Neutron Magic Numbers in sd Shell from Nuclear Charge Radii within Relativistic Hartree Bogoliubov Model

Authors: Rong, Dr. Yu-Ting, Liu, Mr. Ping-Mo, Yang, Ms. Dan, Rong, Dr. An, Rong, Dr. Yu-Ting

Date: 2025-07-17T15:31:27+00:00

Abstract

Charge radii are sensitive indicators to identify the nuclear structure phenomena throughout the whole nuclide chart. In particular, the shrunken trend of changes of charge radii along a long isotopic chain is intimately associated with the shell quenching effect. In this work, the systematic evolution of charge radii along the proton numbers $Z = 8$, 10, 12, 14, 18 isotopes is investigated by a relativistic Hartree Bogoliubov model. A ansatz about neutron-proton correlation around Fermi surface is considered for describing the abnormal behavior of nuclear charge radii. Our results show that the neutron-proton pairing corrections around the Fermi surface lead to a sudden strengthening of the charge radii of these isotopic chains at $N = 8$, 20 and 28, reflecting the fact that this correction enhances the shell closure across $N = 8$, 20 and 28. The reproduction of the $N = 14$ charge radius in the Mg isotopes is affected by the way in which pairing correlations are handled, with BCS theory overestimating the shell effect of $N = 14$, and the Bogoliubov quasiparticle transformation suggests a stronger pairing correlation near the proton Fermi surface, which is more consistent with experimental results. An analysis of the deviations from the theoretical and available experimental data for the charge radii of the 24 selected even-even nuclei shows that the neutron-proton pairing correction around the Fermi surface has an improved effect on the calculation of the charge radii using the meson-exchange effective interactions, but it does not help to significantly improve the results calculated by the density-dependent effective interactions.

Full Text

Preamble

Neutron Magic Numbers in the sd Shell from Nuclear Charge Radii within the Relativistic Hartree-Bogoliubov Model

Yu-Ting Rong,† Ping-Mo Liu, and Dan Yang
Department of Physics, Guangxi Normal University, Guilin 541004, China
Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China

Rong An‡
School of Physics, Ningxia University, Yinchuan 750021, China
Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China
Key Laboratory of Beam Technology of Ministry of Education, School of Physics and Astronomy, Beijing Normal University, Beijing 100875, China

Charge radii serve as sensitive probes for identifying nuclear structure phenomena across the entire nuclide chart. In particular, the contracted trend in charge radius evolution along long isotopic chains is intimately associated with shell quenching effects. In this work, we investigate the systematic evolution of charge radii along isotopic chains with proton numbers Z = 8, 10, 12, 14, and 18 using a relativistic Hartree-Bogoliubov model. An ansatz regarding neutron-proton correlations around the Fermi surface is introduced to describe the anomalous behavior of nuclear charge radii. Our results demonstrate that neutron-proton pairing corrections near the Fermi surface lead to sudden enhancements in the charge radii of these isotopic chains at N = 8, 20, and 28, reflecting that these corrections strengthen shell closure across these neutron numbers. The reproduction of the N = 14 charge radius in Mg isotopes is sensitive to the treatment of pairing correlations: BCS theory overestimates the N = 14 shell effect, while the Bogoliubov quasiparticle transformation suggests stronger pairing correlations near the proton Fermi surface, yielding better agreement with experimental results.

An analysis of deviations between theoretical predictions and available experimental data for the charge radii of 24 selected even-even nuclei reveals that neutron-proton pairing corrections around the Fermi surface improve charge radius calculations using meson-exchange effective interactions, but do not significantly enhance results obtained with density-dependent effective interactions.

INTRODUCTION

Shell structure represents a distinctive feature of nuclear many-body systems, characterized by the existence of proton and neutron magic numbers. These magic numbers reveal the chemical stability and intrinsic structure of atomic nuclei, holding profound significance for understanding nuclear physics, nucleosynthesis, and practical applications. Magic nuclei manifest through various phenomena, including sudden increases in binding energy [?], high excitation energies, low quadrupole transition probabilities [?], localized abrupt changes in charge radius [?, ?] and proton radius [?, ?], and reduced neutron/proton capture cross sections [?] compared to neighboring nuclei. Modern radioactive beam facilities have extended studies to exotic nuclei, potentially revealing new magic numbers or revising traditional shell models [?].

The systematic evolution of bulk properties from oxygen to argon isotopic chains contains rich information about shell structure, particularly regarding the disappearance of traditional magic numbers and the emergence of new ones. Along the oxygen isotopes, the high excitation energy and low B(E2) value of the 2^+_1 state [?, ?], small quadrupole deformation parameter β_2 [?], reduced proton radius [?], large inclusive cross sections, and broad momentum distributions from quasifree (p, pN) scattering [?] provide strong evidence for an N = 14 subshell closure in ^{22}O. Additionally, a large shell gap is clearly identified for N = 16 in ^{24}O from various measurements [?, ?, ?], establishing ^{24}O as a doubly magic nucleus. Mass, charge radius, and Coulomb excitation measurements of neutron-rich Ne, Na, and Mg isotopes [?, ?] have suggested the breakdown of the traditional magic number N = 20 [?]. At higher masses, the observation of a low-lying 2^+ state in ^{42}Si provides transparent evidence for the collapse of the N = 28 shell closure [?].

