

Bayesian Neural Network Evaluation of Neutron-Induced Fission Product Yields of ^{232}Th

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Abstract

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Full Text

Preamble

Bayesian Neural Network Evaluation of Neutron-Induced Fission Product Yields of ^{232}Th

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Research on neutron-induced fission product yields of ^{232}Th is crucial to understanding the competition between symmetric and asymmetric fission in actinide nuclei. However, obtaining complete isotopic yield distributions over a wide range of neutron energies remains experimentally challenging. In this work, a Bayesian neural network (BNN) model is developed to predict both independent (IND) and cumulative (CUM) fission yields of ^{232}Th under neutron irradiation at various incident energies. To address the limited availability of experimental data for the analysis of IND mass distributions, we substituted mass-number-based yields with yields of specific isotopes. Furthermore, physical phenomena or quantities such as the odd-even effect and isospin are introduced as constraints to enhance the physical consistency of the predictions. The impact of these constraints is evaluated through the mass-chain yield distributions and their dependence on energy. Incorporating physical constraints significantly improves the prediction accuracy, yielding more reliable and physically meaningful fission yield data for applications in nuclear physics and reactor design.

Keywords: Bayesian neural network, ^{232}Th , independent fission yield, cumulative fission yield, odd-even effect, isospin

INTRODUCTION

Nuclear fission, as a fundamental phenomenon in nuclear physics, plays a crucial role in a wide range of applications, including nuclear energy, national defense, and medical isotope production. It is also essential for studies on the synthesis of superheavy nuclei [?, ?], the analysis of reactor antineutrino energy spectra [?, ?, ?], and the investigation of nucleosynthesis via the r-process in neutron star mergers [?, ?, ?]. Accurate and comprehensive fission yield data are critical for understanding the dynamic evolution of the fission process. In 2011, the Chinese Academy of Sciences launched the Strategic Priority Science and Technology Project entitled “Future Advanced Nuclear Fission Energy Thorium Molten Salt Reactor (TMSR) Nuclear Energy System,” employing ^{232}Th as the primary fuel [?, ?]. The experimental TMSR reactor achieved first criticality in October 2023. In this system, ^{232}Th serves as a key component, participating mainly in neutron-induced reactions through the Th-U fuel cycle [?].

In particular, high-energy neutrons may induce direct fission of ^{232}Th . Consequently, for accurate burn-up credit analysis, special attention must be given to the fission product yields to reliably assess reactivity feedback and the characteristics of spent fuel. Currently, major international evaluated nuclear data libraries (such as JENDL [?], ENDF [?], CENDL [?], and JEFF [?]) provide comprehensive data for neutron-induced fission yields only at specific energies,

that is, 0.0253 eV (thermal neutron), 0.5 MeV, and 14 MeV. However, the measured neutron-induced fission data are often sparse, with incomplete isotopic coverage, significant uncertainties, and notable discrepancies [?]. Obtaining complete isotopic yield distributions across a continuous range of incident neutron energies remains experimentally challenging. Phenomenological models of nuclear fission (such as the Brosa [?] and GEF [?] approaches) heavily rely on experimental data for parameter calibration and validation. When such data are limited or absent, the underlying assumptions and approximations of these models become weakly constrained, resulting in increased parameter uncertainties and diminished predictive reliability. In the case of ^{232}Th , the lack of systematic fission yield measurements at various neutron energies further hampers the direct application of such semi-empirical models, highlighting the urgent need for alternative approaches to obtain complete yield data.

