper] presents the PDFs of the internal pressure P and symmetry energy _{sym} within neutron stars at 2n_0 and 3n_0. Our key findings are as follows:

1. **Subsaturation Density Functional-Dependent Symmetry Energy Constraint:** For the DD-ME2 and TW99 functionals adopted at n_V < n_0, the PDFs of _{sym} obtained using neutron star observational data of three distinct precisions ($\sigma R = 1.0$, *0.5, 0.2 km) are highly consistent. However, applying higher-precision radius data (*σR = 0.2 km) weakens the constraint on *{sym} (broadening the uncertainty range) under these functionals. Conversely, for the PKDD functional, the* {sym} distribution narrows significantly.

2. **Stiffening of Symmetry Energy with PKDD:** The _{sym} predicted using the PKDD functional at n_V < n_0 is stiffer than that obtained with DD-ME2 or TW99. Notably, increasing the observational precision further stiffens _{sym} within the PKDD framework.

3. **Universal Pressure Reduction and Constraint:** At both 2n_0 and 3n_0, the pressure P decreases and its PDF width narrows (indicating improved constraints) as observational precision improves. This pressure behavior is universal across the different density functionals.

4. **Density-Dependent Sensitivity:** The pressure at 2n_0 exhibits strong sensitivity to the observational precision. In contrast, the PDF of the pressure at 3n_0 remains nearly unchanged between σ_R = 1.0 km and σ_R = 0.5 km.

5. **Comparison with Multi-Messenger Constraints:** For comparison, we show results from Ref. [61], which utilize neutron star data from the NICER and LIGO/Virgo collaborations within piecewise-polytropic (PP) and constant speed of sound (CS) models, incorporating constraints from chiral effective field theory (EFT) for densities n ≤ 1.5n_0. Despite our results being derived solely under the constraint $R_{1\cdot 4}$ = 11.9 km, we find remarkable agreement, particularly at σ_R = 0.5 km, with the findings of Ref. [61].

Figure 7 [Figure 7: see original paper] displays the 90% credible intervals for the proton fraction (y_p) and squared sound speed (v_s^2) of NS matter as functions of nucleon density. Owing to the direct correlation between y_p and the nuclear symmetry energy, the density-dependent uncertainty range of y_p obtained with high-precision neutron star radius data (*σR = 0.2 km) aligns closely with the constrained behavior of* {sym} shown in Fig. 5. By definition, v_s = √(dP/dE) (where P is the pressure and E the energy density), implying that larger v_s^2 signifies a stiffer EOS. The plots reveal that high-precision data yield an overall downward shift and reduced central values of v_s^2, consistent with the pressure reduction observed in Fig. 5. Strikingly, a reversal behavior emerges in the high-density region n_V > 4n_0, characterized by a broadening of the v_s^2 uncertainty band. Furthermore, while adopting the DD-ME2 and TW99 functionals at n_V < n_0 yields similar results, adopting the PKDD functional exhibits heightened sensitivity to observational precision. Notably, at σ_R = 0.2 km, the credible intervals for both v_s^2 and y_p undergo significant contraction, indicating substantially improved constraints.

IV. Conclusion

We have performed a Bayesian analysis of the density-dependent relativistic mean-field model using neutron star radius data for a 1.4M star with three levels of observational precision: σ_R = 1.0, 0.5 and 0.2 km. Our primary aim was to quantify how improvements in radius measurements affect the constraints on high-density coupling parameters, the corresponding EOS of neutron star matter, and derived stellar properties.

The results show that a modest improvement from σ_R = 1.0 km to 0.5 km yields only limited changes in the posterior distributions, whereas a further reduction to σ_R = 0.2 km leads to substantial tightening of constraints, particularly for the isoscalar couplings α''*S(2n_0) and* α''V(2n_0), whose uncertainties are reduced by up to 90%. The isovector coupling α''*{TV}(2n_0) remains weakly constrained adopting the functionals DD-ME2 and TW99 at n_V < n_0 but shows notable improvement for PKDD. While the high-precision data generally narrow the credible intervals of the isoscalar couplings and soften the pressure of neutron star matter, their effect on the symmetry-energy-related coupling* α{TV} is functional-dependent, i.e., broadening if DD-ME2 and TW99 are adopted at n_V < n_0, but narrowing if PKDD is adopted.

At the EOS level, σ_R = 0.2 km data favor a softer neutron star matter EOS, with lower central pressures and reduced uncertainty bands across most densities. The EOS of symmetric nuclear matter is particularly sensitive to observational precision, while the symmetry energy exhibits opposite trends depending on the underlying functional at n_V < n_0. The pressure at 2n_0 is highly responsive to improved constraints, whereas the pressure at 3n_0 changes little between σ_R = 1.0 and 0.5 km. High-precision measurements also lead to better constraints on the proton fraction and sound speed, although the latter shows increased uncertainty above 4n_0.

Our findings demonstrate that future high-precision radius measurements, at the level of σ_R 0.2 km as anticipated from upcoming X-ray timing missions and third-generation gravitational-wave detectors, will significantly sharpen constraints on the high-density behavior of the EOS and the associated RMF couplings. Such measurements will be essential for disentangling the density dependence of the symmetry energy of nuclear matter, thereby improving our understanding of neutron-rich matter under extreme conditions.

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References

[1] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).
[2] B.-A. Li, L.-W. Chen, and C. M. Ko, Phys. Rep. 464, 113 (2008).
[3] J. M. Lattimer and M. Prakash, Phys. Rep. 621, 127 (2016).

