Study on a Gauss-Seidel-Based Acceleration Method for Solving CRAM Burnup Equations

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Date: 2025-08-27T00:00:00+00:00

Abstract

Burnup calculations for reactors are essential for the safe operation and fuel management of nuclear power plants. In recent years, the Chebyshev Rational Approximation Method (CRAM) has been one of the main methods for solving burnup equation systems. When employing CRAM for solution, the computation of complex matrices typically uses Sparse Gaussian Elimination (SGE), including symbolic and numeric elimination, offering limited improvements in computational efficiency. Based on the independently developed burnup calculation program AMAC, this study develops an accelerated method for solving CRAM burnup equation systems based on Gauss-Seidel (GS). Using three burnup databases of different scales (containing 71, 221, and 1487 nuclides), computational analyses were performed for a light water reactor test case in terms of both computational accuracy and efficiency. In terms of computational accuracy, with the linear subchain analytical method serving as the reference solution, the PFD and IPF forms based on GS exhibit comparable accuracy. For calculations involving short-lived nuclides, the IPF form demonstrates superior computational accuracy compared to the PFD form. In terms of computational efficiency, the GS method is significantly better than the SGE method, achieving a maximum efficiency improvement of up to 80.17% across the three burnup libraries, which facilitates enhanced efficiency in high-fidelity burnup calculations. This study recommends employing the IPF-form burnup equation system based on GS for actual burnup calculations, as it achieves a balance between computational accuracy and efficiency.

Full Text

Preamble

Study on the Accelerated Method for Solving the CRAM Burnup Equations Based on Gauss-Seidel

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Abstract

[Background]: Reactor burnup calculation is crucial for the safe operation and fuel management of nuclear power plants. In recent years, the Chebyshev Rational Approximation Method (CRAM) has become one of the primary approaches for solving burnup equations. When applying CRAM, complex matrix calculations typically employ Sparse Gaussian Elimination (SGE), which includes symbolic and numerical elimination phases, yet offers limited improvement in computational efficiency. [Purpose]: This study aims to develop a Gauss-Seidel (GS)-based acceleration method for solving CRAM burnup equations to enhance the computational efficiency of the burnup equation solver. [Methods]: Based on the self-developed burnup calculation code AMAC, this work developed a GS-based acceleration method for solving CRAM burnup equations. Three burnup databases of different scales (containing 71, 221, and 1487 nuclides) were utilized to analyze a light-water reactor test problem from both computational accuracy and efficiency perspectives. For accuracy evaluation, the linear sub-chain analytical method served as the reference solution. The calculation accuracy of GS-based Partial Fraction Decomposition (PFD) and Incomplete Partial Fractions (IPF) formulations was found to be comparable, though the IPF formulation demonstrated superior accuracy for short-lived nuclides. [Results]: In terms of computational efficiency, the GS method significantly outperformed the SGE method, achieving maximum efficiency improvements of up to 80.17% across the three burnup databases. [Conclusions]: This study recommends adopting the GS-based IPF formulation for practical burnup calculations, as it effectively balances computational accuracy and efficiency.

Keywords: Burnup equations; Chebyshev Rational Approximation Method; Gauss-Seidel iteration; Sparse Gaussian Elimination; Acceleration method

1 Burnup Equations

A burnup system typically contains hundreds to thousands of nuclides. For a burnup system comprising n different nuclides, the matrix form of the burnup equation required to determine nuclide density variations is as follows:

$$\frac{d\mathbf{n}(t)}{dt} = \mathbf{A}\mathbf{n}(t)$$

where $\mathbf{n}(t)$ represents the nuclide density vector at time t, and $\mathbf{A}$ denotes the burnup matrix. The matrix exponential solution to Equation (1), given an initial nuclide density vector $\mathbf{n}(0)$, is:

$$\mathbf{n}(t) = e^{\mathbf{A}t}\mathbf{n}(0)$$

The burnup matrix $\mathbf{A}$ exhibits strong sparsity and stiffness due to significant orders-of-magnitude differences in half-lives among radioactive nuclides [11]. To visually illustrate its distribution characteristics, Figure 1 [Figure 1: see original paper] shows the transmutation relationships among nuclides based on the ORIGEN burnup database. Only non-zero elements in the burnup matrix are color-coded in the figure. Row and column indices are arranged according to ZAI order (ZAI = 10000×Z + 10×A + I, where Z is the proton number, A is the mass number, and I is the excitation state) to enable rapid indexing and positioning of different nuclides [12-13]. As shown in Figure 1, matrix elements are primarily concentrated near the diagonal and on the right side, with a few elements at the top. Diagonal elements result from decay or transmutation reactions, right-side elements represent fission products, and top elements are mainly byproducts of certain reactions such as H and He.