With the development of artificial intelligence, machine learning (ML) techniques have found extensive applications in nuclear physics research [?, ?, ?], such as prediction and analysis of atomic nuclear mass [?, ?, ?, ?, ?, ?, ?, ?, ?], charge radius [?, ?, ?, ?, ?], α -decay and β -decay half-lives [?, ?, ?], ground-state spin [?], fission product yields [?, ?, ?, ?, ?], and reaction cross sections [?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. ML algorithms are capable of identifying hidden patterns in large, complex datasets and extracting underlying physical correlations. Among them, the Bayesian neural network (BNN) has demonstrated strong potential in evaluating fission product yields [?] and investigating their dependence on neutron energy [?]. Unlike traditional neural networks, BNN treats model parameters as probability distributions rather than fixed values. This allows not only for yield predictions, but also for quantifying the associated uncertainties [?]. By incorporating prior knowledge and posterior inference, BNN imposes meaningful constraints on model parameters and effectively mitigates overfitting risks [?]. Moreover, BNN naturally integrates experimental uncertainties and missing data during network training, thereby enhancing both the robustness and predictive accuracy of the model [?, ?].

In this work, a Bayesian neural network (BNN) framework is employed to model and then to predict the fission product yields of ^{232}Th [?]. Guided by the findings in Ref. [?], the hyperbolic tangent (tanh) activation function is selected for its superior capability in capturing yield trends and reducing prediction errors. To balance model complexity and accuracy, a two-hidden-layer architecture is adopted, which offers improved performance on complex fission datasets with a manageable number of trainable parameters. Therefore, a two-hidden-layer BNN architecture with tanh activation functions is adopted in this work. The network structure is further optimized through an analysis of error distributions to determine the appropriate number of neurons in each layer.

In scenarios where experimental data are sparse, the inclusion of relevant physical constraints, such as the odd-even effect and isospin, has been shown to significantly enhance predictive accuracy [?, ?, ?, ?]. The BNN predictions incorporating the odd-even effect and isospin showed significant improvement

compared to the uncorrected case. Based on the established BNN framework, this study investigates the mass distributions of IND for ^{232}Th at incident neutron energies of 1 MeV and 13 MeV, and further explores the energy dependence of CUM for selected nuclides. The remainder of this paper is organized as follows. Section II introduces the theoretical framework of the BNN method. Section III provides detailed descriptions of the network construction and analysis of results, with Section III A focusing on IND and Section III B providing a comprehensive analysis of CUM. Finally, Section IV summarizes the key findings and methodological innovations of this work.

II. BNN MODEL

A Bayesian neural network (BNN) is adopted in this study to predict fission product yields by combining Bayesian inference with the structure of traditional neural networks [?]. Unlike conventional neural networks, where model parameters are fixed after training and produce a single deterministic output, BNN represents model parameters as probability distributions. This probabilistic formulation enables BNN to provide both predictions and reliable estimates of associated uncertainties.

The core of BNN lies in the posterior probability distribution of the model parameters,

$$p(\theta|x, t) = \frac{p(x, t|\theta)p(\theta)}{\int p(x, t|\theta)p(\theta)d\theta}$$

where $p(x, t|\theta)$ is the likelihood function, describing the probability of the observed data given the model parameters θ , and $p(\theta)$ is the prior distribution representing prior beliefs about these parameters. The denominator $\int p(x, t|\theta)p(\theta)d\theta$ serves as the normalization constant to ensure that the posterior is a valid probability distribution. As training data accumulates, the influence of the prior diminishes in favor of the likelihood.

The likelihood function $p(x, t|\theta)$ is given by

$$p(x, t|\theta) = \exp(-\chi^2/2)$$

The discrepancy between the target and predicted values is quantified by the χ^2 loss function,

$$\chi^2(\theta) = \sum_i \frac{(t_i - f(x_i, \theta))^2}{\Delta t_i^2}$$

where t_i is the target value, $f(x_i, \theta)$ is the network prediction for input x_i , and Δt_i is the associated experimental uncertainty or noise term [?]. Minimizing χ^2 during training corresponds to maximizing the likelihood.

In Bayesian neural networks, a feed-forward neural network is used,

$$f(x, \theta) = a + \sum_j b_j \tanh \left(\sum_i d_{ji} x_i + c_j \right)$$

where $x = (x_1, x_2, \dots, x_l)$ is the input vector consisting of l characteristics, typically including the proton number (Z), neutron number (N), and mass number (A) of the fissioning nuclei and fission fragments, together with the excitation energy (E) of the compound nucleus. $\theta = \{a, b_j, c_j, d_{ji}\}$ represents the values of the model parameters, where d_{ji} is the connection weight between the input layer and the hidden layer, indicating the influence of the input characteristics on the hidden neurons. c_j is the bias of the hidden layer, determining the baseline level when there is no contribution from the input layer to the activation function. b_j is the connection weight between the hidden layer and the output layer, representing the contribution weight of each hidden neuron to the final output. a is the bias of the output layer. The nonlinear tanh activation function enables accurate approximation of complex continuous functions.

