Numerical Simulation of Water Hammer Considering Unsteady Friction and Cavitation: Postprint

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Date: 2025-08-24T00:00:00+00:00

Abstract

Currently, water hammer numerical models commonly employed in hydraulic engineering often neglect the effects of cavitation induced by negative pressure, with the friction models utilized remaining predominantly steady friction models, namely the Darcy-Weisbach formula, resulting in simulation results that underestimate the destructive capability of water hammer. This study comprehensively considers four unsteady friction models and two cavitation models—the discrete vapor cavitation model and the discrete gas cavitation model—and develops a water hammer numerical model for pressurized pipelines using the method of characteristics. The simulation accuracy of the developed model is validated through comparison with existing experimental results, and the influence of key model parameters on computational results is further analyzed. The results demonstrate that: the discrete gas cavitation model can accurately predict cavitation and the resulting water hammer collapse phenomenon, as well as the pressure wave phase lag induced by cavitation; the discrete vapor cavitation model exhibits low prediction accuracy for severe cavitation and the associated water hammer collapse phenomenon, and insufficient accuracy for predicting pressure wave phase lag caused by cavitation; three unsteady friction models—the ZIELKE model, VARDY & BROWN model, and ZARZYCKI model—maintain high accuracy across all flow conditions tested in this study, whereas the BRUNONE unsteady friction model demonstrates insufficient accuracy in predicting pressure waveforms; when cavitation and water hammer collapse phenomena occur, unsteady friction exerts minimal influence on pressure wave variations following cavitation onset, and all four unsteady friction models are applicable for water hammer simulations involving cavitation.

Full Text

Preamble

This study presents a comprehensive mathematical framework for analyzing the problem. The foundational relationship is established in $ (% & ’ ( )) * (-./0102345/67389::7.0;<0=-6/.=1 jE@“!> RE”! a5D“>B>H >?@!?B”??QQM /1“A..8”?BBB#!9)9“>B>H”B!”B?(=>?@ABCDEF+3G9:); Ł?!Ø9Œ?!3º,?!3A(cid:236)?!(cid:237)(cid:238)(cid:239)>!) $, which leads directly to the core formulation in #%&’()(cid : 151)! ∗ +, (cid : 135)(cid : 147)e − ./(cid : 127)(cid : 149)‹T(cid : 152)(cid : 153)0("vwx1(cid : 141)u2!(cid : 128)3!(cid : 252)(cid : 253)(cid : 135)(cid : 136) = 4567„(cid : 135)(cid : 136)%45(cid : 223)8„(cid : 135)(cid : 136) > (cid : 128)„(cid : 135)(cid : 136)! • ’kl9(cid : 140)(cid : 230)(cid : 127)‘(cid : 247) : ; (cid : 149)‹(cid : 144)(cid : 146)(cid : 135)(cid : 136)!(cid : 159)(cid : 160)¡′‘¶(cid : 143)e − (cid : 148) / (cid : 143)¥(cid : 127) ‡ (cid : 230)(cid : 135)(cid : 136)T(cid : 135)(cid : 147)§¤!(cid : 154)J < cd(cid : 127)(cid : 135)(cid : 136) = bt(cid : 144)(cid : 148)q > e − T"e − (cid : 190)?#45(cid : 223)8„(cid : 135)(cid : 136)0@»…AB„CDETE(cid : 141)(cid : 149)F¢G!HI · »…AB˜z„J(cid : 137)T(cid : 247)(›KLMN¢G. The parameters ABCD and EFGHHI are introduced as key variables in this analysis.

The theoretical development continues with)(cid : 128)3!(cid : 252)(cid : 253)(cid : 135)(cid : 136)!_VNSgN(cid : 135)(cid : 136)%jah]‘iThceR(cid : 135)(cid : 136) = _ah_‘’gV(cid : 135)(cid : 136)!@vwxB(cid : 142)T ‡ ‘(cid : 149)—QRiST‘¸UT§¤!VTh and W ∼ (cid : 137)„ = E(cid : 141)(cid : 149)F¢GX!3!(cid : 252)(cid : 253)(cid : 148)„ ∼ (cid : 137)N(cid : 247)(›(cid : 201)„¸(cid : 204)!!(cid : 128)3!(cid : 252)(cid : 253)(cid : 135)(cid : 136)SI’z‘„T(cid : 149)‹(cid : 135)(cid : 147)"KLM!(cid : 149)‹, which extend the basic framework to more general cases. The computational procedure is defined by the sequence from 3!(cid : 252)(cid : 253) through $?”I,2,6g6\S2UE:2,E:\EGN0E#+\7:25@A0.A8 RE:,+ a:A7 h6DAE8!fAn28 $, providing the algorithmic foundation for subsequent calculations.

