Critical Behavior of Odd-Order Net Baryon Number Generalized Susceptibility

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Date: 2025-08-22T00:00:00+00:00

Abstract

Based on the universality of critical behavior and assuming thermal equilibrium in the QCD system, this study maps variables from the three-dimensional Ising model onto the QCD phase plane to investigate the density distributions of the fifth-, seventh-, and ninth-order generalized susceptibilities of net baryon number on the phase plane, as well as their dependence on net baryon chemical potential along the chemical freeze-out line. When only leading-order critical contributions in the Ising model are considered, approaching the critical point from the region of low net baryon chemical potential yields a structure in susceptibilities of all orders characterized by a negative dip immediately followed by a positive peak. Upon inclusion of next-to-leading-order critical contributions, the existence of the negative dip depends on the distance to the phase boundary and the magnitude of scaling exponents in the critical region, whereas the positive peak structure persists regardless of these factors. Compared with the negative dip, the positive peak structure constitutes a more robust signature of the critical point.

Full Text

Preamble

Vol. 00, No. 0
Aug. 2025

Nuclear Physics Review
Article ID: 1007-4627(2025) 00-0000-10

ChinaRxiv

Critical Behavior of Odd-Order Generalized Susceptibilities of the Net-Baryon Number

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Abstract

Based on the universality of critical behavior and assuming that the QCD system reaches thermal equilibrium, this work maps the variables of the three-dimensional Ising model onto the QCD phase plane to investigate the density distributions of the fifth-, seventh-, and ninth-order generalized susceptibilities of the net-baryon number on the phase plane, as well as their dependence on the net-baryon chemical potential along the chemical freeze-out line. When only the leading-order critical contribution from the Ising model is considered, approaching the critical point from the low net-baryon chemical potential region yields a characteristic structure in each order of susceptibility consisting of a negative dip followed immediately by a positive peak. Upon incorporating next-to-leading-order critical contributions, the existence of the negative dip becomes dependent on the distance to the phase boundary and the magnitude of the critical region scaling exponents, while the positive peak structure persists regardless of these factors. Compared to the negative dip, the positive peak structure represents a more stable signature of the critical point.

Keywords: three-dimensional Ising model; QCD critical point; generalized susceptibilities of net-baryon number

1. Linear Mapping from the Ising Model to Quantum Chromodynamics

In the parametric representation of the Ising model, the magnetization (M) and reduced temperature (t) are parameterized using two variables R and θ [47-48]:

$$M = m_0 R^{\beta}\theta, \quad t = R(1-\theta^2).$$

The equation of state can be expressed in terms of R and θ as:

$$h = h_0 R^{\beta\delta}\tilde{h}(\theta).$$

Here, m_0 and h_0 in equations (1) and (2) are normalization constants determined by the conditions $M(t=-1, h=+0)=1$ and $M(t=0, h=1)=1$, yielding values of 0.605 and 0.364, respectively. The critical exponents $\beta \simeq 0.326$ and $\delta \simeq 4.8$ belong to the three-dimensional Ising universality class [49]. The function $\tilde{h}(\theta) = \theta(1-0.76201\theta^2+0.00804\theta^4)$. The variables R and θ have ranges $R \geqslant 0$ and $|\theta| \leqslant 1.154$.

The free energy density is defined as:

$$\mathcal{F}(M,t) = h_0 m_0 R^{2-\alpha} g(\theta),$$

where $\alpha \simeq 0.11$ is another critical exponent of the three-dimensional Ising universality class satisfying the relation $2-\alpha = \beta(\delta-1)$. The function $g(\theta)$ takes the form:

$$g(\theta) = c_0 + c_1(1-\theta^2) + c_2(1-\theta^2)^2 + c_3(1-\theta^2)^3,$$

with coefficients:

$$c_0 = \frac{(1+a+b)}{2-\alpha},$$
$$c_1 = -\frac{1}{2(\alpha-1)}[(1-2\beta)(1+a+b) - 2\beta(a+2b)],$$
$$c_2 = -\frac{1}{2(\alpha+1)}[2\beta b - (1-2\beta)(a+2b)],$$
$$c_3 = -\frac{b(1-2\beta)}{2(\alpha+1)}.$$