Numerous nuclear structure models have been employed to unveil the underlying mechanisms governing the emergence of new magicity and shell quenching phenomena. Shell model calculations attribute the appearance of new magic numbers N = 14 and N = 16 to strong neutron-proton tensor interactions [?, ?, ?, ?]. Ab initio calculations with modern two- and three-nucleon forces achieve considerable improvement in simultaneously describing binding energies, charge radii, and matter radii for stable oxygen isotopes, though deficiencies persist for the most neutron-rich systems [?]. The collapse of the N = 20 shell closure is attributed to population of the neutron pf shell in the presence of sd orbitals at substantial prolate deformation, a phenomenon known as the island of inversion [?, ?]. Coupled-cluster methods based on nucleon-nucleon and three-nucleon potentials qualitatively reproduce evolutionary trends in charge radii and the N = 14 neutron number in Ne and Mg isotopes after angular momentum projection, though isotope shifts remain challenging [?]. In mean-field theories, the ground state of ^{32}Mg is spherical and only becomes deformed after increasing the spin-orbit strength by 20% based on the Skyrme SLy4 force [?]. While deformed ground states can be achieved by adjusting neutron and proton pairing gaps, the magic number N = 20 persists for the ground state of ^{32}Mg in relativistic mean-field (RMF) theory [?]. Angular momentum projection approaches based on the HFB model [?, ?] and RMF model [?] transform the spherical mean-field ground state of ^{32}Mg into a deformed state with β_2 close to the measured value. A similar picture emerges from the projected shell model [?].

As noted above, the contracted trend in charge radius evolution along an isotopic chain serves as a signature for identifying shell closure effects. Charge radii are influenced by various mechanisms, including pairing correlations [?], deformation [?, ?], cluster structure [?, ?], shell evolution [?], and center-of-mass correlations [?]. The modified RMF plus BCS equation ansatz (RMF(BCS)* model), which incorporates neutron-proton pairing correlations around the Fermi surface into the charge radius formula [?, ?], successfully describes odd-even staggering and inverted parabolic behavior. This method provides a good description of charge radii for most O, Ne, and Mg isotopes. However, it predicts odd-even staggering in the O isotopic chain with a sudden increase at N = 14, underestimates charge radii for N = 14 and N > 18 nuclei in Mg isotopes, and overestimates the charge radius of ^{18}Ne (N = 8). Since the BCS approximation is unsuitable for nuclei far from the β-stability line [?, ?], and the magic numbers 14, 16, and 20 in the sd shell appear in neutron-rich regions and even at the drip line, it is necessary to further investigate charge radii using the same ansatz but with a more appropriate treatment of pairing correlations.

The Bogoliubov transformation is considered superior to the BCS method for treating pairing correlations in nuclei far from β-stability. In this context, we recently introduced neutron-proton pairing correlations extracted from quasiparticle states around the Fermi surface into the multidimensionally-constrained relativistic Hartree-Bogoliubov (MDC-RHB) model [?]. This approach successfully reproduces charge radii of Ca and Ni isotopes and N = 28, 30, 32, and 34 isotones. Therefore, in this work, we revisit the problems encountered in the RMF(BCS)* approach and examine the shell closure effects around neutron numbers N = 14, 16, and 20 through the lens of nuclear charge radii, elucidating the relationship between shell structure in these isotopes and neutron-proton pairing.

This paper is organized as follows. In Sec. II, we briefly introduce the MDC-RHB model and the neutron-proton pairing correlation extracted from quasiparticle states around the Fermi surface. In Sec. III, we investigate charge radii for Z = 8, 10, 12, 14, and 18 isotopes using this method and discuss shell structure phenomena around N = 14, 16, and 20. A summary is presented in Sec. IV.

THEORETICAL FRAMEWORK

In the MDC-RHB model, the RHB equation in coordinate space can be written as follows [?, ?]:

$$\begin{pmatrix} h-\lambda & -\Delta^* \\ \Delta & -h+\lambda \end{pmatrix} \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

where λ is the Fermi energy, Δ is the pairing tensor field, E_k is the quasiparticle energy, and $(U_k(r), V_k(r))^T$ is the quasiparticle wave function. The single-particle Hamiltonian h can be expressed as:

$$h = \alpha \cdot p + \beta[M + S(r)] + V(r) + \Sigma_R(r)$$

where M is the nucleon mass, and $S(r)$, $V(r)$, and $\Sigma_R(r)$ are the scalar, vector, and rearrangement potentials, respectively.

For meson-exchange interactions:

$$S(r) = g_\sigma \sigma, \quad V(r) = g_\omega \omega_0 + g_\rho \rho_0 \cdot \tau_3 + e\frac{1-\tau_3}{2}A_0, \quad \Sigma_R(r) = \frac{\partial \alpha_S}{\partial \rho_S}\rho_S + \frac{\partial \alpha_V}{\partial \rho_V}\omega_0 + \frac{\partial \alpha_{TV}}{\partial \rho_{TV}}\rho_0$$

where g_σ, g_ω, and g_ρ are coupling constants of σ, ω_0, and ρ_0 meson fields, A_0 is the time-like component of the Coulomb field mediated by photons, e is the charge unit for protons, and $\tau_3 = \pm 1$ distinguishes neutrons from protons. ρ_S and ρ_V are the isoscalar and isovector densities, respectively.

