Theoretical Method for Rockburst Prevention and Control Based on Prestressed Optimal-Axial-Ratio Elliptical Tunnels - Postprint

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Abstract

Rockburst, as a severe geological hazard, often poses significant threats to tunnel stability and construction safety. Under non-hydrostatic pressure conditions, conventional circular tunnels tend to develop localized stress concentration zones in the surrounding rock mass, which can easily induce rockburst. Elliptical tunnels with specific axial ratios have the potential to avoid this problem and have become one of the research hotspots in rockburst prevention and control. This paper analyzes the stress distribution characteristics of conventional circular and elliptical tunnels, as well as the local concentration features of tangential stress. In this context, a theoretical methodology for rockburst prevention based on prestressed elliptical tunnels with optimal axial ratio is proposed, and a theoretical solution for the optimal axial ratio of prestressed elliptical tunnels is presented. Taking a plateau tunnel as an engineering case study, numerical calculation results confirm that designing the prestressed tunnel geometry according to the optimal axial ratio can completely eliminate local tangential stress concentration, achieving a uniform distribution state and significantly reducing the overall tangential stress level of the tunnel. Theoretically, it also possesses the advantage of enabling excavation stress compensation. This work provides a theoretical reference for stress control and rockburst prevention in high-stress tunnel engineering.

Full Text

Research on the Theory and Method of Rockburst Prevention for Prestressed Elliptical Tunnels with Optimal Aspect Ratio

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Abstract

Rockbursts, as a severe geological hazard, often pose a significant threat to tunnel stability and construction safety. Traditional circular tunnels under non-hydrostatic pressure conditions are prone to forming localized stress concentration zones in surrounding rock, which can easily trigger rockbursts. In contrast, elliptical tunnels with specific axis ratios have the potential to avoid this issue, making them a focal point in rockburst prevention research.

This paper analyzes the stress distribution and localized tangential stress concentration characteristics in conventional circular and elliptical tunnels. Based on this analysis, a theoretical method for rockburst prevention using prestressed elliptical tunnels with optimal aspect ratio is proposed, and a theoretical solution for determining this optimal axis ratio is derived. Using a high-altitude tunnel as a case study, numerical results confirm that designing the prestressed tunnel shape according to the optimal aspect ratio can completely eliminate localized tangential stress concentration, achieve a uniformly distributed stress state, and significantly reduce the overall tangential stress level of the tunnel. Theoretically, this approach also offers the advantage of achieving excavation stress compensation. This provides a theoretical reference for stress control and rockburst prevention in high-stress tunnel engineering.

Keywords: high-stress tunnels; rockburst prevention; tunnel cross-section shape; optimization design

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1. Introduction

Rockburst, as a severe geological hazard, typically poses significant threats to tunnel stability and construction safety, severely impacting production, damaging equipment, and potentially causing casualties [1]. The occurrence of rockbursts is closely related to stress redistribution after excavation, surrounding rock strength, and rock brittleness. In hard and brittle rock formations, tunnel shape and surrounding stress distribution critically influence rockburst initiation. Under non-hydrostatic pressure conditions, the tangential stress σ distribution around conventional circular tunnels is non-uniform, particularly under strong tectonic conditions where pronounced relative stress concentration zones develop at the tunnel periphery. These stress concentration zones typically reach plasticity and damage first, becoming potential rockburst sources. Elliptical tunnels have become a research hotspot for rockburst prevention due to their favorable stress distribution characteristics [2].

Numerous scholars have investigated surrounding rock failure in elliptical tunnels. Guo Jiaqi et al. [3] analyzed plastic zone distribution around elliptical tunnel openings using complex variable function methods and applied this to stability evaluation of karst tunnels and rock pillars in small-scale concealed karst cavities. Li Guichen et al. [4] noted that for high-stress roadways, appropriate circular or elliptical tunnel cross-section shapes should be selected based on stress values and principal stress directions, with elliptical shapes generally better suited to high-stress environments. Ma Depeng et al. [5] comprehensively analyzed energy release characteristics, stress distribution, and plastic zone patterns in surrounding rock, examining the influence of horizontal tectonic stress on tunnel stress states. Their results demonstrated that elliptical tunnels exhibit superior post-excavation mechanical behavior with the lowest risk of rockburst. Fan Jiyue et al. [6], considering the interaction between lining and surrounding rock, indicated that when the major-to-minor axis ratio of elliptical tunnels falls between certain values, the lining experiences relatively reasonable stress distribution with high structural safety. Yu Xuefu [7,8] pointed out that elliptical tunnels can alleviate surrounding rock pressure, and optimizing traditional circular tunnel shapes to elliptical forms can effectively mitigate stress concentration, achieve more uniform stress distribution, and reduce rockburst risk. Moreover, rockburst-prone tunnels typically involve hard brittle rock characterized by failure at small deformations. He Manchao [9] argued that tangential stress around tunnels primarily originates from stress redistribution caused by excavation unloading. Timely stress compensation after tunnel excavation can reduce the magnitude of stress adjustment, thereby effectively decreasing tangential stress formation.