[Figure 1: see original paper] shows the schematic structure of the single-hidden-layer Bayesian neural network used for fission yield evaluation. Given a new input x_n , the network prediction is the expected value over the posterior,

$$\langle f(x_n, \theta) \rangle = \int f(x_n, \theta) p(\theta | x, t) d\theta$$

Each prediction corresponding to a specific set of parameters is weighted by its posterior probability, and the integration yields the final predictive output of the BNN, which incorporates both the central estimate and its uncertainty.

III. RESULTS AND DISCUSSION

A. Independent Fission Yield

Based on the fission yield dataset selected in this work, BNN models with both single and double hidden layers were constructed and evaluated. Comparative analysis revealed that the two-hidden-layer architecture provides better generalization performance and results in lower prediction errors than the single-hidden-layer. To further optimize the architecture and prevent either overfitting from excessive complexity or underfitting from insufficient model capacity, the number of neurons per layer was systematically examined. For both simplicity and consistency, the same number of neurons was assigned to each of the two hidden layers. This symmetric configuration not only ensures balanced parameter distribution across the network, but also reduces the number of hyperparameters to tune during model optimization. The neuron count was varied from 16 to 40 in increments of four. Each configuration was trained under identical conditions using a fixed dataset and hyperparameter set. During training, total error on the training dataset was monitored across iterations to evaluate convergence behavior and prediction stability. The resulting error trends, shown in [Figure 2: see original paper], provide a basis for selecting the optimal network configuration used in subsequent fission yield predictions.

[Figure 2: see original paper] illustrates the variation of the mean square error (MSE) during training for networks with different neuron counts per hidden

layer. Under a fixed network architecture, the error gradually decreases with the increase in training iterations and stabilizes as the number of iterations approaches 10^5 , indicating that convergence has been reached. At a given number of training iterations, increasing the number of neurons per hidden layer leads to a noticeable reduction in prediction error. However, this trend begins to plateau when the neuron count reaches 28, beyond which the additional reduction in error becomes marginal. This observation suggests that increasing model complexity beyond this point does not lead to significant performance gains and may potentially introduce the risk of overfitting. To balance predictive accuracy and model generalization, the configuration with two hidden layers and 28 neurons per layer is selected as the optimal network architecture.

In the modeling of IND, the mass distribution of the fission product yields is of central importance. However, experimental data directly reporting yield as a function of mass number are limited. To address this issue and maximize the use of available data, the training input was designed to include the neutron number of the fissioning nuclei, the proton number and mass number of the fission fragments, the excitation energy of the compound nucleus, and the corresponding IND of individual nuclides. The dataset used in IND includes 4,128 evaluated data points extracted from the JENDL database [?] and 112 experimental data points obtained from the EXFOR database [?]. The mass-yield data from EXFOR are employed for validation purposes. To further enhance the predictive performance of the BNN, additional physical constraints were introduced. Specifically, the odd-even effect is incorporated by assigning a value of +0.2 for even-proton-number fragments and -0.2 for odd-proton-number fragments, following the methodology described in Ref. [?]. In addition, isospin symmetry is accounted for by including the third component of isospin, defined as $(Z - N)/2$, as an input feature.

Once the BNN predicts the yields of individual nuclides, the mass yield distribution is reconstructed by summing the predicted yields of nuclides with the same mass number. This enables the generation of independent fission product mass distributions induced by incident neutrons on thorium isotopes, even when direct experimental data on mass yields are unavailable.