When all eigenvalues of the burnup matrix are distributed near the negative real axis, CRAM can provide an optimal rational approximation of the matrix exponential on the interval $(-\infty, 0]$ [7]. The rational approximation function $r_k(z)$ can be expressed as:

$$r_k(z) = \frac{p_k(z)}{q_k(z)} \approx e^z$$

where $p_k(z)$ and $q_k(z)$ are rational polynomials of order less than or equal to k; k is the rational expansion order. When the order reaches 16, relatively accurate results can be obtained:

$$\mathbf{n}(t) \approx \mathbf{n}_0 + 2\mathrm{Re}\left\{\sum_{i=1}^{k/2} \alpha_i (\mathbf{A} - \theta_i \mathbf{I})^{-1} \mathbf{A}\mathbf{n}_0\right\}$$

where $\mathbf{A}$ represents the burnup matrix; $\mathbf{I}$ is the identity matrix of the same order as the burnup matrix; $\mathbf{n}_0$ is the initial nuclide density vector; α_i and θ_i are scalar coefficients; and $\mathbf{n}_\infty$ is the function limit value as time approaches infinity. To enhance CRAM computational accuracy, one must consider increasing the expansion order or employing refined burnup databases.

Further research revealed that when nuclide densities decrease sharply within a burnup time step, the computational accuracy of the PFD-form CRAM deteriorates or produces erroneous results [8]. Consequently, Pusa proposed the Incomplete Partial Fractions rational approximation function expansion form, which offers significantly higher computational accuracy and numerical stability compared to the PFD form. The IPF formulation is expressed as:

$$\mathbf{n}(t) \approx \mathbf{n}_0 + 2\mathrm{Re}\left\{\alpha_1 \mathbf{A}(\mathbf{A}-\theta_1\mathbf{I})^{-1}\left[\alpha_2 \mathbf{A}(\mathbf{A}-\theta_2\mathbf{I})^{-1}\left[\cdots\left[\alpha_{k/2}\mathbf{A}(\mathbf{A}-\theta_{k/2}\mathbf{I})^{-1}\mathbf{n}_0\right]\right]\right]\right\}$$

where α_i and θ_i are IPF coefficients. Comparing Equations (4) and (5) reveals that the IPF-form CRAM burnup equation system requires $k/2$ complex matrix inversion and multiplication operations as the expansion order k increases, resulting in lower computational efficiency.

2 GS-Based Iterative Method for Solving CRAM Burnup Equations

As shown in Equations (4) and (5), the key to solving different forms of CRAM burnup equations lies in efficiently and accurately computing the coefficient matrix inverse $(\mathbf{A}-\theta_i\mathbf{I})^{-1}$. Different solution methods are typically selected based on the matrix size and sparsity. Direct matrix inversion methods are commonly used for matrices with low order and sparsity, but these are computationally inefficient for large sparse matrices. Therefore, most current codes employ the SGE method to solve CRAM burnup equations.

The SGE-based solution process comprises symbolic decomposition and numerical elimination [14]. Symbolic decomposition analyzes potential fill-in elements (non-zero elements generated during elimination) in the coefficient matrix to reduce storage overhead during numerical elimination. Numerical elimination performs actual Gaussian elimination based on symbolic decomposition results, including forward elimination and back substitution. During forward elimination, a non-zero element is selected as the pivot from unprocessed rows, which is then used to eliminate elements in the same column of other rows, making them zero. Repeating this process yields an upper triangular matrix. The back substitution phase solves for unknowns starting from the last equation of the upper triangular matrix, ultimately obtaining the nuclide density vector. Notably, due to nuclide production and depletion during burnup, the positions of non-zero elements in the coefficient matrix change, requiring symbolic decomposition to be repeated at each burnup step before numerical elimination.

In contrast, the GS method is an iterative approach for solving linear systems that exhibits rapid convergence for diagonally dominant matrices. Its core concept involves continuously updating each component of the solution vector to gradually approach the exact solution without matrix preprocessing. Consequently, the GS method avoids repeated symbolic decomposition of the coeffi-

cient matrix across different burnup steps, yielding higher computational efficiency. Additionally, the GS method offers storage advantages, requiring only the coefficient matrix, vectors, and the iterative solution vector without storing numerous intermediate results, thus reducing computational overhead. The method's relatively straightforward computational process facilitates easier code implementation and maintenance. Considering the sparse and stiff characteristics of burnup matrices, combined with the magnitude of coefficients θ_i in PFD and IPF forms, the coefficient matrix exhibits diagonal dominance. Therefore, this study developed a GS-based acceleration method for solving CRAM burnup equations to enhance computational efficiency.

