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Date: 2025-08-04T17:25:19+00:00

Abstract

Based on the characteristics of small deformation theory, a novel method for stability analysis of slope retaining walls under different rotation axis conditions is proposed (small deformation theory solution). For the slope: based on the stress distribution satisfying the stress equilibrium differential equations, compatibility equations, force boundary conditions, and macroscopic equilibrium, the theoretical solution for slope stress distribution is obtained; for the retaining wall: using the same method, the theoretical solution for stress distribution under moment equilibrium conditions at different rotation points can be obtained, and a point failure criterion is proposed. Taking the waste transfer station in Jinguoping Town, Badong County as an example, stress analysis of the slope and retaining wall demonstrates that: using the stress calculation method for retaining walls and slopes proposed in this paper, respective stress solutions can be obtained; the new stability evaluation method based on point strength criteria can correctly assess the stability state of each point. Years of operation of this retaining wall indicate that this design analysis method is feasible.

Full Text

Preamble

Stress Distribution Characteristics of Retaining Walls Considering Different Rotation Points

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Abstract

Based on the characteristics of small deformation theory, this paper proposes a novel method for stability analysis of slope retaining walls under different rotational axis conditions (small deformation theory solution). For the slope body: theoretical solutions for stress distribution are obtained based on satisfaction of the stress equilibrium differential equations, compatibility equations, force boundary conditions, and macroscopic equilibrium. For the retaining wall: using the same method, theoretical solutions for stress distribution under moment equilibrium conditions at different rotation points can be obtained, and a point failure criterion is proposed. Taking the solid waste transfer station in Jinguoping Town, Badong County as an example, stress analysis of the slope and retaining wall demonstrates that the proposed calculation method can obtain respective stress solutions for both the retaining wall and slope body. The new stability evaluation method based on point strength criteria can correctly assess the stable state of each point. Years of operation of this retaining wall have shown that the proposed design and analysis method is feasible.

Keywords: Theoretical solution of small deformation; Stress differential equilibrium equation; Retaining wall; Rotating axis; Stress distribution

1. Introduction

Retaining walls are structures designed to support roadbed fill or hillside soil masses, preventing deformation and instability of the fill or soil body. Retaining walls can stabilize road embankments and cut slopes, and are frequently used to remediate landslides and slope failures [1]. Classical Coulomb earth pressure theory assumes the slip surface is planar, the backfill is cohesionless soil, earth pressure follows a triangular distribution, and derives the active earth pressure calculation formula. Rankine earth pressure theory assumes a semi-infinite soil mass, a rigid wall, vertical and smooth wall back, horizontal fill surface, triangular earth pressure distribution, and obtains corresponding theoretical solutions from limit equilibrium. Terzaghi [3] demonstrated that active earth pressure on rigid retaining walls is related to wall movement, showing significant differences among three distinct movement modes: translation, rotation about the wall top, and rotation about the wall base. Mao Yisheng [4] explained fundamental defects in Coulomb earth pressure theory from a mechanical perspective. Kezdi [5] investigated the rotation mode of retaining walls about the wall base. Handy [6] studied stress distribution in retaining walls using the soil arching principle combined with finite element methods.

Current research on retaining walls primarily focuses on theory, numerical analysis, and model testing. Chen Zhongda [1] introduced design essentials and mechanical characteristics of retaining walls, summarizing their advantages and disadvantages. Dawson et al. [2] established an integrated calculation model combining retaining walls with slopes. Active earth pressure calculation for retaining walls is a classical topic; however, problems persist in current active earth pressure calculations. Traditional theory oversimplifies the problem, as noted by Huang Yuewen [7]. With the development of numerical analysis methods, different calculation approaches have been proposed. Recently, the partial strength reduction method for progressive slope failure processes has been widely applied. References [9-10] propose a new numerical calculation method that can consider different boundary conditions and can be generalized and applied.

2. Methodology

2.1 Fundamental Approach

For any material considering boundary effects, when the geometry is determined, its theoretical solution is definite. As boundary conditions change, the geometry also changes, and the solution changes accordingly.