The Gibbs free energy density relates to the free energy density through:

$$\mathcal{G}(h,t) = \mathcal{F}(M,t) - Mh.$$

Since pressure equals the negative of the Gibbs free energy ($P = -\mathcal{G}$), the pressure in the three-dimensional Ising model can be written as:

$$P_{\text{Ising}}(R,\theta) = h_0 m_0 R^{2-\alpha}[\theta\tilde{h}(\theta) - g(\theta)].$$

The parametric representation of the n-th order susceptibilities of magnetization and energy in the three-dimensional Ising model can be obtained by differentiating the pressure with respect to h and t:

$$\chi_{M,E}^{n_1,n_2} = \frac{\partial^{n_1+n_2} P_{\text{Ising}}}{\partial h^{n_1} \partial t^{n_2}}, \quad n = n_1 + n_2.$$

When $n_2 = 0$, equation (7) represents the n-th order susceptibility of magnetization (χ_M^n), while when $n_1 = 0$, it represents the n-th order susceptibility of energy (χ_E^n).

The n-th order susceptibility of the net-baryon number (χ_B^n) can be obtained at fixed temperature by taking the n-th derivative of the QCD pressure with respect to the net-baryon chemical potential:

$$\chi_B^n(T, \mu_B) = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}.$$

The complete QCD pressure can be reconstructed following the method in Ref. [51] by separating it into two parts: one unrelated to the Ising model and another related to it:

$$P(T, \mu_B) = T^4 c_{\text{Non-Ising}}(T) + P_{\text{QCD}}(T, \mu_B),$$

where the first term represents the Taylor series expansion of the pressure unrelated to the Ising universality class, and the second term $P_{\text{QCD}}(T, \mu_B)$ denotes the mapping of the Ising model pressure to QCD. Details of the mapping can be found in Refs. [45,51].

In this work, we focus exclusively on the critical behavior of the net-baryon generalized susceptibilities, so the pressure can be expressed as [51]:

$$P(T, \mu_B) = T^4 C P_{\text{Ising}}(R(T, \mu_B), \theta(T, \mu_B)).$$

The derivatives $\partial R/\partial h$, $\partial\theta/\partial h$, $\partial R/\partial t$, and $\partial\theta/\partial t$ can be obtained through implicit differentiation of equations (1) and (2).

To map Ising model results to QCD, a linear relationship containing six mapping parameters is typically assumed between Ising and QCD variables [50-51]:

$$\begin{aligned} T - T_C &= w(\rho t \sin\alpha_1 + h \sin\alpha_2), \\ \mu_B - \mu_{BC} &= w(-\rho t \cos\alpha_1 - h \cos\alpha_2), \end{aligned}$$

where T_C and μ_{BC} are the temperature and net-baryon chemical potential at the QCD critical point mentioned in the Introduction. The parameters w and ρ are two scaling parameters related to the critical region in the mapping from the Ising model to QCD, whose values remain undetermined. The angles α_1 and α_2 were introduced in the Introduction.

Following the approach in Ref. [46] to reduce mapping parameters, we assume the QCD phase transition line can be expressed as:

$$T = T_0 \left[1 - \kappa \left(\frac{\mu_B}{T_0} \right)^2 - \lambda \left(\frac{\mu_B}{T_0} \right)^4 \right],$$

where T_0 is the transition temperature at $\mu_B = 0$, set to 156.5 MeV based on recent lattice QCD results [52]. The curvature κ at $\mu_B = 0$ is set to 0.015, which is consistent with recent lattice calculations, functional renormalization group results, and Dyson-Schwinger equations within error bars [24-26,52-54]. Using the critical point from Ref. [24], $(T_C, \mu_{BC}) = (107, 635)$ MeV, the parameter λ is determined to be 0.000256 to ensure the transition line passes through this critical point, a value also compatible with lattice QCD calculations [52,54].

Since the t-axis is tangent to the QCD first-order transition line at the critical point after mapping to the $T-\mu_B$ plane, the angle α_1 is determined to be 10.8°. If the h-axis is assumed perpendicular to the t-axis, α_2 becomes 100.8°. Only the scaling parameters w and ρ remain undetermined.