In [Figure 3: see original paper], BNN0 denotes the baseline model without additional physical constraints; BNN1-1 incorporates the odd-even effect; BNN1-2 includes isospin; and BNN2 incorporates both the odd-even effect and isospin simultaneously. In panel (a), corresponding to thermal neutron-induced fission of ^{227}Th , BNN1-1 reduces the unphysical negative values present in BNN0 but introduces an abnormal upturn near mass number 170. Similarly, BNN1-2 suppresses the negative values and the upturn but fails to reproduce the fine structure in the light fragment region. In contrast, BNN2 not only eliminates these unphysical features but also retains the peak structure originally predicted by BNN0, indicating that the simultaneous inclusion of both constraints results in a more precise representation of the fission mechanism and improves model predictive accuracy.

In panel (b), for thermal neutron-induced fission of ^{229}Th , BNN1-1 and BNN1-2 exhibit similar behaviors as in panel (a), whereas BNN2 successfully reproduces the peak structure of the heavy fragment for the first time, aligning well with the evaluated data. This suggests that the formation of heavy fission fragments is significantly influenced by the combined effects of the odd-even pattern and isospin. These results underscore the necessity of jointly incorporating both constraints to advance theoretical modeling of the production of heavy fission fragments.

In panel (c), which shows neutron-induced fission of ^{232}Th at 0.5 MeV, BNN0 exhibits an abnormal upturn around mass number 70 and significantly underestimates the yields in the 138-144 mass range relative to the evaluated data. BNN1-1 corrects the heavy fragment underestimation, while BNN1-2 mitigates the abnormal upturn and reduces negative yields. However, only BNN2 resolves all these deficiencies simultaneously, demonstrating the best overall performance in reproducing the fission fragment mass yield distribution.

In panel (d), corresponding to 14 MeV neutron-induced fission of ^{232}Th , BNN2 provides the most accurate description of the symmetric fission region and matches the evaluated data well in the light fragment region (mass 80-100), outperforming the other models in both respects. Across all four reaction systems, compared to BNN0, BNN1-1 results in a slightly narrower confidence band, while BNN1-2 shows a modest increase in uncertainty. In contrast, BNN2 yields a substantial reduction in confidence band width, indicating significantly improved predictive reliability. This reduction in uncertainty further suggests that the combination of both physical constraints enhances the generalizability of the model. In summary, while the odd-even effect and isospin each contribute to improving BNN model performance, they have individual limitations. Their simultaneous incorporation in BNN2 leads to the most substantial improvement in both yield prediction accuracy and uncertainty reduction, closely aligning with evaluated data and demonstrating the robustness of the combined-constraint model. Despite the incorporation of additional physical constraints, YBNN still exhibit unphysical negative values. Reference [?] demonstrates that applying a penalty on negative values improves both the representation of yield peak structures and the overall predictive accuracy. This penalty mechanism will be considered in future network designs.

B. Cumulative Fission Yield

The training dataset includes both the proton and neutron numbers of the fissioning nuclei and fission fragments, the excitation energy of the compound nucleus, and the cumulative fission yields. It comprises 4,128 evaluated data points from the JENDL database [?] and 1,258 experimental data points from the EXFOR database [?]. Based on the collected cumulative fission yield data, various BNN architectures, including single-hidden-layer and two-hidden-layer networks with varying numbers of neurons per layer, were systematically evaluated. Optimal performance was achieved with a two-hidden-layer architecture

with 20 neurons per layer, which was adopted as a baseline model. Four configurations were tested: (1) without additional physical constraints (BNN0), (2) with only the odd-even effect (BNN1-1), (3) with only the isospin effect (BNN1-2), and (4) with both physical constraints simultaneously (BNN2). The implementation of these physical constraints follows the same approach as in IND [?]. Comparative analysis against the evaluated data shows that the BNN2 model, which incorporates both effects, achieves the best predictive accuracy and generalization capability. As a result, only the BNN2 model was retained in the following analysis.