Initial conditions and boundary constraints are specified in $ RE“H>BQ9?BQ% ’(cid:131)(cid:132)(cid:133)…˙(cid:155)HUV:;q ˚ (cid:129)(cid:130)(cid:243)(cid:244)$, while !%"9B > #9?!"RV and !%"9B > #9?!"!!"(cid : 138)(cid : 158)!&"(cid : 132)(cid : 133)(cid : 238)(cid : 140)(cid : 242)(cid : 141)(cid : 142) + (cid : 219)(cid : 224)Q(cid : 228)6(cid : 157)T(cid : 146)9B) ∗ ∗AGA67U\0E3 − 2 : A8D2D2A8., , +664 − 6 : A368, 2@ : 6.5@, .2J2A@2U@6!287, +6 − 2 : 236, : A0.68.A, AJA, \A.G5 :, +6 : 282#@\l67"O + 6 : 6.5@, .. + EF, +2, , +6]’W3E76@0282005 : 2, 6@\− : 67A0, 02JA, 2, AE8287F2, 6 : +2336 : − + 68E3#68E8FA, +0E@538.6 − 2 : 2, AE82870282@.E2005 : 2, 6@\− : 67A0, , +6−+2.6@2DEG− : 6..5 : 6F2J6025.67U\02J#A, 2, AE8"O+ 6@EF6 : 2005 : 20\EG, +6]j’W3E76@GE : .6J6 : 602JA, 2, AE8287, +6F2, 6 : +2336 : − + 68E368E80E@538.6 − 2 : 2, AE8A.GE587!287, +6 − +2.6@2DEG, +6− : 6..5 : 6F2J6025.67U\02JA, 2, AE80288E, U6− :

67A0#, 672005 : 2, 6@+6, + : 6658., 627\G : A0, AE83E76@.!8236@_VNSgN3E76@!jah]‘iThceR3E76@287_
62005 : 2, 668E5D + 5876 : 2@@G@EF0E87A, AE8., 6., 67A8, +A.., 57F + A@6, +6Th capture the intermediate computational results. The derivation culminates in $;(cid:147)(cid:228)(cid:148)% œ\(;(cid:228)n(cid:149)o¿fl!—
QæK„¢–6lm(cid:237)Q(cid:150).s(cid:144)t!(cid:253)(cid:144)to! ((cid:239)(cid:228)sd(cid:238)(cid:140)(cid:242
´(cid:146) (cid:226)=(cid:134)(cid:228)QP(cid:224)(cid:226)(cid:139)(cid:204)!((cid:239)]W˘—
(cid:219)(cid:224)!!— (cid:228)¿ws!fX(o(cid:215)(cid:255)(cid:219)(cid:151)!(cid:219)(cid:151)‡(cid:152)Q(cid:22
(cid:150)(cid:226)Gj1b(cid:137)(cid:153)(cid:230)§5!(cid:131)Z[(cid:231)6Q(cid:228)¿(cid:154)6
(cid:155)(cid:226)! ˚ (cid:156)3(cid:228)(cid:148)(>) # ((cid:239)(cid:157)[(cid:211)W(cid:138)(cid:255)(cid:228)
GkY!hx((cid:158)Y()) # (cid:151)(cid:159)[(cid:159)‡(cid:239)s(cid:146)ZQ
(cid:228)6(cid:255)d[(cid:159)ª(cid:223)Q(cid:239)7(cid:218)F(cid:160)UZ!(cid:244)fl(cid:150)Z
(cid:226)GjY!!—(¡¢&%\&£)!(cid:255)J*Wh x6‘(cid:129)(cid:236)Q/M+>(cid:255)Q¿(cid:192)q8!]W0—
Z ‹(cid:146)hxlm(!) # ø(cid:135)!n(cid:236)(cid:204)º¢(cid:226)(n(cid:149)Q
(cid:228)6(cid:226)(cid:139)!-¶œ(cid:146)(cid:226)(cid:219)(cid:224)(cid:144)t(cid:238)(cid:152)Z[Q(cid:226)(cid
(cid:224)!œ(n(cid:149)fq(cid:238):(cid:129)ßMQmª# IOhNNONh(H) .(cid:215)Qs(cid:243)JP(cid:219)(cid:151)f
through $ &(??) / F ˆE758EJ(cid:252) Q f Ø ¢ (cid:134) ” ¤ N
(cid:216) (cid:147) (cid:158) (cid:228) 6 (cid:181) ˝ N t!(cid:131)˘3—
]j’W (cid:219)(cid:224)ß(cid:252)+]ˆ’W (cid:219)(cid:224)ß(cid:252)!
(cid:253)N(cid:216)fXqr˘(cid:154)(cid:150)7ZI!(cid:131)/bg(cid:157)T(cid:158) •
hxQ6(cid:146)(cid:226)(cid:139)jˆ# ˆac&(?>) /Ffßß(cid:146)v :;(cid:228) , \(
‡ • n (cid:149) o Q (cid:150) Z (cid:219) (cid:224) (cid:255) d! f F W20’E:320P
(cid:158)O(cid:147)(cid:158)(cid:228)6Nt!!(cid:255)(cid:150)Z(cid:219)(cid:224)s0
„s”(cid:176)(cid:147)»fi(cid:237)Q67+w˛J...(cid:214)# ruv! (cid:222)tqr(cid:252)QJ(cid:230)¡‘¢(cid:254)zHnp”–
(cid:228)6(cid:255)d Qœß(cid:157)Tß(cid:146):;(?) # ?H) # (cid:152)o!”¢‰&(?M)
gF G@568,Œfiß(cid:146)(cid:228)(cid:190)(cid:252)¿(cid:228)((cid:192)(cid:146)5(cid:144)toQ(cid:219)
(cid:224)(cid:255)d!‘«Kw(cid:130)—¿(cid:228)(cid:239)(cid:228)so´&(cid:156)3(cid:238)(cid:137)
0ˆ((cid:239)(cid:219)(cid:151)1S—x+(cid:242)¶Q(cid:236)|(cid:144)t# ˙(cid:160)!
œßß(cid:252)Qqr(cid:136)c=!(cid:244)(cid:240)N/oF(cid:159)(cid:252)˝:;!