The odd-order ($2n+1$ order) susceptibilities of the net-baryon number are then given by:

$$\chi_B^{2n+1} = T^4 C \sum_{k=0}^{2n+1} \binom{2n+1}{k} \left(\frac{\partial h}{\partial \mu_B}\right)^{2n+1-k} \left(\frac{\partial t}{\partial \mu_B}\right)^{k} \chi_{M,E}^{2n+1-k,k},$$

where $n = 0, 1, 2, 3, ...,$ and $\binom{2n+1}{k} = (2n+1)!/(k!(2n+1-k)!)$. The derivatives $\partial h/\partial \mu_B$ and $\partial t/\partial \mu_B$ can be obtained from equation (11):

$$\frac{\partial h}{\partial \mu_B} = -\frac{\sin(\alpha_1)}{T_C w \sin(\alpha_1 - \alpha_2)},$$
$$\frac{\partial t}{\partial \mu_B} = \frac{\sin(\alpha_2)}{T_C w\rho \sin(\alpha_1 - \alpha_2)}.$$

If only leading-order critical contributions are considered (i.e., only $k = 0$ terms in equation (16)), the $(2n+1)$-th order susceptibility of the net-baryon number becomes:

$$\chi_{B,L}^{2n+1} = T^4 C \left(\frac{\partial h}{\partial \mu_B}\right)^{2n+1} \chi_M^{2n+1}.$$

The combined contributions from terms with $k \neq 0$ in equation (16) are collectively referred to as next-to-leading-order critical contributions in this work.

2. Density Distributions of Net-Baryon Generalized Susceptibilities in the Phase Plane

When investigating the density distributions of net-baryon generalized susceptibilities, we consider three sets of values for the scaling parameters w and ρ: $(w, \rho) = (0.4, 0.8)$, $(0.8, 0.8)$, and $(0.8, 0.4)$. The fifth-, seventh-, and ninth-order generalized susceptibilities of the net-baryon number are calculated in the phase plane region defined by $480 \leqslant \mu_B \leqslant 680$ MeV and $90 \leqslant T \leqslant 130$ MeV. For susceptibilities of the same order, the color function is identical. Due to large differences in magnitude between different orders, the color mapping is designed such that for the same color in the density plots, the seventh-order susceptibility value is 1000 times that of the fifth-order, while the ninth-order value is 1 million times that of the fifth-order. Red, yellow, and green regions indicate positive susceptibility values (with red representing the maximum and green the minimum), while blue regions indicate negative values (darker blue corresponds to more negative values).

When only leading-order critical contributions are considered, the density distributions of the fifth-order ($\chi^5_{B,L}$), seventh-order ($\chi^7_{B,L}$), and ninth-order ($\chi^9_{B,L}$) generalized susceptibilities are shown in the left, middle, and right columns of [Figure 2: see original paper], respectively. Different rows correspond to different critical region scaling parameters. In each subplot, the purple solid line represents the QCD transition line described by equation (12), and the purple solid dot marks the critical point.

As shown in Figure 2: see original paper, when $w = 0.4$ and $\rho = 0.8$, the distribution of $\chi^5_{B,L}$ exhibits alternating red and blue lobes around the critical point. Below the transition line, $\chi^5_{B,L}$ becomes negative when approaching the critical point from the smooth crossover region. Changing the scaling parameters w and ρ, as in Figure 2: see original paper and (g), does not alter these qualitative features. However, as w increases, the primary region composed of dark blue and red lobes narrows in the T direction, while decreasing ρ narrows it in the μ_B direction. The density distributions of $\chi^7_{B,L}$ and $\chi^9_{B,L}$ in the middle and right columns of [Figure 2: see original paper] share essentially the same characteristics as $\chi^5_{B,L}$, with the main difference being that higher-order susceptibilities exhibit more blue and red lobes, increasing the alternation frequency. Consequently, the number of sign changes in the susceptibility increases when approaching the critical point from the smooth crossover region.

When next-to-leading-order critical contributions are included, the fifth-order (χ^5_B), seventh-order (χ^7_B), and ninth-order (χ^9_B) generalized susceptibilities are shown in the left, middle, and right columns of [Figure 3: see original paper], respectively. Comparison with [Figure 2: see original paper] reveals that next-to-leading-order contributions dramatically alter the density distributions. Three main effects can be summarized: First, the area comprising red and dark blue lobes increases and is mostly located above the transition line. Second, when approaching the critical point from the smooth crossover region, the dark blue lobes disappear in the χ^5_B density distribution (left column of [Figure 3: see original paper]). For χ^7_B and χ^9_B, only minimal dark blue lobes appear very close to the transition line (middle and right columns of [Figure 3: see original paper]). Third, additional alternating red and blue lobes emerge in the high baryon chemical potential region below the critical point, with more extra lobes appearing at higher susceptibility orders. As shown in the middle row of [Figure 3: see original paper] where $w = \rho = 0.8$, these additional lobes appear green or light blue, indicating smaller magnitudes. As w or ρ decreases, these extra lobes become darker in color (top and bottom rows of [Figure 3: see original paper]).