For point-coupling interactions:

$$S(r) = \alpha_S \rho_S + \alpha_{TS}\rho_{TS}\tau_3 + \beta_S \rho_S^2 + \delta_S \Delta \rho_S + \delta_{TS}\Delta \rho_{TS}\tau_3$$

$$V(r) = \alpha_V \rho_V + \alpha_{TV}\rho_{TV}\tau_3 + \gamma_V \rho_V^3 + \gamma_S \rho_S^3 + \delta_V \Delta \rho_V + \delta_{TV}\Delta \rho_{TV}\tau_3 + e\frac{1-\tau_3}{2}A_0$$

$$\Sigma_R(r) = \frac{\partial \alpha_{TV}}{\partial \rho_{TV}}\rho_{TV}$$

where α_S, α_V, α_{TS}, α_{TV}, β_S, γ_S, γ_V, δ_S, δ_V, δ_{TS}, and δ_{TV} are coupling constants for different channels, and ρ_{TS} and ρ_{TV} are time-like components of isoscalar and isovector currents, respectively.

The pairing field Δ is calculated from the effective pairing interaction V and the pairing tensor κ as:

$$\Delta(r_1\sigma_1, r_2\sigma_2) = \int d^3r_1' d^3r_2' \sum_{\sigma_1'\sigma_2'} V(r_1\sigma_1, r_2\sigma_2, r_1'\sigma_1', r_2'\sigma_2')\kappa(r_1'\sigma_1', r_2'\sigma_2')$$

In this work, we adopt a separable pairing force of finite range with pairing strength $G = 728$ MeV $\cdot$ fm^3 and effective range $a = 0.644$ fm [**?**].

The modified root-mean-square (rms) charge radius r_{ch} is given by [**?**]:

$$r_{ch}^2 = \langle r_p^2 \rangle + 0.7056 \text{ fm}^2 + \Delta_D \text{ fm}^2 + \frac{a_0}{A^\delta}$$

where the first term represents the charge distribution of point-like protons, the second term accounts for the finite size of protons, A is the mass number, and a_0 is a normalization constant. The term $\Delta_D = |D_n - D_p|$ measures neutron-proton correlations around the Fermi surface:

$$D_{n,p} = \sum_k u^k_{n,p} v^k_{n,p}$$

where $u^k_{n,p}$ and $v^k_{n,p}$ are the occupation amplitudes of the kth quasiparticle orbital for neutrons or protons, with $v^{k^2}_{n,p} = 1 - u^{k^2}_{n,p}$. In practice, quasiparticle levels satisfying $|E_k - \lambda| < 20$ MeV are included in the sum. The parameters $a_0 = 0.561$ and $\delta = 0.355$ (0.000 for even-even, odd-even, and even-odd nuclei) are taken from Ref. [?].

The MDC-RHB model allows for multipole moments under V4 symmetry [?, ?]. In this work, we restrict calculations to axial and reflection symmetries, considering only quadrupole deformation β_{20}. To avoid parameter-dependent biases, we employ the effective interactions PK1 [?], NL3 [?], DD-ME2 [?], and DD-PC1 [?].

RESULTS AND DISCUSSION

As mentioned in the introduction, N = 14 and 16 shell effects are observed in O, N, Ne, and Mg isotopes. To investigate these characteristic magicities, we calculate the systematic evolution of charge radii along O, Ne, Mg, Si, and Ar isotopic chains using the MDC-RHB model. For clarity, results obtained with and without neutron-proton correlations around the Fermi surface are denoted as RHB* and RHB models, respectively.

Figure 1 [Figure 1: see original paper] shows charge radii for O, Ne, Mg, Si, and Ar isotopic chains calculated with the DD-ME2, NL3, DD-PC1, and PK1 effective interactions. RHB calculations yield charge radii for $^{16-22}$O of comparable magnitude, with the PK1 force providing the most accurate estimate for ^{16}O. However, all four forces underestimate the charge radius of ^{18}O. As shown in Fig. 1(b), the RHB* model suppresses the systematic trend of charge radius changes at N = 8 and 16, implying enhanced shell quenching at these neutron numbers after implementing the neutron-proton correction. This correction increases the charge radius of ^{18}O, improving its description by the PK1 and NL3 parameter sets. The double-magic nature of ^{16}O largely eliminates surface proton-neutron correlations, leaving its charge radius unchanged by the correction.

Charge radii for Ne isotopes are shown in Figs. 1(c) and 1(d). RHB results align with the experimental isotopic evolution trend, with density-dependent interactions (DD-ME2 and DD-PC1) yielding values closer to experiment, while meson-exchange interactions (NL3 and PK1) slightly underestimate experimental values. Kinks in charge radii appear at N = 8 and N = 14 shell closures. After including the neutron-proton surface pairing correction, charge radii increase, and those calculated with meson-exchange interactions better match experimental data. The smallest charge radius at N = 14 is predicted by DD-ME2, NL3, and PK1, while DD-PC1 gives the smallest value at N = 16. Furthermore, the

RHB model predicts no abrupt change in charge radii across N = 20, whereas the RHB* model anticipates a sudden increase in the charge radius of ^{32}Ne due to proton-neutron surface pairing, thereby manifesting the magicity of N = 20. Notably, the RHB* model performs suboptimally on the proton-rich side, overestimating the charge radius of ^{18}Ne.