Assuming a continuous medium, the solution must satisfy stress boundary conditions, equilibrium equations, and compatibility equations. When stress distributions are unequal, discontinuous stress solutions can be obtained, thus solving the problem of stress discontinuity during failure. The approach is as follows:

1. Determine the macroscopic geometric characteristics of the research object through precise measurement, and establish geometric characteristic description equations associated with the research object;
2. Analyze the unit weight distribution of the research object and establish unit weight distribution equations;
3. Analyze the stress characteristics of the research object, and establish corresponding stress equations for different boundary conditions;
4. When the research object satisfies the corresponding equilibrium equations, stress boundary condition equations, and compatibility equations, select the representation form of the stress equations and calculate the corresponding constant coefficients;
5. Conduct specific analysis of the force characteristics of the research object, combine with current strength criteria, and determine its failure characteristics.

2.2 Two-Dimensional Slope-Retaining Wall Example

Using the above fundamental concepts, this section illustrates the theoretical solution for a two-dimensional plane-strain retaining wall in a slope:

For step 1, precise measurement determines macroscopic geometric characteristics. Geometric characteristic description equations are established for the research object as shown in Figure 1 [Figure 1: see original paper], using appropriate forms to represent the geometric characteristic description equations for boundaries AD, DC, BC, BF, FH, HG, GE, and EA.

For step 2, the unit weight distribution equation is established as:

For step 3, stress characteristics are analyzed. Based on different boundary conditions, corresponding stress equations are established. For plane problems, the expressions for horizontal stress (σ) and vertical stress (σ) on boundary AD are:

The unit weight distribution equation is given by: $\gamma =$ ...

For step 4, when the research object satisfies the stress differential equilibrium equations and stress boundary conditions, the representation form of the stress equations is selected and the corresponding constant coefficients are calculated. Under two-dimensional conditions, the stress expressions are taken as:

The corresponding unit weight equation is: $\gamma =$ ...

With the Y-axis vertical, under gravity conditions, the equilibrium equations are satisfied:

The condition that each coefficient term equals zero is a necessary condition for the stress equilibrium equations. From these equations, the following relationships can be obtained:

From these relationships, we can derive:

When the unit weight is constant (γ), the corresponding solutions satisfy boundary conditions, equilibrium equations, and compatibility equations, allowing all constant coefficients to be solved. Substituting these into the stress expressions yields the corresponding stress solutions from equations (3-5).

For step 5, when analyzing the force characteristics of the research object, current strength criteria are combined to determine failure characteristics. Constitutive models are selected to study deformation characteristics, which are then compared with field observations to further clarify associated behavioral features.

3. Case Study

3.1 Computational Element Division and Equations

Based on the theoretical stress solutions obtained above, this section illustrates the method using the slope-retaining wall at the solid waste transfer station in Jinguoping Town, Badong County. A slope model is established with ADCBFHGEA from Figure 1 as the research object. Taking the stress expression from constant term to complete form yields 63 constant coefficients, which can be reduced to 33 coefficients through equilibrium equations (8-9). The corresponding constants can then be determined based on given different boundary stress conditions.

For Element I: Stress boundary conditions on DA boundary are zero, yielding 12 equations under strong constraint conditions. Stress boundary conditions on

DC boundary are:

On GE boundary, stress boundary conditions are satisfied with equation forms identical to those for AE and AD boundaries. However, EF boundary condition requires equal stress boundary conditions for sliding mass and retaining wall:

where σ represents stresses at EF boundary for sliding mass and retaining wall respectively. In this study, the retaining wall is fully connected to the foundation, meaning no failure occurs between the retaining wall and slope body, and stresses are continuous. The equilibrium equations for EFHG retaining wall are:

Horizontal force equilibrium:

Vertical force equilibrium:

where F represents horizontal and vertical forces per unit width on retaining wall (GHEF) from EF side.

Moment equilibrium equation: For Element III rotating about its axis, the moment equilibrium equation about point ( ) is:

where (X, Y) are coordinates of point D. Under strong constraint conditions, equation (16) provides 18 equations, while DC and DA boundary conditions provide 30 equations.

The slope in this study is backfilled with clay, and CF is the rock layer dip angle. According to traditional assumptions, this surface has already failed, with discontinuous tangential stress but continuous normal stress. Using the strength reduction method, the relationship between tangential and normal stresses is:

where τ and σ represent tangential and normal stresses on BC boundary respectively; c, ϕ, and f represent soil cohesion, friction angle, and strength reduction factor. Stresses on AB boundary are continuous, and the equilibrium equations for ABCDA sliding mass are:

Horizontal force equilibrium:

Vertical force equilibrium:

where W represents weight per unit width of ABCDA sliding mass.