The matrix inversion calculation can be expressed as:

$$(\mathbf{A} - \theta_i \mathbf{I})^{-1}\mathbf{b} = \mathbf{x}$$

where coefficients $\mathbf{A}$ and $\mathbf{b}$ correspond to $(\mathbf{A} - \theta_i\mathbf{I})$ and $\mathbf{A}\mathbf{n}_0$ in PFD and IPF forms, respectively. By defining the iteration matrix $\mathbf{M} = \mathbf{A} - \theta_i\mathbf{I}$ and substituting the matrix expression, Equation (6) can be transformed into iterative form:

$$\mathbf{M}\mathbf{x}^{(m+1)} = \mathbf{b} - \mathbf{N}\mathbf{x}^{(m)}$$

where $\mathbf{M}$ and $\mathbf{N}$ correspond to the lower triangular and strictly upper triangular matrices of $(\mathbf{A}-\theta_i\mathbf{I})$, respectively; $\mathbf{x}^{(m+1)}$ is the $(m+1)$-th iteration result; $\mathbf{x}^{(m)}$ is the m-th iteration result; and $\mathbf{x}^{(0)} = \mathbf{n}_0$ is the initial boundary condition.

When using GS iteration, the component-wise iterative format is:

$$x_i^{(m+1)} = \frac{1}{m_{ii}}\left(b_i - \sum_{j=1}^{i-1} m_{ij}x_j^{(m+1)} - \sum_{j=i+1}^{n} m_{ij}x_j^{(m)}\right), \quad i = 1, 2, \cdots, n$$

where m_{ii} represents the diagonal element of coefficient matrix $\mathbf{M}$; m_{ij} represents the element in row i, column j of the coefficient matrix; b_i represents the i-th element in vector $\mathbf{b}$; and m is the iteration number.

Substituting Equation (8) into Equation (7) and iterating until convergence, the final iterative forms for PFD and IPF can be obtained by combining Equations (4) and (5):

$$\mathbf{n}(t) \approx \mathbf{n}_0 + 2\mathrm{Re}\left\{\sum_{i=1}^{k/2} \alpha_i \mathbf{x}_i^{(m)}\right\}$$

$$\mathbf{n}(t) \approx \mathbf{n}_0 + 2\mathrm{Re}\left\{\alpha_1 \mathbf{A}\mathbf{x}_1^{(m)}\right\}$$

where each iteration result represents the nuclide density at the current iteration count. This study employs a convergence criterion based on the nuclide density norm:

$$\frac{\|\mathbf{x}^{(m+1)} - \mathbf{x}^{(m)}\|_2}{\|\mathbf{x}^{(m+1)}\|_2} < \epsilon$$

where ϵ is a convergence value set according to actual computational accuracy requirements.

The GS-based CRAM burnup equation solving acceleration method was implemented in the burnup calculation code AMAC. AMAC is a self-developed code for burnup and radioactive decay calculations, designed to compute material nuclide compositions and radiation characteristics. The program integrates multiple burnup solution methods and can be flexibly coupled with transport calculation codes, demonstrating good computational accuracy and efficiency in validation analyses against international codes such as ORIGEN and FISPACT [15-16]. In transport-burnup coupling calculations, transport computation accounts for over 90% of the total computational cost [17]. To improve efficiency and reduce computational expense, AMAC has developed a model-order-reduction-based transport-burnup coupling framework, including: (1) a dynamic mode decomposition-based transport-burnup coupling method to significantly reduce transport calculation frequency and improve efficiency [18]; and (2) a Grassmann manifold-based burnup reduction model to substantially decrease burnup matrix construction costs during coupling calculations [19].

The AMAC program architecture, shown in Figure 2 [Figure 2: see original paper], consists of three main components: pre-processing, solver, and post-processing. Pre-processing reads input cards to obtain material composition, time steps, flux levels, and other relevant parameters, while also reading and storing decay constants, microscopic cross-sections, fission yields, and other parameters from the burnup database to construct the burnup matrix for the solver. Considering the sparse nature of burnup matrices, AMAC employs sparse matrix storage formats to reduce storage overhead and improve computational efficiency. The solver includes four solution methods: TTA, MMPA, CRAM, and QRAM. The CRAM method is further divided into PFD and IPF formulations based on solution form. Post-processing formats the calculation results, outputting data covering nuclide densities, radioactivity, decay heat, decay photon spectra, etc. Based on these computational methods and procedures, the method was implemented in AMAC, with the computational flow shown in Figure 3 [Figure 3: see original paper].