For the Mg isotopic chain (Figs. 1(e) and 1(f)), RHB calculations agree well with experimental values for proton-rich isotopes but underestimate charge radii of neutron-rich isotopes, exhibiting kinks at N = 14 and 20. The Fermi-surface correlation between protons and neutrons enhances charge radii for isotopes with neutron numbers greater than 14, yielding RHB* calculations in better agreement with experiment than RHB calculations. Thus, neutron-proton surface pairing vibrations may underlie the observed charge radius changes on the neutron-rich side of Mg isotopes. Compared to Ref. [**?**], which showed excessive charge radius depression at N = 14 using the same correction method, the RHB* model with NL3 and PK1 interactions yields charge radii of 3.0434 fm and 3.0415 fm, respectively, very close to the experimental value of 3.0340(26) fm [**?**]. This indicates that characterizing N = 14 magicity significantly influences the treatment of correlations, necessitating investigation of the pairing force form within the same mean-field Hamiltonian and thorough analysis of the microscopic mechanism producing these results.

Charge radii of Si isotopes are displayed in Figs. 1(g) and 1(h). The RHB model predicts a larger charge radius for ^{28}Si (N = 14) than for ^{30}Si (N = 16), contrary to experimental data. Including neutron-proton correlations around the Fermi surface brings the charge radii of these nuclei into good agreement with experiment, with PK1 and NL3 interactions giving theoretical values comparable to experimental ones. More pronounced kinks appear at N = 20 and 28 after considering the correction term, indicating enhanced shell closures at these neutron numbers.

Figures 1(i) and 1(j) show charge radii of Ar isotopes calculated by RHB and RHB* models, respectively. RHB calculations exhibit a parabolic dependence on neutron number, showing larger deviations from experimental results. RHB* calculations are generally in good agreement with experiment on the neutron-rich side, except for N = 22 where the charge radius is somewhat underestimated. On the proton-rich side, the charge radius of the N = 14 isotope deviates significantly from experiment after including the correction term.

As noted above, local variations in nuclear charge radii are influenced by the treatment of pairing correlations. Figure 2 [Figure 2: see original paper] shows charge radii along the Mg isotopic chain calculated using BCS and Bogoliubov quasiparticle transformations, respectively, with the same single-particle Hamiltonian h. Given the concordance of corrected charge radii with experimental values, these calculations employ the NL3 effective interaction. As shown in Fig. 2(a), the charge radius determined by BCS pairing correlation exhibits a pronounced trough at ^{26}Mg, both before and after neutron-proton surface correction, suggesting strong N = 14 magicity as reported in the literature [**?**]. In

Fig. 2(b), applying the Bogoliubov quasiparticle transformation to treat pair correlation reveals that the charge radii of $^{24-29}$Mg after neutron-proton correction show strong agreement with experimental data, demonstrating that this correction facilitates reproduction of the charge radius at N = 14.

To understand the N = 14 magicity and its microscopic mechanism, we analyze the depression of the charge radius and the effect of surface pairing corrections through the single-particle energy levels of ^{26}Mg in Fig. 3 [Figure 3: see original paper]. Figures 3(a) and 3(b) show neutron and proton single-particle levels calculated using the RMF+BCS method. These figures reveal that neutron and proton single-particle levels are predominantly occupied below the Fermi surface, with negligible occupation above it, indicating relatively small pairing gaps for both N = 14 and Z = 12. This suggests enhanced nuclear stability and small charge radii. According to Eq. (6), neutron-proton pairing correlation around the Fermi surface is determined by Δ_D, which in turn depends on non-integer occupation numbers of protons and neutrons. BCS calculations indicate predominantly integer occupation, resulting in small Δ_D and insignificant correction effects. In contrast, single-particle levels from RHB calculations (Figs. 3(c) and 3(d)) show neutron levels consistent with BCS results, but reveal clear fractional occupations in several proton levels near the Fermi surface. This enhanced proton pairing correlation yields a slightly larger charge radius from RHB compared to BCS. Simultaneously, the increase in D_p in Eq. (6) and the enhanced correction term for neutron-proton surface pairing correlation cause the ^{26}Mg charge radius from RHB* to be substantially larger than that from RHB, in better agreement with experimental measurements.

The absence of N = 20 shell closure in Mg isotopes is typically attributed to the island of inversion. Level inversion produces substantial deformation in the ground state of ^{32}Mg, leading to dissolution of the N = 20 shell closure. However, the RHB model fails to reproduce this phenomenon, as all four parameter sets yield spherical ground states. Beyond-mean-field calculations produce reasonably large deformation in ^{32}Mg ground states [?] and could address the discrepancy between calculated and measured charge radii. In this study, we demonstrate that RHB* calculations also yield more accurate charge radii for ^{32}Mg compared to RHB calculations, particularly with density-dependent interactions DD-PC1 and DD-ME2. Beyond-mean-field calculations based on the present framework are discussed in Refs. [?, ?], and further study in this direction is warranted.