Moment equilibrium equation: For Element I rotating about its centroidal axis (point) ( ), the moment equilibrium equation is:

where (X, Y) are coordinates of rotation point O_1. For Element II, the equations are identical to Element I, but stresses on AB boundary must be equal.

For the retaining wall (Element III), a computational model is established. Boundary conditions for the retaining wall model in Figure 1 are as follows:

Using the same method as for ADCBA, corresponding constant coefficients can be obtained.

3.2 Case Study Details

3.2.1 Project Overview The solid waste transfer station in Jinguoping Town, Badong County is located 192° from Badong County city center at a direct distance of approximately 84.2 km. The project is situated adjacent to Jinguoping town center, connected by a gravel road. The site covers an area of 1,711.50 m^2, with the waste transfer station platform at elevation 612 m, retaining wall base at elevation 611-613 m, and slope height of approximately 10 m (see Figure 2 [Figure 2: see original paper]). The right side of the waste transfer station is backfilled with red clay, forming a cross-section corresponding to profile I-I. Below the transfer station is moderately weathered sandstone of T2b2 formation with high strength (uniaxial compressive strength 40-60 MPa) and rock dip angle of 47°. The retaining wall foundation rests on a plain concrete clay cushion (C20) and moderately weathered sandstone.

3.2.2 Computational Model Based on profile I-I of the Jinguoping Town waste transfer station, a computational model is established. The backfilled clay unit weight is taken as 19.35 kN/m^3, cohesion c = 20 kPa, and friction angle $\phi = 20°$. The basic dimensions of slope body ADCFEA are: FE = 5 m, AD = 4.28 m, DC = 0.3 m, CF = 12.35 m, EA = 5.91 m. The retaining wall EFHG is C25 plain concrete with basic dimensions: EF = 5 m, FH = 4.04 m, HG = 5.26 m, GE = 0.8 m. For comparison with current methods, the finite element quadrilateral mesh models for sliding mass and retaining wall are shown in Figure 2, with computational parameters as described.

3.2.3 Model Boundary Conditions and Failure Criteria For the computational model, boundary conditions are: DC boundary stress conditions follow equation (12), AD, AE, and GE boundary conditions are:

For GH boundary:

Failure criterion: The Mohr-Coulomb criterion is adopted for stress strength, using point failure as the judgment condition. For both slope body and retaining wall, based on calculated principal stresses and using experimental friction angle values, the corresponding cohesion values are back-calculated. When the back-calculated cohesion exceeds the material test value, that point is judged to have failed. The friction angles for slope body and retaining wall are taken as:

3.2.4 Stress Calculation Results for Slope and Retaining Wall For the retaining wall, elastic modulus is 3,000 MPa and Poisson's ratio is 0.11. Based on the theoretical solution, a computational model is established to determine coordinates of all points and geometric boundary description equations. Under slope safety factor (f = 1.20) and centroidal rotation conditions, the coefficients for the sliding mass are solved and substituted into equations (3-5) to obtain stress distributions at corresponding coordinate points (see Figure 3 [Figure 3: see original paper]), principal stress distributions, and values (see Figure 4 [Figure 4: see original paper]). For the retaining wall under rotation about

F and H axes, stress distributions, principal stresses, and cohesion values are shown in Figures 5 [Figure 5: see original paper] through 8 [Figure 8: see original paper].

3.2.5 Comparative Study of Calculation Methods Given that finite element method rotation equilibrium essentially means zero moment about the centroidal axis, comparison between finite element method and the proposed method can only be made for sliding mass and retaining wall moment equilibrium calculations about the centroid. Under equal moment and boundary conditions, using ANSYS software, the results show that the difference between finite element ANSYS calculations and the proposed method is less than 8.05%, located at the retaining wall base as shear stress (see Figure 9 [Figure 9: see original paper]).

3.3 Results Analysis

3.3.1 Analysis of Slope Calculation Results Horizontal stresses in the slope are small with no tensile stresses generated. Vertical stresses are less than the product of unit weight and depth, and shear stresses are also small, less than vertical stresses. Boundary stresses are minimal and satisfy boundary conditions. Corresponding first and third principal stresses are small with no tensile stresses observed. Back-calculated cohesion values are small. Under a safety factor of 1.2, all points in the slope are basically within the strength range. Results indicate that if the slope fails first, the failure point would be at the soil-rock interface corresponding to the retaining wall height (see Figure 4).