[Figure 4: see original paper] shows the predicted fission fragment mass distributions for ^{232}Th induced by 1 MeV and 13 MeV neutrons, with both energy points excluded from the training data. YBNN2 are compared with YBNN0 and the YGEF [?]. In panel (a), BNN2 eliminates the unphysical negative yields observed in BNN0 around mass numbers $A = 150 - 160$. Moreover, in the mass range $A = 90 - 100$, YBNN2 shows an improved agreement with YGEF. In panel (b), YBNN2 also suppresses the negative values in the mass range $A = 100 - 120$ and produces results significantly closer to YGEF in the $A = 120 - 140$ region. In both panels, the confidence intervals of YBNN2 are noticeably narrower compared to those of YBNN0, indicating reduced predictive uncertainty and improved generalization performance. These results suggest that the simultaneous inclusion of the odd-even effect and isospin enhances the physical realism of the model and underscores the importance of incorporating both effects in fission yield modeling.

[Figure 5: see original paper] compares the cumulative fission yields evaluated (YJENDL) with the predictions of the BNN without additional physical constraints (YBNN0) and the physically constrained model (YBNN2) for four representative reactions. In thermal neutron-induced fission of thorium isotopes (panels (a) and (b)), YBNN0 exhibits a systematic overestimation compared to YJENDL, particularly in low-yield (blue) regions, and fails to reproduce characteristic yield peaks. In contrast, BNN2 provides a much improved prediction, especially for light fragments, where YBNN2 closely matches YJENDL. In particular, two pronounced peaks (red spots) corresponding to ^{137}Ba and ^{140}Ce appear in both YBNN2 and YJENDL, demonstrating the model's ability to capture key heavy fragment yields. In panel (c), corresponding to a different actinide target, BNN0 again overpredicts the yields in the low-value regions. This trend of overestimation is significantly mitigated in BNN2, which is in better agreement with the evaluated distribution. Panel (d), corresponding to fission at 14 MeV incident neutron energy, shows that BNN2 substantially improves the yield predictions in the symmetric fission region, aligning more closely with YJENDL than BNN0. In general, across the four reactions, YBNN2 demonstrates significantly better agreement with YJENDL than YBNN0, particularly in reproducing the heavy fragment yield peaks. These results demonstrate that incorporating the odd-even effect and isospin enhances the physical reliability and predictive precision of the model. The mean squared error is reduced from 0.565 to 0.364, further confirming the effectiveness of the added physical con-

straints.

[Figure 6: see original paper] presents the cumulative fission yields for heavy fragments with mass numbers $A = 134 - 144$ from four neutron-induced thorium fission reactions at different neutron energies. As fission products reach stability primarily through β -decay, these chain yields reflect the population of final stable or long-lived nuclides along the decay chains. The evaluated data (YJENDL) show a clear odd-even staggering, with yields at odd mass numbers notably higher than those of neighboring even- A isotopes. In addition, a pronounced enhancement is observed at $A = 140$, corresponding to the production of ^{140}Ce , which is a β -stable nuclide with a magic neutron number ($N = 82$), and one of the yield peaks identified in [Figure 5: see original paper]. BNN predictions without physical constraints (YBNN0) show a smoother trend, failing to capture the odd-even structure and underestimating the peak at $A = 140$. In contrast, the constrained model (BNN2), which incorporates both the odd-even effect and isospin, reproduces the evaluated pattern well (including the odd-even staggering and the sharp rise at $A = 140$). These results demonstrate that the integration of physical features into the BNN framework significantly improves its ability to capture decay chain effects and local nuclear structure signatures. The BNN2 model thus offers a more accurate and physically interpretable prediction of the cumulative yields of the fission product.