To facilitate understanding of N = 14 magicity, Fig. 4 [Figure 4: see original paper] presents Δr_{ch} defined as $\Delta r_{ch}(Z,N) = r_{ch}(Z,N) - r_{ch}(Z-2,N)$ along N = 14 isotones. The calculations with NL3 and PK1 are essentially identical. The empirical formula $r_{ch} = r_0 A^{1/3}$ (with $r_0 = 1.2$ fm), shown as the green line, depends only on mass number and cannot reproduce shell structure. RHB calculations show very small $\Delta r_{ch}(Z)$ for Z = 8, implying strong N = 14 magicity in ^{22}O, consistent with measured proton radii [?]. After including neutron-proton pairing corrections, the N = 14 shell closure in ^{22}O is weakened while that in ^{28}Si is enhanced. Combined with experimental values for ^{26}Mg, the RHB* model provides a better description of available experimental results than the RHB model.

Finally, Table 1 lists the average deviation $\bar{\chi}^2$ and root-mean-square deviation Δ between experimental and calculated charge radii for 24 even-even nuclei from O to Ar isotopes, defined as:

$$\bar{\chi}^2 = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{r_{ch,i}^{\mathrm{exp}} - r_{ch,i}^{\mathrm{cal}}}{\Delta r_{ch,i}^{\mathrm{exp}}}\right)^2}$$

$$\Delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(r_{ch,i}^{\mathrm{exp}} - r_{ch,i}^{\mathrm{cal}}\right)^2}$$

where $\Delta r_{ch,i}^{\mathrm{exp}}$ is the experimental uncertainty of the measured charge radius $r_{ch,i}^{\mathrm{exp}}$ for the ith nucleus. For DD-ME2, NL3, and PK1, the $\bar{\chi}^2$ of RHB* is smaller than that of RHB, while the opposite holds for DD-PC1. In terms of root-mean-square deviation Δ, the values for meson-exchange interactions NL3 and PK1 decrease after correction, while those for density-dependent interactions DD-ME2 and DD-PC1 increase slightly. These results indicate that incorporating neutron-proton pairing corrections around the Fermi surface is necessary for mean-field calculations with meson-exchange effective interactions to systematically describe charge radii from even-even O to Ar nuclei. Density-dependent effective interactions do not require this correction, as RHB calculations already yield larger charge radii than meson-exchange interactions.

SUMMARY

In this work, we systematically investigate charge radius evolution across oxygen to argon isotopic chains using the RHB framework. A refined charge radius formula incorporating neutron-proton pairing correlations near the Fermi surface is employed to describe anomalous behavior. We analyze the emergence of new magic numbers at N = 14 and 16 and the quenching of the traditional N = 20 shell closure.

Notably, including neutron-proton pairing correlations induces abrupt enhancements in charge radii at N = 8, 20, and 28, demonstrating that such correlations amplify shell closure effects at these neutron numbers. The Mg isotopic chain provides critical insights into the interplay between pairing treatments and shell structure. While conventional BCS theory artificially enhances the N = 14 shell effect, the Bogoliubov transformation approach predicts enhanced proton surface pairing correlations, yielding results closer to experimental charge radii. Further validation against experimental data for 24 even-even nuclei confirms that neutron-proton pairing corrections significantly improve charge radius descriptions with meson-exchange effective interactions, though benefits diminish for density-dependent interactions.

Future studies extending this framework to heavier isotopic chains are in progress. Additionally, the interaction-dependent performance of neutron-proton pairing corrections calls for a unified theoretical approach to reconcile meson-exchange and density-dependent interactions.