Therefore, combining two measures—optimized tunnel shape design and prestressed support—can provide internal pressure through prestressed support while calculating the elliptical tunnel axis ratio that yields uniform tangential stress distribution based on internal pressure and lateral pressure coefficient. This approach enables determination of the optimal tunnel cross-section shape according to geological conditions, further improving tunnel stability and reducing rockburst risk. This study theoretically proposes this method and discusses its feasibility, particularly for special tunnels in strong tectonic regions such as shafts, ventilation tunnels, water diversion tunnels, and even some traffic tunnels with less stringent clearance requirements. To this end, we examine the tangential stress distribution and local stress concentration in circular tunnels, circular prestressed support tunnels, and elliptical tunnels with optimal axis ratio based on "axis transformation theory." Based on this analysis, we present the optimal shape for prestressed elliptical tunnels and investigate the advantages of this method and its effectiveness in eliminating localized high stress concentration.

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2. Stress Distribution in Conventional Circular and Elliptical Tunnels

This study addresses linear tunnel engineering where length far exceeds width and height, satisfying the plane strain assumption. Considering that tunnel siting must favor safety and stability, construction typically occurs in intact, non-fractured, and good-quality rock masses with strong brittle characteristics. Therefore, surrounding rock can be assumed as a homogeneous, isotropic elastic body approximated using elastic theory. To ensure tunnel stability, the tunnel axis direction is assumed to align with the maximum horizontal principal stress, with tunnel cross-section stresses being the minimum horizontal principal stress and vertical stress. Furthermore, prestressed rock bolts are assumed to be uniformly and densely distributed along the tunnel cross-section, with each bolt applying equal prestress, which is simplified as uniform internal pressure acting on the tunnel periphery in theoretical calculations.

2.1 Local Stress Distribution in Unreinforced Circular Tunnels After tunnel excavation, rock mass disturbance decreases with distance from the excavation face, gradually approaching the in-situ stress state. During construction, the tunnel wall is the most failure-prone region and thus requires special attention. For the most widely used circular tunnels without active prestressed support, assuming a circular tunnel radius R_0, rock mass elastic modulus E, Poisson's ratio , vertical stress p_0, and lateral pressure coefficient λ (assuming $\lambda > 1$), the secondary stress distribution in surrounding rock has been extensively studied. Under elastic conditions, the tangential stress σ (where is defined as the angle between a peripheral point and the tunnel crown, positive clockwise) and radial stress σr are expressed as [10]:

$$\sigma = p_0(1 + \lambda) + 2p_0(1 - \lambda)\cos 2 \quad \sigma r = 0$$

Analysis shows that σ reaches maximum and minimum values at the crown and sidewall respectively, with a difference of:

$$\Delta\sigma = 4p_0(\lambda - 1)$$

This demonstrates that under non-prestressed conditions, tangential stress distribution around circular tunnel walls is non-uniform. The difference between maximum and minimum values is $4p_0(\lambda - 1)$. The greater λ deviates from 1, the larger this difference becomes, leading to more significant stress concentration at the crown and invert. The higher σ at the crown may satisfy high stress concentration conditions, causing the crown and invert to become the first compressive stress damage zones and potential rockburst sources.