[4] F. Özel and P. Freire, Annu. Rev. Astron. Astrophys. 54, 401 (2016).
[5] A. L. Watts et al., Rev. Mod. Phys. 88, 021001 (2016).
[6] L. Baiotti, Prog. Part. Nucl. Phys. 109, 103714 (2019).
[7] M. Dutra et al., Phys. Rev. C 85, 035201 (2012).
[8] M. Dutra et al., Phys. Rev. C 90, 055203 (2014).
[9] C.-J. Xia et al., Phys. Rev. D 102, 023031 (2020).
[10] K. Hebeler, Phys. Rep. 890, 1 (2021).
[11] S. Shlomo, V. M. Kolomietz, and G. Colò, Eur. Phys. J. A 30, 23 (2006).
[12] B.-A. Li and X. Han, Phys. Lett. B 727, 276 (2013).
[13] M. Oertel et al., Rev. Mod. Phys. 89, 015007 (2017).
[14] Y. Zhang et al., Phys. Rev. C 101, 034303 (2020).
[15] R. Essick et al., Phys. Rev. Lett. 127, 192701 (2021).
[16] S. Huth et al., Nature 606, 276 (2022).
[17] E. S. Fraga and P. Romatschke, Phys. Rev. D 71, 105014 (2005).
[18] E. S. Fraga, A. Kurkela, and A. Vuorinen, Astrophys. J. 781, L25 (2014).
[19] P. B. Demorest et al., Nature 467, 1081 (2010).
[20] E. Fonseca et al., Astrophys. J. 832, 167 (2016).
[21] J. Antoniadis et al., Science 340, 1233232 (2013).
[22] LIGO Scientific and Virgo Collaborations, Phys. Rev. Lett. 121, 161101 (2018).
[23] T. E. Riley et al., Astrophys. J. 887, L21 (2019).
[24] T. E. Riley et al., Astrophys. J. 918, L27 (2021).
[25] M. C. Miller et al., Astrophys. J. 887, L24 (2019).
[26] M. C. Miller et al., Astrophys. J. 918, L28 (2021).
[27] D. Choudhury et al., Astrophys. J. 971, L20 (2024).
[28] P. Bedaque and A. W. Steiner, Phys. Rev. Lett. 114, 031103 (2015).
[29] Y.-L. Ma and M. Rho, Phys. Rev. D 100, 114003 (2019).
[30] C.-J. Xia et al., Chin. Phys. C 45, 055104 (2021).
[31] M. Hippert, E. S. Fraga, and J. Noronha, Phys. Rev. D 104, 034011 (2021).
[32] H. Tan et al., Phys. Rev. Lett. 128, 161101 (2022).
[33] E. Annala et al., Nat. Commun. 14, 8451 (2023).
[34] M.-Z. Han et al., Sci. Bull. 68, 913 (2023).
[35] C.-J. Xia, W.-J. Xie, and M. Bakhiet, Phys. Rev. D 110, 114009 (2024).
[36] B.-A. Li, P. G. Krastev, D.-H. Wen, and N.-B. Zhang, Eur. Phys. J. A 55, 117 (2019).
[37] D. Blaschke et al., Universe 6, 81 (2020).
[38] B.-A. Li, B.-J. Cai, W.-J. Xie, and N.-B. Zhang, Universe 7, 182 (2021).
[39] C. D. Capano et al., Nature Astronomy 4, 625 (2020).
[40] A. J. Dittmann et al., Astrophys. J. 974, 295 (2024).
[41] T. Salmi et al., Astrophys. J. 941, 150 (2022).
[42] T. Salmi et al., Astrophys. J. 974, 294 (2024).
[43] S.-N. Zhang et al. (eXTP), Sci. China Phys. Mech. Astron. 62, 29502 (2019).
[44] P. S. Ray et al., arXiv:1903.03035 [astro-ph.IM] (2019).
[45] B. Sathyaprakash et al., Class. Quantum Grav. 29, 124013 (2012).
[46] R. Abbott et al., Astrophys. J. 896, L44 (2020).

[47] W.-J. Xie and B.-A. Li, Astrophys. J. 883, 174 (2019).
[48] W.-J. Xie and B.-A. Li, Astrophys. J. 899, 4 (2020).
[49] W.-J. Xie, Z.-W. Ma, and J.-H. Guo, Nucl. Sci. Tech. 34, 91 (2023).
[50] J. Meng, ed., Relativistic Density Functional for Nuclear Structure, International Review of Nuclear Physics, Vol. 10 (World Scientific Pub Co Pte Lt, 2016).
[51] G. A. Lalazissis et al., Phys. Rev. C 71, 024312 (2005).
[52] S. Typel and H. Wolter, Nucl. Phys. A 656, 331 (1999).
[53] W.-H. Long, J. Meng, N. V. Giai, and S.-G. Zhou, Phys. Rev. C 69, 034319 (2004).
[54] C.-J. Xia et al., Commun. Theor. Phys. 74, 095303 (2022).
[55] R. C. Tolman, Proc. Nat. Acad. Sci. 20, 169 (1934).
[56] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).
[57] B. P. Abbott et al., Phys. Rev. Lett. 121, 161101 (2018).
[58] W.-J. Xie and B.-A. Li, J. Phys. G: Nucl. Part. Phys. 48, 025110 (2021).
[59] W.-J. Xie and B.-A. Li, Phys. Rev. C 103, 035802 (2021).
[60] W.-J. Xie, B.-A. Li, and N.-B. Zhang, Phys. Rev. D 110, 043025 (2024).
[61] N. Rutherford et al., Astrophys. J. 971, L19 (2024).

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