[1] M. Wang, W. J. Huang, F. G. Kondev et al., The AME 2020 atomic mass evaluation (II). Tables, graphs and references. Chin. Phys. C 45, 030003 (2021). doi:10.1088/1674-1137/abddaf.
[2] B. Pritychenko, M. Birch, B. Singh et al., Tables of E2 transition probabilities from the first 2+ states in even-even nuclei. At. Data Nucl. Data Tables 107, 1 (2016). doi:10.1016/j.adt.2015.10.001.
[3] I. Angeli and K. Marinova, Table of experimental nuclear ground state charge radii: An update. At. Data Nucl. Data Tables 99, 69 (2013). doi:10.1016/j.adt.2011.12.006.
[4] I. Angeli and K. P. Marinova, Nuclear charge radii as signature for structural changes. J. Phys.: Conf. Ser. 724, 012032 (2016). doi:10.1088/1742-6596/724/1/012032.
[5] S. Bagchi, R. Kanungo, W. Horiuchi et al., Neutron skin and signature of the N = 14 shell gap found from measured proton radii of 17−22N. Phys. Lett. B 790, 251 (2019). doi:10.1016/j.physletb.2019.01.024.
[6] S. Kaur, R. Kanungo, W. Horiuchi et al., Proton Distribution Radii of 16−24O: Signatures of New Shell Closures and Neutron Skin. Phys. Rev. Lett. 29, 142502 (2022). doi:10.1103/PhysRevLett.129.142502.
[7] P. Díaz Fernández, H. Alvarez-Pol, R. Crespo et al., Quasifree (p, pN) scattering of light neutron-rich nuclei near N = 14. Phys. Rev. C 97, 024311 (2018). doi:10.1103/PhysRevC.97.024311.
[8] O. Sorlin and M.-G. Porquet, Nuclear magic numbers: New features far from stability. Prog. Part. Nucl. Phys. 61, 602 (2008). doi:10.1016/j.ppnp.2008.05.001.
[9] X. F. Yang, S. J. Wang, S.G. Wilkins et al., Laser spectroscopy for the study of exotic nuclei. Prog. Part. Nucl. Phys. 129, 104005 (2023). doi:10.1016/j.ppnp.2022.104005.
[10] Y. Ye, X. Yang, H. Sakurai et al., Physics of exotic nuclei Nat. Rev. Phys. 7, 21 (2025). doi:10.1038/s42254-024-00782-5.
[11] P. Thirolf, B. Pritychenko, B. Brown et al., Spectroscopy of the 2+ 1 state in 22O and shell structure near the neutron drip line. Phys. Lett. B 485, 16 (2000). doi:10.1016/S0370-2693(00)00720-6.
[12] M. Stanoiu, F. Azaiez, Zs. Dombrádi et al., N = 14 and 16 shell gaps in neutron-rich oxygen isotopes. Phys. Rev. C 69, 034312 (2004). doi:10.1103/PhysRevC.69.034312.
[13] E. Becheva, Y. Blumenfeld, E. Khan et al., N = 14 Shell Closure in 22O Viewed through a Neutron Sensitive Probe. Phys. Rev. Lett. 96, 012501 (2006). doi:10.1103/PhysRevLett.96.012501.

[14] A. Ozawa, T. Kobayashi, T. Suzuki et al., New magic number, N = 16, near the neutron drip line. Phys. Rev. Lett. 84, 5493 (2000). doi:10.1103/PhysRevLett.84.5493.
[15] R. Kanungo, I. Tanihata, A. Ozawa et al., Observation of new neutron and proton magic numbers. Phys. Lett. B 528, 58 (2002). doi:10.1016/S0370-2693(02)01206-6.
[16] T. Otsuka, R. Fujimoto, Y. Utsuno et al., Magic Numbers in Exotic Nuclei and Spin-Isospin Properties of the NN Interaction. Phys. Rev. Lett. 87, 082502 (2001). doi:10.1103/PhysRevLett.87.082502.
[17] C. Hoffman, T. Baumann, D. Bazin et al., Evidence for a doubly magic 24O, Phys. Lett. B 672, 17 (2009), doi:10.1016/j.physletb.2008.12.066.
[18] R. Kanungo, C. Nociforo, A. Prochazka et al., One-Neutron Removal Measurement Reveals 24O as a New Doubly Magic Nucleus. Phys. Rev. Lett. 102, 152501 (2009). doi:10.1103/PhysRevLett.102.152501.
[19] K. Tshoo, Y. Satou, H. Bhang et al., N = 16 Spherical Shell Closure in 24O. Phys. Rev. Lett. 109, 022501 (2012). doi:10.1103/PhysRevLett.109.022501.
[20] A. Chaudhuri, C. Andreoiu, T. Brunner et al., Evidence for the extinction of the N = 20 neutron-shell closure for 32Mg from direct mass measurements. Phys. Rev. C 88, 054317 (2013). doi:10.1103/PhysRevC.88.054317.
[21] K. Marinova, W. Geithner, M. Kowalska et al., Charge radii of neon isotopes across the sd neutron shell. Phys. Rev. C 84, 034313 (2011). doi:10.1103/PhysRevC.84.034313.
[22] D. T. Yordanov, M. L. Bissell, K. Blaum et al., Nuclear Charge Radii of 21–32Mg. Phys. Rev. Lett. 108, 042504 (2012). doi:10.1103/PhysRevLett.108.042504.
[23] C. Thibault, R. Klapisch, C. Rigaud et al., Direct measurement of the masses of 11Li and 26–32Na with an on-line mass spectrometer. Phys. Rev. C 12, 644 (1975). doi:10.1103/PhysRevC.12.644.
[24] T. Motobayashi, Y. Ikeda, K. Ieki et al., Large deformation of the very neutron-rich nucleus 32Mg from intermediate-energy Coulomb excitation. Phys. Lett. B 346, 9 (1995). doi:10.1016/0370-2693(95)00012-A.
[25] B. Pritychenko, T. Glasmacher, P. Cottle et al., Role of intruder configurations in 26,28Ne and 30,32Mg. Phys. Lett. B 461, 322 (1999). doi:10.1016/S0370-2693(99)00836-3.
[26] Y. Yanagisawa, M. Notani, H. Sakurai et al., The first excited state of 30Ne studied by proton inelastic scattering in reversed kinematics. Phys. Lett. B 566, 84 (2003).
[27] B. Bastin, S. Grévy, D. Sohler et al., Collapse of the N = 28 shell closure in 42Si. Phys. Rev. Lett. 99, 022503 (2007). doi:10.1103/PhysRevLett.99.022503.
[28] T. Otsuka, T. Suzuki, R. Fujimoto et al., Evolution of nuclear shells due to the tensor force. Phys. Rev. Lett. 95, 232502 (2005). doi:10.1103/PhysRevLett.95.232502.
[29] V. Lapoux, V. Somà, C. Barbieri et al., Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces. Phys. Rev. Lett. 117, 052501 (2016). doi:10.1103/PhysRevLett.117.052501.
[30] A. Poves and J. Retamosa, Onset of deformation at the N = 20 neutron shell closure far from stability. Phys. Lett. B 184, 311 (1987). doi:10.1016/0370-