3 Numerical Verification and Analysis

This study selected three burnup databases of different scales to analyze a light-water reactor pin-cell test problem, with an irradiation flux of 1.5×10^{14} cm^{-2}s^{-1},

200 days of irradiation followed by 10 years of cooling. The geometric model and material composition are shown in Figure 4 [Figure 4: see original paper] and Table 1 , respectively, comprising three material-filled regions from inner to outer: fuel, cladding, and moderator [20-21]. The three selected burnup databases contain 71, 221, and 1487 nuclides (hereinafter referred to as Database-71, Database-221, and Database-1487). The validation strategy for the GS-based CRAM burnup equation solving method encompasses two aspects: first, computational accuracy, where GS and SGE methods are employed under PFD and IPF formulations to solve burnup matrices of different scales for comparative numerical precision analysis. Considering that the TTA method is an analytical solution-based burnup calculation approach capable of precisely calculating every nuclide in the burnup chain, it is commonly used as a reference solution for various numerical methods [22]. Therefore, this study selected TTA results as the reference solution to verify the correctness of the GS method, with a convergence value ϵ of 1.0×10^{-20}. Second, computational efficiency, where efficiency analyses of CRAM equations under both solution forms were conducted across the three databases, considering the impact of different expansion orders on computational efficiency to validate the effectiveness of the GS method. The computational hardware used in this study was a computer with an AMD Ryzen 7 7745HX with Radeon Graphics (3.60 GHz) processor.

3.1 Computational Accuracy

This study first calculated the test problem using GS and SGE methods under CRAM' s two solution forms (PFD and IPF) across the three burnup databases. Database-71 contains 71 nuclides, including 21 fissile nuclides, 49 fission products, and 1 pseudo-nuclide. Database-221 contains 221 nuclides, including 28 fissile nuclides and 193 fission products. Both databases include eight reaction types: decay, isomeric transition, electron capture, (n,γ), $(n,2n)$, $(n,3n)$, (n,p), (n,α), and fission reactions [23-24], which have been extensively validated and applied in software such as MVP [25] and MOSRA [26]. Database-1487 integrates ORIGEN-S and ORIGEN-2 databases, containing 1487 nuclides, 23 neutron reaction cross-sections, 11 decay reactions, and 30 fission yields for heavy nuclides.

Table 2 presents the number of short-lived nuclides, minimum nuclide lifetime, maximum nuclide lifetime, and number of non-zero elements in the burnup matrix construction for the three databases. The selection criterion for short-lived nuclides follows the ORIGEN program [12,22], as shown in Equation (12):

$$T_{1/2} \leq 0.1\Delta t$$

where $T_{1/2}$ is the nuclide half-life (s) and Δt is the time step (s).

To evaluate the numerical precision performance of GS and SGE, this study employs Maximum Nuclide Absolute Relative Difference (MNARD) and Average Nuclide Absolute Relative Difference (ANARD) for comparative analysis.

MNARD refers to the maximum absolute relative deviation among all calculated nuclide densities, while ANARD represents the average absolute relative deviation across all calculated nuclide densities.

Table 3 shows MNARD and ANARD for nuclide densities in Database-1487, using SGE calculation results as the reference solution. The results indicate that PFD-16 achieves MNARD and ANARD on the order of 1.0×10^{-10} and 1.0×10^{-12}, respectively, with GS having minimal impact on PFD formulation accuracy. Meanwhile, IPF-16, IPF-32, and IPF-48 achieve MNARD and ANARD on the order of 1.0×10^{-13} and 1.0×10^{-14}, respectively, demonstrating that GS possesses good computational accuracy for solving IPF-form burnup equations.

Tables 4 and 5 present MNARD and ANARD for Databases-221 and 71, respectively. In Database-221 calculations, PFD-16 achieves MNARD and ANARD on the order of 1.0×10^{-10} and 1.0×10^{-12}, while the three IPF formulations achieve MNARD and ANARD on the order of 1.0×10^{-13} and 1.0×10^{-14}. Further reducing the database scale to Database-71, IPF-16 achieves MNARD and ANARD on the order of 1.0×10^{-10} and 1.0×10^{-11}, respectively, while IPF formulations reach the order of 1.0×10^{-14}, approaching the effective digit limit of double-precision floating-point numbers and indicating excellent numerical precision of the GS method.