2.2 Stress Distribution in Prestressed Circular Tunnels For prestressed circular tunnels, assuming uniformly distributed prestress acting perpendicular to the tunnel wall with magnitude ps, the boundary conditions yield:

$$\sigma = p_0(1 + \lambda) + 2p_0(1 - \lambda)\cos 2 - ps \ \sigma r = ps \ \tau r = 0$$

Equation (4) shows that σ values decrease to some extent after prestress support application, demonstrating certain effectiveness in reducing high stress concentration at the crown and invert. σ still reaches maximum and minimum values at the crown and sidewall: $p_0(3\lambda - 1)$ and $p_0(3 - \lambda)$ respectively. The difference $\Delta\sigma$ remains $4p_0(\lambda - 1)$, indicating that the relative difference between stresses remains unchanged. Moreover, this approach further increases tensile stress risks at the sidewalls. Since rock typically has very weak tensile strength, tensile stress occurrence is extremely detrimental to tunnel stability and safety. For $\lambda > 3$, tensile stress appears at the sidewall even without prestress; for $1 < \lambda \leq 3$, compressive stress at the sidewall is minimal and decreases with increasing λ. Therefore, while circular active prestress support can reduce highly concentrated maximum tangential stress to some degree, it does not affect the difference between maximum and minimum tangential stress and increases the risk of tensile stress at minimum stress locations.

2.3 Stress Distribution in Unreinforced Elliptical Tunnels with Optimal Aspect Ratio Optimizing tunnel shape to adjust the stress field is an economical and efficient approach, particularly suitable for non-traffic openings without cross-sectional clearance requirements. This study can draw upon "axis transformation theory," focusing more on reducing maximum compressive stress and weakening stress concentration effects while ensuring no tensile stress occurs in the tunnel. Assuming an elliptical tunnel with horizontal semi-axis a and vertical semi-axis b, aspect ratio $m = b/a$, and lateral pressure coefficient $\lambda > 1$.

The stress distribution around elliptical tunnels without prestressed support is [7]:

$$\sigma = p_0[(m^2\sin^2 + \cos^2) + \lambda(m^2\cos^2 + \sin^2)] / (m^2\sin^2 + \cos^2)$$

When designed with aspect ratio $m = 1/\lambda$, the elliptical tunnel periphery experiences not only zero tensile stress but also uniform compressive stress distribution. This state is most favorable for roadway stability, and this aspect ratio is defined as the optimal ratio [8].

The uniformly distributed compressive stress is:

$\sigma = p_0(1 + \lambda/m)$

The difference between the maximum tangential stress in circular tunnels $\sigma = p_0(3\lambda - 1)$ and this value is $2(\lambda - 1)p_0 > 0$, indicating that the uniformly distributed tangential stress under optimal aspect ratio reduces the maximum compressive stress concentration region compared to circular tunnels by $2(\lambda - 1)p_0$. The difference between Equation (10) and the minimum tangential stress in circular tunnels $\sigma = p_0(3 - \lambda)$ is $2(2\lambda - 1)p_0 > 0$, showing this value increases compared to the circular tunnel minimum, reducing the risk of localized tensile stress. Thus, the obtained uniform stress distribution eliminates stress concentration, but the stress reduction magnitude is limited. According to Equation (10), when λ and p_0 are large, σ remains substantial. Furthermore, without active prestressed support, excavation stress cannot be compensated, making it difficult to control excavation face deformation. Rockbursts may occur before significant surrounding rock deformation, particularly in tunnels with high rockburst risk.

All above formulas use $\lambda > 1$ as an example, though related theories and principles apply to cases where $\lambda < 1$ as well.

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3. Prestressed Elliptical Tunnel with Optimal Aspect Ratio

3.1 Calculation Method for Optimal Aspect Ratio of Prestressed Elliptical Tunnels Analysis of the three methods above reveals that none can simultaneously satisfy multiple requirements: eliminating localized stress concentration at tunnel walls, comprehensively reducing stress levels, achieving stress compensation, and resolving the contradiction between passive support and rockburst potential. Therefore, their effectiveness is limited. The design concept for prestressed elliptical tunnels with optimal aspect ratio can combine advantages of both unreinforced elliptical tunnels with optimal aspect ratio and circular prestressed support, potentially meeting all these requirements. This approach can overcome static rockburst conditions and, through prestress compensation forming internal pressure support, anchor rock masses that might experience elastic rebound, controlling dynamic rockburst conditions.