As previously analyzed in [Figure 6: see original paper], the BNN2 model more accurately captures the mass-dependent features of the fission product yields, particularly the odd-even staggering and the notable deviation around $A = 140$. To quantitatively assess the improvement from the incorporation of physical constraints, [Figure 7: see original paper] compares the ratios of the model predictions to the JENDL evaluations of four neutron-induced fission reactions in thorium isotopes. The results show that the yield ratios predicted by BNN2 (YBNN2/YJENDL) are consistently closer to unity than those from the original BNN0 model (YBNN0/YJENDL), indicating a significant enhancement in prediction accuracy. This improvement is particularly evident at several mass numbers. For instance, at $A = 137$, the YBNN2 consistently align more closely with the YJENDL in all four cases, corresponding to the enhanced production of ^{137}Ba observed in [Figure 5: see original paper]. Around $A = 140$, the YBNN2 are nearly identical to the YJENDL in panels (a) and (d), and also exhibit notable improvements over the YBNN0 in panels (b) and (c). However, at $A = 136$, both YBNN0 and YBNN2 show notable deviations from YJENDL, suggesting that additional physical effects may influence yields in this region and warrant further investigation. In summary, the overall trend of YBNN2/YJENDL being consistently closer to unity than YBNN0/YJENDL clearly demonstrates the benefit of incorporating the odd-even effect and isospin into the BNN framework. These physical enhancements significantly improve the model's ability to reproduce evaluated cumulative fission yields for thorium-induced reactions, emphasizing their indispensable role in refining fission yield predictions.

[Figure 8: see original paper] presents the cumulative fission yields of four representative fission products— ^{95}Zr , ^{99}Mo , ^{132}Te , and ^{131}I —as a function of incident neutron energy from 0 to 14 MeV. These nuclides were selected for their significance in nuclear applications, environmental safety, and human health. Specifically, ^{95}Zr is a precursor of ^{95}Nb , a widely used redox indicator in reactor monitoring. ^{99}Mo , with a high decay branching ratio (88%) to $^{99\text{m}}\text{Tc}$, plays a critical role in nuclear medicine. ^{132}Te promotes corrosion, shortening the service life of structural materials, while ^{131}I is a radiotoxic nuclide with substantial biological impact. For ^{95}Zr [panel (a)], the yield initially increases, then gradually decreases at higher neutron energies (4–14 MeV). The BNN0 model consistently underestimates the experimental data, whereas BNN2 offers more accurate predictions across the energy range, with only a slight overestimation at 14 MeV. In the case of ^{99}Mo [panel (b)], the yield remains nearly constant from 2 to 12 MeV, with a small drop at 14 MeV. The BNN2 predictions accurately capture this behavior and also correct the underestimation seen in BNN0 at thermal energy (0.0253 eV). For ^{132}Te [panel (c)], the yield shows a pronounced increase at low energies, followed by a gradual decline beyond 8 MeV. The BNN2 model matches experimental trends more closely than BNN0, particularly in the 0–4 MeV region. As shown in panel (d), the yield of ^{131}I rises with increasing neutron energy. The predictions of BNN2 align well with the experimental and evaluated data above 5 MeV and at thermal energy, but both BNN predictions slightly overestimate the experimental results between 2 and 4 MeV. Overall, all experimental points fall within the 95% confidence intervals of the BNN predictions. Notably, the confidence bands associated with BNN2 are narrower than those of BNN0, demonstrating that the incorporation of odd-even effects and isospin corrections not only improves predictive accuracy but also reduces model uncertainty.

IV. SUMMARY

In this work, Bayesian neural network (BNN) was employed to evaluate both independent (IND) and cumulative fission product yields (CUM), with the goal of improving predictive accuracy beyond the limitations posed by sparse and inconsistent experimental data available only at a few neutron energies (thermal, 0.5 MeV, and 14 MeV). In the evaluation of IND, input features were optimized by incorporating the proton and mass numbers of fission fragments, along with their corresponding yields. This enabled the use of a broader set of experimental data during model training. A two-hidden-layer architecture with 28 neurons per layer was adopted based on error minimization analysis. The incorporation of the odd-even effect and isospin significantly enhanced the predictive performance of the model. When both physical constraints were applied simultaneously, the model effectively suppressed unphysical negative yields, mitigated abnormal upturns, and reproduced fine structures in the heavy fragment region. The uncertainty of the prediction, as reflected in the 95% confidence intervals, was also substantially reduced. For CUM evaluation, a two-hidden-layer architecture with 20 neurons per layer was selected after comprehensive

