

Statistical Principles for Elastic Particles in Numerical Phase Space

Authors: Liang Zhongcheng

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Abstract

By introducing the path subspace, this study extends the energy phase space to a digital phase space (Λ space), thereby completing the digital statistical description of elastic particle equilibrium systems. Unlike the Γ phase space (6N-dimensional) of the Gibbs statistical ensemble, the Λ phase space (18N-dimensional) is divided into three regions: liquid, solid, and gas, and its energy probability distribution function possesses cyclic symmetry. The Λ space employs association degree to characterize particle interactions, thereby avoiding the potential energy integration difficulty of the Γ space. This paper presents the macroscopic and mesoscopic equilibrium conditions for elastic particle systems, and derives the partition functions and state functions for the three regions. Furthermore, by extending the definition of the entropy function, the thermodynamic differential equations are generalized to the entire range of states of matter. The results demonstrate that the structural information of matter can be decoded using four independent parameters: one digital variable and three scaling bases.

Full Text

2.3 Derived Energy

The derived energy of the system is defined through the energy operator. For the kinetic component, we define the derived energy as:

$$\tilde{r}U$$

where the derived energy is expressed as:

$$\begin{aligned} \tilde{U} = \{ & E_z := E_{z2} + E_z \{ E_k = L_k + H_k, E_l = H_l + K_l, E_h = K_h + L_h \} J_z := \\ & E_{z3} - E_z \{ J_k = H_k - K_k, J_l = K_l - L_l, J_h = L_h - H_h \} Q_z := E_{z1} + E_z \{ Q_k = \\ & K_k + L_k, Q_l = L_l + H_l, Q_h = H_h + K_h \} G_z := E_{z2} - E_z \{ G_k = L_k - \\ & H_k, G_l = H_l - K_l, G_h = K_h - L_h \} U_z := Q_z - E_z \{ U_k = Q_k - H_k, U_l = \end{aligned}$$

$$Q_l - K_l, U_h = Q_h - L_h \} Y_z := Q_z + G_z \{ Y_k = Q_k + G_k, Y_l = Q_l + G_l, Y_h = Q_h + G_h \} a_z (cU/U) b_z (U/U) a_z := b_z := \{ K_k, K_k, \{ \} \} V_{zL} \}$$

The probability distribution in phase space yields the normalization condition:

$$\tilde{U} \{ \{ \{ V_l, V_l, \} \} \} 3cU$$

with the momentum vector defined as $P_z := (P_z^1, P_z^2, P_z^3) = P_z e_0$. The magnitude satisfies:

$$\tilde{r}P_s = E_s/V_s$$

The derived energy operator thus connects the microscopic variables to macroscopic thermodynamic quantities through the partition function.

4. Thermodynamic Relations

4.1 L2-Norm and Derived Energies

The L2-norm formulation provides the foundation for derived energies in the system. The derived energies for different regions are summarized in Table 2, which shows the relationships between the digital variables and thermodynamic quantities. The mesoscopic equilibrium conditions require that the derived energies satisfy specific normalization constraints.

4.2 Parameter Configuration

The fundamental parameter vector is defined as $k^* = (k, l, h) = (1, 1, 1)$, establishing the baseline configuration for the three-phase system. This symmetric assignment ensures equal weighting of the liquid, solid, and gas regions in the Λ -space.

4.3 Differential Relations

The thermodynamic differential relations emerge from the derived energy framework. Starting from the fundamental relation $\lambda = \theta - \eta$, we obtain:

$$d\lambda = d(\theta - \eta) = d(\sigma E_s - P_2 V_s) = (E_s d\sigma - P_2 dV_s) - (-\sigma dE_s + V_s dP_2)$$

Similarly, from $\lambda = v - \gamma$, we derive:

$$dv = E_s d\sigma - P_2 dV_s, d\gamma = -\sigma dE_s + V_s dP_2$$

The Gibbs free energy differential follows as:

$$dG = d(C\gamma) = C d\gamma + \gamma dC = C(-\sigma dE_s + V_s dP_2) + \gamma dC = -S dE_s + V dP_2 + \gamma dC$$

This yields the generalized thermodynamic relation:

$$dG = -S dE_s + V dP_2 + \gamma dC$$

4.4 Total Differential Equations

The complete set of thermodynamic differential equations for the elastic particle system is presented in Table 3. These equations generalize traditional thermodynamics to the full range of matter states in Λ -space. The key relations include:

- Energy conservation: $dE_2 - d(P_2V) = dQ - d(SE_s) = 0$
- Fundamental relation: $SdE_s - VdP_2 + Cd\gamma = 0$

The total differential equations connect the derived energies (E_1, E_2, E_3), thermodynamic potentials (G, Q, U, Y), and state variables (S, V, C).