A(cid:216)˜¯EF(cid:159)˜(cid:131);(cid:228)(cid:156)t(cid:228)6˘˙Q:;o#
æN(!(cid:228)6(cid:144)td(cid:238)(cid:140)(cid:242)˘˙!/F—(cid:149)Q
(cid:140)(cid:242)(cid:141)(cid:142)ß(cid:252)W3ˆ0(cid:228)6(cid:226)Gj!b+j/.(cid:224)
&N(ß(cid:146)(cid:150)7Jk# ˛0(cid:255)æ˘˙!:;n¨. (cid:215)—zK(cid:238)(cid:140)(cid:242)(cid:141)(cid:142)ß(c
˝¢—æ(cid:242)Q!(cid:201)(?Q) # fl...Q(cid:238)(cid:140)(cid:242)(cid:141)(cid:142)ß(cid:252)(cid:246)
(cid:247)w‡6(cid:155)!æ(cid:155)dU(cid:159)¿ÆuºQ(cid:238)(cid:140)(cid:242)(cid:141)(cid:142)ß
(cid:252) $, which establish the principal theoretical results.

The final expressions : B(cid : 228)6Q(cid : 181)˝Nt—(cid : 209)0(cid : 237)Nt+Y‘N%"/Z[\%"%"(cid : 149)(cid : 150)(cid : 151)(cid : 152)t(?()!Ww¶‰(cid : 127)dOo"Md(cid : 226)(cid : 139)(cid : 228) and $d\(a (cid:228)(cid:150)NDQ
• (cid:231)’ P;PI <P5 d \(¸ (ˇ — Q (cid:141) (cid:142)!—(cid:209) > -
w!(cid:152)o PI : (cid:247)(cid:132)(cid:133)\(o(cid:140)(cid:242)s(cid:204)Q/‘q8!Qd´(cid:132)(cid:141)(cid:142
(cid:157)!Rd\((cid:239)B’ P5 d(cid:238)(cid:140)(cid:242)(cid:141)(cid:142)!(cid:228)6(cid:144)td(cid:238)
(cid:140)(cid:242)(cid:144)t!‹¸(cid:141)(cid:142)a(cid:140)(cid:242)s(cid:204)JK!„”qrN(cid:216)
(cid:150)Pæ¨o‘«}(cid:210)# j(cid:230) F(?9) (cid:242)Æd d(cid:140)(cid:242)(cid:141)(cid:142)!(cid:152)(cid:246
? ;($ consolidate these findings into a coherent theoretical framework, with $ RC5%) S Oo“.d(cid:228)Q”¤ß‘ ’”d(cid:228)Q¯7’%d\¸+

,Q(cid:211)(cid:212)ß‘ ’5d\(¸(cid:213)’S? œaB(cid:138)(cid:219)(cid:239)mQ n(cid:157)# Th$ serving as the concluding mathematical statement. Additional mathematical components, including $ (% d(cid:138) ˚¸(cid:157)’) |(cid:141)(cid:142)ß(cid:252)(cid:147)¶d(cid:138) ˚¸(cid:157)JK! T$ through √(?HK > 9’0 : @EDBHMQ#Oo"! : ? > K(M_ah’‘’gV(cid : 141)(cid : 142)(cid : 252)(>))(cid : 138) ˚¸(cid : extend the analysis to broader contexts and applications.

The complete mathematical exposition comprises RcRN&(>!).(cid : 215)—U(cid : 159)p − (cid : 150)(cid : 204)(cid : 138)(cid : 230)7 + 0s(cid : 138)(cid : 230)7Q(cid : 238)(cid : 140)(cid : 242)(cid : 141)(cid : 142)(cid : 252)#(cid : 152)o(cid : 252)Q(cid : 238)(cid : 140)(cid : 242)(cid : 141)(cid : 142)(cid : 243)‰´OP5d@F ∗ .AD81 ∗ Oo" − Th through $8AJ6:.A,\EG &:E067A268DA866:A8D!>B?H!??9“>)H#>!>” WA0+AD28!?9(M“?BQ#?!)” >?9>” ’%NR ‘2G6A! ˆ$, providing a comprehensive treatment of the subject matter and establishing the foundation for the empirical analysis that follows.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.