As shown in Tables 3 through 5, MNARD and ANARD for IPF formulations using the GS method across the three databases all reach above 1.0×10^{-13}, while PFD formulations achieve above 1.0×10^{-10}. Notably, there exists an order-of-magnitude difference between the two formulations, primarily because the rational approximation function in PFD form is more sensitive to rounding errors, whereas IPF form employs an incomplete partial fraction rational expansion that is less sensitive to rounding errors, offering better numerical stability and precision. These results verify that the GS method demonstrates good numerical precision for solving both IPF and PFD burnup equations across different database scales, meeting the accuracy requirements for practical burnup calculations.

3.1.1 Database-1487 Among the three databases employed, Database-1487 contains the most comprehensive nuclide types and reaction types, with the largest burnup matrix scale. When stored in sparse format by AMAC, the number of non-zero elements reaches 37,847, as shown in Table 2. Additionally, the magnitude difference between the longest-lived and shortest-lived nuclides reaches 10^{31}, making the burnup matrix the most stiff. Using the TTA method as the reference solution with a truncation value of 1.0×10^{-20}, Figure 5 [Figure 5: see original paper] presents the relative deviations of nuclide densities calculated using the GS method based on Database-1487. The results show that relative deviations for both GS-based PFD and IPF formulations are less than 1.0×10^{-7}, with PFD-16 achieving an ANARD of 1.347×10^{-9}, while IPF demonstrates overall better accuracy than PFD. As the IPF expansion order increases, computational accuracy gradually improves, with IPF-48 reaching an

accuracy of 1.0×10^{-15} for nuclide calculations.

Considering that Database-1487 includes 484 short-lived nuclides that directly affect the calculation accuracy of end-of-chain nuclides, Table 6 further presents the relative deviations of short-lived nuclides at the end of the irradiation period under PFD and IPF formulations. The results show that compared to PFD, the GS-based IPF formulation achieves improved accuracy for short-lived nuclides, with most reaching relative deviations of 1.0×10^{-15}. The nuclide ^{126}Sb exhibits the maximum relative deviation of 3.963×10^{-8}, corresponding to a density of 1.930×10^{-15} atoms/cm^3, which has negligible impact on the burnup system. These results verify the correctness of the GS-based CRAM burnup equation solving method for detailed burnup chain calculations.

3.1.2 Database-221 In Database-221 calculations, the number of short-lived nuclides decreases to 51 compared to Database-1487, and the magnitude difference between longest-lived and shortest-lived nuclides decreases from 10^{31} to 10^6, significantly reducing burnup matrix stiffness. The burnup matrix involves 5,703 non-zero elements. Figure 6 [Figure 6: see original paper] shows the relative deviations of nuclide densities calculated using the GS method. As the expansion order increases, IPF computational accuracy gradually improves, with IPF-16, IPF-32, and IPF-48 all achieving relative deviations below 1.0×10^{-10}. For PFD results, except for ^{237}U with a relative deviation of 4.055×10^{-10}, all nuclides show relative deviations below 1.0×10^{-10}. Table 7 further presents the relative deviations of short-lived nuclides at the end of the irradiation period under both formulations. The IPF formulation demonstrates improved accuracy for short-lived nuclides compared to PFD, with relative deviations below 1.0×10^{-15}. These results indicate that IPF offers higher computational accuracy for short-lived nuclides and verify the applicability and effectiveness of the GS method for medium-scale burnup database calculations.

3.1.3 Database-71 Database-71 represents a simplified burnup chain containing only 19 short-lived nuclides, with a magnitude difference of 1.0×10^3 between longest-lived and shortest-lived nuclides, the lowest burnup matrix stiffness, and 1,149 non-zero elements. Tracking this minimal-scale burnup system evaluates the correctness of the GS method for simplified chain calculations. Figure 7 [Figure 7: see original paper] shows the relative deviations of nuclide densities calculated using the GS method, with PFD and IPF results showing good agreement with the reference solution. PFD achieves an ANARD of 1.578×10^{-11}, while IPF reaches 1.0×10^{-13} across three expansion orders, demonstrating higher accuracy. For short-lived nuclides, Table 8 presents the relative deviations of ^{237}U and ^{242}Cm at the end of the irradiation period, confirming that IPF formulation again outperforms PFD in accuracy.