4.5 Partition Function and State Functions

The partition function for each region is constructed from the degree of association C_n and the microscopic parameters. For the liquid region (k), the partition function yields:

$$\tilde{U}L_1$$

The derived state functions follow from logarithmic derivatives of the partition function. The energy scale bases are determined by:

- $K_k = NE_k^s \equiv C\kappa_k$
- $L_l = NE_l^s \equiv C\lambda_l$
- $H_h = NE_h^s \equiv C\eta_h$

These relations establish the connection between microscopic elastic particle properties and macroscopic thermodynamic observables.

Table 1 Digitization of energy and pressure parameters showing the relationship between digital variables (a_s, b_s), scale bases, and derived quantities for each region (k, l, h).

Table 2 Derived energies of the system and midson (intermediate states) showing how digital variables combine to produce macroscopic thermodynamic quantities.

Table 3 Energy relations and total differential equations for the complete thermodynamic description in Λ -space.

1. Physical Constants

The Avogadro constant $N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1}$ serves as the fundamental scaling factor. The molar volume at reference temperature T_2 is denoted $V_A(T_2)$. The characteristic length scale is:

$$r_s = \sqrt[3]{V_s} = \sqrt[3]{V_A(T_2)}$$

with corresponding time and inertia scales:

$$t_s = \sqrt{I_s/E_s}, I_s = m_s r_s^2$$

2. Thermodynamic Functions

The digital variable C represents the normalized volume:

$$C = \tilde{V} = V_A(T)/V_A(T_2) \leq N_A$$

At the reference temperature $T = T_2$, $C = N$. The temperature-dependent parameters are defined as:

$$a(T) := V_A(T_2)/[2V_A(T)] \quad b(T) := V_A(T)/V_A(T_2)$$

These parameters encode the thermal expansion behavior and phase transition characteristics.

3. State Equation

The fundamental state equation emerges as:

$$P^*V = ckT$$

where the effective pressure $P^* = (2a^2 + 1) \cdot kT \cdot N_A/V_A(T_2)$. The system exhibits critical behavior when $da/dT = 0$, corresponding to the phase transition point T_0 where $V'[a(T_0)] = 0$ and $V''[a(T_0)] > 0$. This occurs in the temperature range $30^\circ\text{C} \sim 4^\circ\text{C}$, consistent with known anomalous expansion phenomena.

5. Conclusion

The statistical principle of elastic particles in digital phase space provides a unified framework for describing matter across all states. The Λ -space formulation avoids the potential energy integration problem of traditional Γ -space by employing the degree of association as the interaction measure. The cyclic symmetry of the three regions (liquid, solid, gas) enables decoding of structural information through four independent parameters: one digital variable and three scale bases. This approach establishes a foundation for the final theory of physics based on barycenter reference frames.

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The Statistical Principle of Elastic Particles in Digital Phase Space

Liang Zhong-Cheng¹)[†]

¹(College of Electronic and Optical Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210023, China)

Abstract

By introducing the path subspace, this study extends the energy phase space to a digital phase space (Λ -space), thus accomplishing the digital statistics of the elastic particle equilibrium systems. Unlike the Γ phase space ($6N$ dimensions) of Gibbs statistical ensemble, the Λ -space ($18N$ dimensions) is divided into three regions: liquid, solid, and gas, and their energy probability distributions have cyclic symmetry. The interaction between particles in Λ -space is characterized by the degree of association, thereby avoiding the problem of potential energy integration in Γ -space. This article presents the macroscopic and mesoscopic equilibrium conditions for elastic particle systems and derives the partition functions and state functions for three regions. In addition, the thermodynamic differential equations are generalized to the entire range of matter states by extending the definition of the entropy function. The results indicate that the structural information of matter can be decoded using four independent parameters: one digital variable and three scale bases.

Keywords: statistical physics, elastic particles, statistical phase space, partition function, thermodynamical functions, structure of matter

[†] Corresponding author. E-mail: zcliang@njupt.edu.cn

Note: Figure translations are in progress. See original paper for figures.

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