In a previous patent [11], the authors used theoretical analysis to derive the optimal aspect ratio m for elliptical tunnels under combined external and internal pressure:

$m = (p_0\lambda - ps) / (p_0 - ps)$

Elliptical shapes calculated using this aspect ratio yield uniform tangential stress around the tunnel periphery, all compressive. This formula provides the optimal aspect ratio for prestressed elliptical tunnels. Designing tunnels according to this ratio produces shapes closest to uniform stress distribution, maximizing stress concentration elimination.

3.2 Numerical Validation of Effectiveness The effectiveness of the proposed prestressed elliptical tunnel design method and its advantages compared to three alternative methods can be directly demonstrated through engineering case studies. Numerical computation provides an effective validation approach. Different tunnel design methods can be input into numerical software to calculate their respective plastic zones, which can then be compared in terms of range and magnitude to verify each method's effectiveness in mitigating localized stress concentration. Plastic zones form due to localized stress concentration in tunnels; rock mass within these zones will experience damage and failure. In rockburst development, the first-appearing plastic zones may become subsequent rockburst sources. Therefore, observing the presence or absence of plastic zones in numerical results for hard rock engineering helps directly determine whether conditions exist for localized stress concentration that could constitute rockburst potential. If no plastic zone exists in a particular case, it indicates that the tunnel wall is unlikely to form subsequent rockburst sources.

Combining the background, a finite element model considering elastoplastic rock mass characteristics was established to simulate four cases: conventional circular tunnel, elliptical tunnel without prestress and with optimal aspect ratio, conventional circular tunnel with prestress, and elliptical tunnel with prestress and optimal aspect ratio determined by the above design method. The differences in plastic zones among these three cross-section designs were compared to evaluate the engineering effectiveness of the proposed method. Using a plateau railway tunnel [12,13] as an example for trial calculations, the tunnel site is located at the great bend of the Yarlung Zangbo River. At a burial depth of 1221 m, measured in-situ stresses are SH = 33.58 MPa, Sh = 31.71 MPa, and Sv = 25.70 MPa. Assuming the most favorable condition where tunnel axis aligns with SH direction, the vertical stress on the tunnel cross-section is p_0 = 25.70 MPa, horizontal stress is λp_0 = 31.71 MPa, giving λ = 1.234. Assuming a circular tunnel radius R_0 = 6 m (note: aspect ratio m is independent of R_0 according to Equation (11)), the cross-sectional area S = 113.04 m^2. Under the condition of constant cross-sectional area, specific dimensions for each case are shown in Table 1 . The elliptical internal pressure support value uses ps = 10 MPa as a representative case.

Table 1 Tunnel dimensions for four simulation cases

Case	Active support internal pressure ps (MPa)	(Optimal) aspect ratio m	Horizontal semi-axis a (m)	Vertical semi-axis b (m)
Unreinforced circular tunnel	-	-	6.00	6.00
Unreinforced elliptical tunnel (optimal ratio)	-	0.81	6.67	5.40
Prestressed circular tunnel	10	-	6.00	6.00
Prestressed elliptical tunnel (optimal ratio)	10	0.31	10.79	3.33

The tunnel is located in diorite rock, with basic physical and mechanical parameters shown in Table 2 . A Mohr-Coulomb elastoplastic constitutive model was adopted to consider elastoplastic characteristics. The lateral pressure coefficient and in-situ stresses were consistent with measured data. Computational steps included: initial in-situ stress equilibrium, tunnel excavation, and application of prestressed internal pressure support. The model was then used to calculate the cumulative plastic deformation zones for the three cross-section design methods.

Table 2 Basic characteristics and physical-mechanical parameters of rock mass [12]

Parameter	Value
Unit weight γ (kN/m^3)	26.5
Cohesion c (MPa)	15.2
Internal friction angle ϕ (°)	42.0
Elastic modulus E (GPa)	40.0
Poisson's ratio	0.25