3.2 Computational Efficiency

As described in Section 2, the SGE method requires symbolic decomposition for coefficient matrix preprocessing before numerical elimination when solving CRAM burnup equations. The GS method requires no preprocessing and directly performs iterative solution on the diagonally dominant coefficient matrix. Table 9 presents the computational time for PFD-16 formulation based on SGE and GS methods across the three databases. Runtime statistics were collected with actual execution times presented at a 95% confidence level. For Database-71, the GS method required only 0.857 ms, achieving a 69.27% efficiency improvement over the SGE method' s 2.789 ms. When the database scale increased to Database-221, SGE computation time increased from 2.789 ms to 19.335 ms, while GS time only increased from 0.857 ms to 2.977 ms. With further database scale expansion, the number of non-zero elements in the burnup matrix increased significantly, with SGE computation time for Database-1487 reaching 65.948 ms compared to GS' s 35.305 ms, representing a 46.92% efficiency improvement. For the SGE method, symbolic decomposition accounts for approximately 30-40% of total computation time, with the primary cost being numerical elimination, which grows with non-zero element count. For the GS method, the average number of iterations is 3 for Databases-71 and -221, increasing to 5 for Database-1487 due to the introduction of numerous short-lived nuclides, resulting in increased computation time.

Table 10 further presents computational times for IPF-16 formulation based on SGE and GS methods. The results demonstrate significant efficiency improvements with the GS method. Comparing Tables 9 and 10 reveals that at the same database scale and expansion order, IPF formulation requires slightly more computation time than PFD. This difference stems from their mathematical expressions: as shown in Equations (4) and (5), PFD involves summation operations while IPF involves successive multiplication operations, which are computationally more complex and thus marginally increase computation time. However, as demonstrated in Section 3.1, IPF offers better computational accuracy and numerical stability. This study recommends using the IPF-form CRAM burnup equation solver in practical calculations, as the increased computation time compared to PFD is negligible.

Further increasing the IPF expansion order, Tables 11 and 12 present computational times for IPF-32 and IPF-48 formulations using SGE and GS methods. As the IPF expansion order increases, the number of successive matrix multiplications in Equation (5) increases accordingly, leading to higher computational costs. Additionally, for Database-1487 calculations, the GS method requires 5 iterations compared to an average of 3 for Databases-71 and -221. The introduction of numerous short-lived nuclides increases iteration count, and with 37,847 non-zero elements, each GS iteration requires more time, increasing overall computation time. Nevertheless, the GS method still achieves a 34.30% efficiency improvement over SGE for Database-1487 calculations. In summary, the GS-based CRAM burnup equation solving method developed in this study

demonstrates computational efficiency advantages over the SGE method across three different database scales.

Conclusion

Based on the self-developed burnup calculation code AMAC, this study developed a GS-based acceleration method for solving CRAM burnup equations. Validation and analysis of PFD and IPF-form CRAM burnup equations were completed across three burnup databases of different scales. In terms of computational accuracy, the study first verified that GS and SGE methods achieve comparable numerical precision. Subsequently, using TTA algorithm results as the reference solution, calculations for GS-based PFD-16, IPF-16, IPF-32, and IPF-48 were performed. The results demonstrate that the GS method achieves maximum relative deviations below 1.0×10^{-7} for all nuclides, meeting the accuracy requirements for practical burnup calculations in both PFD and IPF formulations. Regarding computational efficiency, the GS method offers advantages over SGE, with maximum efficiency improvements reaching 80.17%. Considering that high-fidelity full-core burnup calculations may require solving burnup equations billions of times, an 80% reduction in burnup equation solving time translates to hours or days of saved computation time. Future work will integrate this method into the model-order-reduction-based transport-burnup coupling framework for large-scale burnup zone calculations to further enhance overall burnup computational efficiency and reduce costs. This study recommends employing the GS-based IPF formulation for practical burnup calculations to simultaneously balance computational accuracy and efficiency, which is beneficial for improving high-fidelity burnup calculation efficiency.

Author Contributions: SUN Yuqing established the model, researched and analyzed data, and drafted the manuscript; ZHANG Binhang designed the specific research content and direction, proposed reasonable research plans, provided theoretical support, guided program usage and data collection, reviewed the intellectual content, and was responsible for manuscript revision; YUAN Xianbao provided process supervision and results acceptance; TANG Haibo provided technical support.

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Note: Figure translations are in progress. See original paper for figures.