3. _B Dependence of Net-Baryon Generalized Susceptibilities

To study the μ_B dependence of net-baryon generalized susceptibilities along the chemical freeze-out line, we assume the freeze-out line is obtained by shifting the transition line described by equation (12) downward in temperature by ΔT:

$$T_f = T_0 \left[1 - \kappa \left(\frac{\mu_B}{T_0}\right)^2 - \lambda \left(\frac{\mu_B}{T_0}\right)^4\right] - \Delta T.$$

[Figure 4: see original paper] shows the μ_B dependence of $\chi^5_{B,L}$, $\chi^7_{B,L}$, and $\chi^9_{B,L}$ along the chemical freeze-out line with $\Delta T = 1.0$ MeV for different critical region scaling parameters. The vertical green dashed line marks the critical point at $\mu_B = 635$ MeV, while horizontal green dashed lines indicate zero values for each susceptibility. As seen in Figure 4: see original paper, when approaching the critical point from low baryon chemical potential, all curves first exhibit a negative dip followed by a positive peak. The cyan curve shows the widest peak and the purple curve the narrowest, corresponding to ρ values of 1.6 and 0.4, respectively, since larger ρ yields a broader critical region in the μ_B direction. The red curve shows the highest peak and the blue curve the lowest, reflecting w values of 0.4 and 1.6, respectively, as smaller w corresponds to a broader critical region in the T direction and thus closer proximity to the phase boundary for the same ΔT, resulting in higher peaks. Qualitatively, $\chi^7_{B,L}$ and $\chi^9_{B,L}$ in Figure 4: see original paper and (c) behave similarly to $\chi^5_{B,L}$. Quantitatively, the negative dip becomes more pronounced relative to the positive peak at higher orders, with the ratio of dip depth to peak height increasing with susceptibility order. For the red curves in Figure 4: see original paper, (b), and (c), these ratios are 0.098, 0.222, and 0.321, respectively.

The density distributions of $\chi^5_{B,L}$, $\chi^7_{B,L}$, and $\chi^9_{B,L}$ in [Figure 2: see original paper] indicate that the qualitative behavior of their μ_B dependence along the chemical freeze-out line remains the same—a negative dip followed by a positive peak—when varying ΔT in equation (19).

When next-to-leading-order critical contributions are included, the distance to the phase boundary may affect the μ_B dependence. [Figure 5: see original paper] shows the μ_B dependence of χ^5_B, χ^7_B, and χ^9_B along three different freeze-out lines with $\Delta T = 0.2$, 1.0, and 1.5 MeV (top to bottom rows). The chemical freeze-out line moves progressively farther from the phase boundary from top to bottom. Different colored curves correspond to different w and ρ parameters.

As shown in Figure 5: see original paper, when approaching the critical point from low baryon chemical potential, only a positive peak is observed in each curve; the negative dip disappears. Moving gradually away from the phase boundary (Figure 5: see original paper and (g)), no negative dip appears in the low baryon chemical potential region. However, for the red and purple curves with smaller critical region scaling parameters w and ρ, additional positive peaks and negative dips emerge at larger μ_B behind the main positive peak, consistent with the extra lobes below the critical point in Figure 3: see original paper and (g).

For $\Delta T = 0.2$ MeV, the χ_B^7 plot in Figure 5: see original paper shows a negative dip only in the cyan curve when approaching from low baryon chemical potential, but it is insignificant compared to the subsequent positive peak. For χ_B^9 in Figure 5: see original paper, negative dips are faintly visible in curves other than the cyan one, but again remain minor compared to the positive peaks. As the distance from the phase boundary increases (Figure 5: see original paper, (f), (h), and (i)), the negative dips in χ_B^7 and χ_B^9 disappear in the low baryon chemical potential region for all scaling parameters, while additional negative dips and positive peaks appear behind the main peak in the red and purple curves.