The cumulative plastic deformation, reflecting the accumulated plastic zone results, is shown in Figure 1 [Figure 1: see original paper]. In Figure 1(a), the conventional circular tunnel without prestressed support shows plastic deformation around the entire tunnel periphery, indicating rock mass damage, most pronounced near the crown and invert, which are most likely to become high stress concentration zones and potential rockburst sources. In Figure 1(b), the elliptical tunnel with optimal aspect ratio but without prestressed support also shows plastic zones around the periphery, indicating that although stress distribution is very uniform, high stress levels still cause plastic zones throughout the tunnel periphery due to the high stress magnitude. Compared to Figure 1(a), the plastic zone in Figure 1(b) is more uniform with smaller values, with concentration areas at the crown, invert, and sidewalls, consistent with the uniform stress distribution. Therefore, while unreinforced elliptical tunnels with optimal aspect ratio eliminate stress concentration, the overall stress level remains high, easily forming uniformly distributed plastic zones that could still become rockburst sources. In Figure 1(c), after applying 10 MPa prestress to a conventional circular tunnel, the plastic zone significantly decreases while the surrounding stress level also reduces. Prestress application effectively mitigates local stress concentration, reduces plastic deformation range, creates more uniform stress distribution, and lowers rockburst risk. In Figure 1(d), the prestressed elliptical tunnel with optimal aspect ratio shows no plastic deformation throughout the process due to uniform and low stress levels around the tunnel periphery, with rock mass maintaining good original condition. This demonstrates that the design method effectively avoids premature damage in local tunnel regions, and under these secondary stress conditions, no location will become a rockburst source due to strain concentration and prior damage. Thus, the proposed theoretical method for rockburst prevention based on prestressed elliptical tunnels with optimal aspect ratio is validated as effective in solving the problem of high stress concentration in local surrounding rock, providing an optimized design approach for rockburst prevention.

3.3 Discussion on Rockburst Prevention Using Prestressed Elliptical Support The significance of the optimized design method based on prestressed elliptical tunnels with optimal aspect ratio lies in its ability to ensure low and uniformly distributed overall stress levels, eliminating localized high stress concentration conditions for rockburst and preventing any region around the tunnel from becoming potential rockburst sources. In engineering practice, particularly in tectonically active projects where secondary stresses become more concentrated in local regions under tectonic influence, this method becomes more necessary. Additionally, introducing prestressed internal pressure support enables active force application to surrounding rock immediately after excavation, providing stress compensation and resolving the dilemma where passive support requires large rock deformation to develop anchoring force. This avoids the contradiction between rockburst occurrence at small deformations in hard brittle rock and passive support requiring large displacements to generate support force. Therefore, elliptical prestressed support can eliminate localized stress concentration at tunnel walls, comprehensively reduce stress levels, achieve stress compensation, and resolve the contradiction between passive support and rockburst potential, offering a promising theoretical method for rockburst prevention.

Elliptical tunnels calculated using this method can guarantee elimination of stress concentration while maintaining constant cross-sectional area. Primary applications include ventilation tunnels/airways, mine shafts, water diversion tunnels, and traffic tunnels with less stringent clearance requirements. Moreover, mature smooth blasting technology can precisely control tunnel shape without significantly damaging surrounding rock [14], solving construction challenges for elliptical tunnels. Due to construction feasibility, elliptical tunnels have been applied in the underground engineering cross-section design study of the Three Gorges Project [7,10] and in the U1 section of Canada's URL tunnel [15].

From the perspective of active rockburst control, this study proposes a theoretical method for rockburst prevention based on prestressed elliptical tunnels with optimal aspect ratio and validates it through numerical simulation, reaching the following conclusions:

1. Under high in-situ stress conditions where the lateral pressure coefficient deviates from 1, unreinforced circular tunnels, prestressed circular tunnels, and unreinforced elliptical tunnels with optimal aspect ratio exhibit problems of localized tangential stress concentration at tunnel walls and high overall tangential stress levels, creating favorable conditions for rockburst occurrence.
2. The proposed theoretical method for rockburst prevention based on prestressed elliptical tunnels with optimal aspect ratio can precisely calculate tunnel shape based on in-situ stress and prestress. Its effectiveness enables uniform tangential stress distribution around the tunnel periphery while significantly reducing overall tangential stress levels, overcoming rockburst

conditions caused by localized high stress concentration and facilitating stress compensation.

3. Numerical simulation of the engineering case validates the proposed method's effectiveness. Compared with circular tunnels and unreinforced elliptical tunnels, only the prestressed elliptical tunnel with optimal aspect ratio shows no plastic zone in surrounding rock caused by stress concentration during excavation, keeping the rock mass in an elastic state.

The proposed method demonstrates significant theoretical effectiveness, though its assumptions remain somewhat idealized, requiring further research before engineering application.

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