2693(87)90171-7.
[31] E. K. Warburton, J. A. Becker and B. A Brown, Mass systematics for A = 29–44 nuclei: The deformed A 32 region. Phys. Rev. C 41, 1147 (1990). doi:10.1103/PhysRevC.41.1147.
[32] S. J. Novario, G. Hagen, G. R. Jansen et al., Charge radii of exotic neon and magnesium isotopes. Phys. Rev. C 102, 051303 (2020). doi:10.1103/PhysRevC.102.051303.
[33] M. K. Gaidarov, P. Sarriguren, A. N. Antonov et al., Ground-state properties and symmetry energy of neutron-rich and neutron-deficient Mg isotopes. Phys. Rev. C 89, 064301 (2014). doi:10.1103/PhysRevC.89.064301.
[34] Z. Ren, Z. Y. Zhu, Y. H. Cai et al., Relativistic mean-field study of Mg isotopes. Phys. Lett. B 380, 241 (1996). doi:10.1016/0370-2693(96)00462-5.
[35] R. Rodríguez-Guzmán, J. Egido, L. Robledo et al., Angular momentum projected analysis of quadrupole collectivity in 30,32,34Mg and 32,34,36,38Si with the Gogny interaction. Phys. Lett. B 474, 15 (2000). doi:10.1016/S0370-2693(00)00015-0.
[36] M. Borrajo and J. Egido, Ground-state properties of even and odd Magnesium isotopes in a symmetry-conserving approach. Phys. Lett. B 764, 328 (2017). doi:10.1016/j.physletb.2016.11.037.
[37] Y. M. Yao, J. Meng, P. Ring et al., Three-dimensional angular momentum projection in relativistic mean-field theory. Phys. Rev. C 79, 044312 (2009). doi:10.1103/PhysRevC.79.044312.
[38] G. X. Dong, X. B. Wang, H. L. Liu et al., Collectivity of neutron-rich magnesium isotopes investigated by projected shell model calculations. Phys. Rev. C 88, 024328 (2013). doi:10.1103/PhysRevC.88.024328.
[39] P.-G. Reinhard and W. Nazarewicz, Toward a global description of nuclear charge radii: Exploring the Fayans energy density functional. Phys. Rev. C 95, 064328 (2017). doi:10.1103/PhysRevC.95.064328.
[40] U. C. Perera, A. V. Afanasjev and P. Ring, Charge radii in covariant density functional theory: A global view. Phys. Rev. C 104, 064313 (2021). doi:10.1103/PhysRevC.104.064313.
[41] G. A. Miller, A. Beck, S. May-Tal Beck et al., Can long-range nuclear properties be influenced by short-range interactions? A chiral dynamics estimate. Phys. Lett. B 793, 360 (2019). doi:10.1016/j.physletb.2019.05.010.
[42] L. A. Souza, M. Dutra, C. H. Lenzi et al., Effects of short-range nuclear correlations on the deformability of neutron stars. Phys. Rev. C 101, 065202 (2020). doi:10.1103/PhysRevC.101.065202.
[43] W. Cosyn and J. Ryckebusch, Phase-space distributions of nuclear short-range correlations. Phys. Lett. B 820, 136526 (2021). doi:10.1016/j.physletb.2021.136526.
[44] G. A. Lalazissis, M. M. Sharma and P. Ring, Rare-earth nuclei: Radii, isotope-shifts and deformation properties in the relativistic mean-field theory. Nucl. Phys. A 597, 35 (1996). doi:10.1016/0375-9474(95)00436-X.
[45] R. An, X. X. Dong, L. G. Cao et al., Local variations of charge radii for nuclei with even Z from 84 to 120. Commun. Theor. Phys. 75, 035301 (2023). doi:10.1088/1572-9494/acb58b.