Compared to [Figure 4: see original paper], the negative dips before the positive peak become much less pronounced when next-to-leading-order contributions are included, as shown in the bottom row of [Figure 6: see original paper]. However, the influence of extra lobes behind the positive peak becomes more significant. Additional positive peaks and negative dips are not only more prominent in the red and purple curves but also become visible in the black and blue curves. Thus, the effect of extra lobes becomes more pronounced farther from the phase boundary. Nevertheless, only a single positive peak remains visible in the cyan curve. Therefore, before the critical region scaling parameters w and ρ are determined, the positive peak structure is more stable than the negative dip. In experimental searches for the critical point using high-order cumulants of net-proton number, we anticipate observing structures of positive peaks or negative dips in the energy dependence of these cumulants as collision energy decreases.

In summary, when only leading-order critical contributions are considered, the μ_B dependence of all orders of generalized susceptibilities along the chemical freeze-out line exhibits a negative dip followed by a positive peak. However, with next-to-leading-order contributions included, the existence of the negative dip depends on the distance to the phase boundary and the magnitude of critical region scaling parameters, whereas the positive peak persists unaffected. The positive peak structure is therefore a more stable signature for identifying the critical point using high-order susceptibilities of the net-baryon number, consistent with conclusions from our previous study of even-order susceptibilities.

In Ref. [55], experimental data provide the energy dependence of chemical freeze-out parameters:

$$T_f = \frac{158.4}{1 + \exp[2.6 - \ln(\sqrt{s_{NN}})/0.45]},$$
$$\mu_B = \frac{1307.5}{1 + 0.288\sqrt{s_{NN}}},$$

where $\sqrt{s_{NN}}$ is in GeV and temperature and chemical potential are in MeV. At $\mu_B = \mu_{BC} = 635$ MeV, $T_f \approx 90.7$ MeV, giving $\Delta T \approx 16.3$ MeV between the freeze-out temperature and critical temperature. This freeze-out line is therefore far from the transition line described by equation (12). [Figure 6: see original paper] shows the μ_B dependence of $\chi^5_{B,L}$, $\chi^7_{B,L}$, $\chi^9_{B,L}$, χ^5_B, χ^7_B, and χ^9_B along this experimental freeze-out line for various mapping parameters w and ρ. The magnitudes of all susceptibilities decrease significantly when far from the phase boundary.

The top row of [Figure 6: see original paper] reveals that when only leading-order critical contributions are considered, the fifth-, seventh-, and ninth-order susceptibilities all show a negative dip followed by a positive peak when approaching the critical point from low baryon chemical potential, with the negative dip being more pronounced than in [Figure 4: see original paper]. With next-to-leading-order contributions (bottom row of [Figure 6: see original paper]), the negative dip before the positive peak becomes difficult to observe, while the effect of extra lobes behind the peak becomes more significant.

4. Conclusion

Assuming thermal equilibrium in the QCD system, this work maps the reduced temperature and external field variables from the three-dimensional Ising model onto the QCD $T-\mu_B$ phase plane. Under the assumption of perpendicular Ising variables, we investigate how critical behavior in the Ising model contributes to the fifth-, seventh-, and ninth-order susceptibilities of the net-baryon number.

Our findings demonstrate that next-to-leading-order critical contributions dramatically alter the density distributions of susceptibilities in the phase plane. When only leading-order contributions from the Ising model are considered, approaching the critical point from low net-baryon chemical potential along the chemical freeze-out line produces a characteristic structure in the μ_B dependence of the fifth-, seventh-, and ninth-order generalized susceptibilities: a negative dip followed by a positive peak. This structure is independent of the distance between the freeze-out line and the transition line, as well as the critical region scaling parameters.

Upon including next-to-leading-order contributions, the negative dip in the low baryon chemical potential region either disappears or becomes difficult to observe, while the positive peak structure remains robust. When far from the phase boundary with small critical region scaling parameters, additional positive peaks and negative dips appear behind the main peak. Thus, the existence of the negative dip depends on the distance to the phase boundary and the scaling parameters, whereas the positive peak persists regardless. Compared to the negative dip, the positive peak structure constitutes a more stable signature of the critical point, consistent with conclusions from our previous study of even-order susceptibilities.

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Note: Figure translations are in progress. See original paper for figures.