3.3.2 Stability Analysis of Retaining Wall This study obtained solutions for retaining wall rotation about centroidal axis O_3, F axis, and H axis. Solution characteristics are:

Under centroidal axis rotation: maximum σ = 124.83 kPa, σ = 156.78 kPa, τ = 37.98 kPa; minimum values = -37.06 kPa, 0 kPa, -9.86 kPa; maximum c = 40.64 kPa, minimum = 2.442 kPa; maximum principal stresses = 164.04 kPa, 29.49 kPa; minimum = 12.72 kPa, -37.40 kPa. Maximum first and third principal stresses and c values are located at the retaining wall toe. Minimum c value occurs at the middle of the retaining wall free surface, consistent with minimum principal stress, indicating possible bulging failure in the middle, but far below C20 concrete values, making failure unlikely.

Under F-axis rotation: maximum σ = 100.59 kPa, σ = 187.15 kPa, τ = 75.05 kPa; minimum = -38.6 kPa, 0 kPa, -170.7 kPa; maximum c = 89.82 kPa, minimum = 2.443 kPa; maximum principal stresses = 314.87 kPa, 27.153 kPa; minimum = 12.70 kPa, -53.07 kPa. Maximum first and third principal stresses and c values are located at the retaining wall toe, consistent with stress distribution results.

Under H-axis rotation: maximum σ = 126.98 kPa, σ = 156.84 kPa, τ =

36.98 kPa; minimum = -36.52 kPa, 0 kPa, -9.54 kPa; maximum c = 40.26 kPa; maximum principal stresses = 162.25 kPa, 29.68 kPa; minimum = 2.44 kPa, 12.70 kPa, -36.80 kPa. Maximum first and third principal stresses and c values are located at the retaining wall toe.

These results demonstrate that different rotation points produce different stress distributions with significant variations. Maximum and minimum c values are 89.82 kPa and 2.29 kPa respectively, with minimum principal stress of -53.07 kPa, all below C20 concrete and moderately weathered sandstone values. The retaining wall cannot experience local point failure, eliminating possibility of overall failure. Years of operation demonstrate that the retaining wall is stable.

4. Conclusions

Based on the boundary conditions presented herein, a new numerical method is proposed and applied to analyze stress and strain distribution characteristics of the clay slope and retaining wall at the Jinguoping Town waste transfer station in Badong County. From the solution characteristics, the following conclusions are drawn:

1) This paper analyzes scientific issues regarding boundary conditions in numerical analysis and proposes a small deformation theoretical solution. This numerical method can consider effects of different boundary conditions and rotation axes, with calculated stresses and strains showing nonlinear relationships with coordinates.

2) The proposed numerical theoretical solution can obtain stress distributions at different points while satisfying stress differential equations, compatibility equations, and boundary conditions. For soil masses, based on experimental principal stress and principal strain constitutive characteristics, strain distributions in arbitrary directions can be obtained. Additionally, using current strength criteria, locations of initial failure can be determined, thereby deriving applications of point strength design criteria.

3) The proposed numerical method clearly demonstrates that solutions differ under various working conditions, thereby determining the most unfavorable condition. This calculation method provides a theoretical basis for anti-sliding design of retaining walls and other structures, while also providing design references for slope control and monitoring. New control and prevention methods can be derived based on different control forms and materials.

4) Comparison between numerical calculations from this method and finite element calculations shows small deviations of less than 8.05%. The case study demonstrates that the proposed method can be applied in engineering practice. This method can be extended to study dynamic and static loading/unloading analysis and failure processes of related materials for roadbeds, tunnels, dams, etc.

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Figure Captions

Figure 1. Slope and retaining wall computational model
Figure 2. Plan view, profile, and finite element mesh of retaining wall at waste landfill site
Figure 3. Stress distribution of sliding mass and retaining wall at f = 1.20
Figure 4. Principal stress and cohesion distribution of sliding mass and retaining wall at f = 1.20
Figure 5. Retaining wall stress distribution during rotation about point F
Figure 6. [Figure 6: see original paper] Principal stress and cohesion distribution during rotation about point F
Figure 7. [Figure 7: see original paper] Retaining wall stress distribution during rotation about point H
Figure 8. [Figure 8: see original paper] Principal stress and cohesion distribution during rotation about point H
Figure 9. Difference between finite element and proposed method results

Note: Figure translations are in progress. See original paper for figures.