[46] P. Mueller, I. A. Sulai, A. C. C. Villari et al., Nuclear Charge Radius of 8He. Phys. Rev. Lett. 99, 252501 (2007). doi:10.1103/PhysRevLett.99.252501.
[47] W. Geithner, T. Neff, G. Audi et al., Masses and Charge Radii of 17−22Ne and the Two-Proton-Halo Candidate 17Ne. Phys. Rev. Lett. 101, 252502 (2008). doi:10.1103/PhysRevLett.101.252502.
[48] A. E. Barzakh, D. V. Fedorov, V. S. Ivanov et al., Shell effect in the mean square charge radii and magnetic moments of bismuth isotopes near N = 126. Phys. Rev. C 97, 014322 (2018). doi:10.1103/PhysRevC.97.014322.
[49] H. Nakada, Irregularities in nuclear radii at magic numbers. Phys. Rev. C 100, 044310 (2019). doi:10.1103/PhysRevC.100.044310.
[50] T. Day Goodacre, A. V. Afanasjev, A. E. Barzakh et al., Laser Spectroscopy of Neutron-Rich 207,208Hg Isotopes: Illuminating the Kink and Odd-Even Staggering in Charge Radii across the N = 126 Shell Closure. Phys. Rev. Lett. 126, 032502 (2021). doi:10.1103/PhysRevLett.126.032502.
[51] W. Long, J. Meng, N. Van Giai et al., New effective interactions in relativistic mean field theory with nonlinear terms and density-dependent meson nucleon coupling. Phys. Rev. C 69, 034319 (2004). doi:10.1103/PhysRevC.69.034319.
[52] P.-G. Reinhard and W. Nazarewicz, Nuclear charge densities in spherical and deformed nuclei: Toward precise calculations of charge radii. Phys. Rev. C 103, 054310 (2021). doi:10.1103/PhysRevC.103.054310.
[53] Y.-T. Rong, Accuracy of the mean-field theory in describing ground-state properties of light nuclei. Phys. Rev. C 108, 054314 (2023). doi:10.1103/PhysRevC.108.054314.
[54] R. An, L. Geng and S.-S. Zhang, Novel ansatz for charge radii in density functional theories. Phys. Rev. C 102, 024307 (2020). doi:10.1103/PhysRevC.102.024307.
[55] R. An, S.-S. Zhang, L.-S. Geng et al., Charge radii of potassium isotopes in the RMF (BCS)* approach*. Chin. Phys. C 46, 054101 (2022). doi:10.1088/1674-1137/ac4b5c.
[56] J. Dobaczewski, H. Flocard, J. Treiner et al., Hartree-Fock-Bogolyubov description of nuclei near the neutron-drip line. Nucl. Phys. A 422, 103 (1984). doi:10.1016/0375-9474(84)90433-0.
[57] J. Dobaczewski, W. Nazarewicz, T. R. Werner et al., Mean-field description of ground-state properties of drip-line nuclei: Pairing and continuum effects. Phys. Rev. C 53, 2809 (1996). doi:10.1103/PhysRevC.53.2809.
[58] D. Yang, Y.-T. Rong, R. An et al., Potential signature of new magicity from universal aspects of nuclear charge radii. Phys. Rev. C 110, 064314 (2024). doi:10.1103/PhysRevC.110.064314.
[59] P. Ring, Relativistic mean field theory in finite nuclei. Prog. Part. Nucl. Phys. 37, 193 (1996). doi:10.1016/0146-6410(96)00054-3.
[60] H. Kucharek and P. Ring, Relativistic field theory of superfluidity in nuclei. Z. Phys. A 339, 23 (1991). doi:10.1007/BF01282930.
[61] Y. Tian, Z.Y. Ma, P. Ring et al., A finite range pairing force for density functional theory in superfluid nuclei. Phys. Lett. B 676, 44 (2009). doi:10.1016/j.physletb.2009.04.067.
[62] R. An, X. Jiang, N. Tang et al., Improved description of nuclear charge

radii: Global trends beyond N = 28 shell closure. Phys. Rev. C 109, 064302 (2024). doi:10.1103/PhysRevC.109.064302.
[63] B.-N. Lu, J. Zhao, E.-G. Zhao et al., Multidimensionally-constrained relativistic mean-field models and potential-energy surfaces of actinide nuclei. Phys. Rev. C 89, 014323 (2014). doi:10.1103/PhysRevC.89.014323.
[64] S.-G. Zhou, Multidimensionally constrained covariant density functional theories–nuclear shapes and potential energy surfaces. Phys. Scr. 91, 063008 (2016). doi:10.1088/0031-8949/91/6/063008.
[65] G. A. Lalazissis, J. Köppen, P. Ring et al., New parametrization for the Lagrangian density of relativistic mean field theory. Phys. Rev. C 55, 540 (1997). doi:10.1103/PhysRevC.55.540.
[66] G. A. Lalazissis, T. Nikšić, D. Vretenar et al., New relativistic mean-field interaction with density-dependent meson-nucleon couplings. Phys. Rev. C 71, 024312 (2005). doi:10.1103/PhysRevC.71.024312.
[67] T. Nikšić, D. Vretenar, P. Ring et al., Relativistic nuclear energy density functionals: Adjusting parameters to binding energies. Phys. Rev. C 78, 034318 (2008). doi:10.1103/PhysRevC.78.034318.
[68] K. Wang and B.-N. Lu, The angular momentum and parity projected multidimensionally constrained relativistic Hartree–Bogoliubov model. Commun. Theor. Phys. 74, 015303 (2022). doi:10.1088/1572-9494/ac3999.
[69] Y.-T. Rong, X.-Y. Wu, B.-N. Lu et al., Anatomy of octupole correlations in 96Zr with a symmetry-restored multidimensionally constrained covariant density functional theory. Phys. Lett. B 840, 137896 (2023). doi:10.1016/j.physletb.2023.137896.

Note: Figure translations are in progress. See original paper for figures.