testing of various configurations, including single-hidden-layer and two-hidden-layer networks with different neuron counts. Comparative studies were performed, including single-nuclide yield comparisons, analysis of odd-even staggering in mass yields (particularly in the $A = 134 - 144$ region), and yield ratio YBNN/YJENDL analysis. These results demonstrate that the simultaneous incorporation of odd-even effects and isospin constraints significantly improves the reliability of yield predictions. The model more accurately captured features of stable nuclide production and successfully reproduced the anomalous structure near $A = 140$. Overall, our study demonstrates that combining physical constraints, particularly odd-even effect and isospin, significantly enhances the performance and robustness of BNN-based fission yield evaluations. These findings suggest that such physical features should be systematically incorporated into future data-driven models and theoretical approaches to improve the fidelity of fission yield predictions.

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REFERENCES

- [1] J. H. Hamilton, S. Hofmann, Y. Oganessian, Search for super-heavy nuclei. *Annu. Rev. Nucl. Part. Sci.* 63, 383–405 (2013). <https://doi.org/10.1146/annurev-nucl-102912-144535>
- [2] J. C. Pei, W. Nazarewicz, J. A. Sheikh et al., Fission barriers of compound superheavy nuclei. *Phys. Rev. Lett.* 102, 192501 (2009). <https://doi.org/10.1103/PhysRevLett.102.192501>
- [3] G. Mention, M. Fechner, T. Lasserre et al., Reactor antineutrino anomaly. *Phys. Rev. D* 83, 073006 (2011). <https://doi.org/10.1103/PhysRevD.83.073006>
- [4] A. A. Sonzogni, E. A. McCutchan, T. D. Johnson et al., Effects of fission yield data in the calculation of antineutrino spectra for $^{235}\text{U}(n, \text{fission})$ at thermal and fast neutron energies. *Phys. Rev. Lett.* 116, 132502 (2016). <https://doi.org/10.1103/PhysRevLett.116.132502>
- [5] T. A. Mueller, D. Lhuillier, M. Fallot et al., Improved predictions of reactor antineutrino spectra. *Phys. Rev. C* 83, 054615 (2011). <https://doi.org/10.1103/PhysRevC.83.054615>
- [6] S. Goriely, J. L. Sida, J. F. Lemaître et al., New fission fragment distributions and r-process origin of rare-earth elements. *Phys. Rev. Lett.* 111, 242502 (2013). <https://doi.org/10.1103/PhysRevLett.111.242502>
- [7] M. Eichler, A. Arcones, A. Kelic et al., The role of fission in neutron star mergers and its impact on the r-process peaks. *Astrophys. J.* 808, 30 (2015). <https://doi.org/10.1088/0004-637X/808/1/30>
- [8] J. Sadhukhan, S. A. Giuliani, Z. Matheson et al., Efficient method for estimation of fission fragment yields of r-process nuclei. *Phys. Rev. C* 101,

- 065803 (2020). <https://doi.org/10.1103/PhysRevC.101.065803>
- [9] M. H. Jiang, H. J. Xu, Z. M. Dai, Advanced fission energy program-tmsr nuclear energy system. *Bull. Chin. Acad. Sci.* 27, 366-374 (2012). <https://doi.org/10.3969/j.issn.1000->
- [10] X. Z. Cai, Z. M. Dai, H. J. Xu, Thorium molten salt reactor nuclear energy system. *PHYSICS* 45, 578-590 (2016). <https://doi.org/10.7693/wl20160904>
- [11] X.-Z. Cai, Z.-M. Dai, H.-J. Xu, Thorium molten salt reactor nuclear energy system. *PHYSICS* 45, 578-590 (2016). <https://doi.org/10.7693/wl20160904>
- [12] Japanese evaluated nuclear data library (jendl), <https://www.jendl.org/>
- [13] M. Chadwick, M. Herman, P. Obloinský et al., ENDF/B-VII.1 Nuclear data for science and technology: covariances, cross sections, fission product yields and decay data. *Nucl. Data Sheets* 112, 2887-2996 (2011). <https://doi.org/10.1016/j.nds.2011.11.002>
- [14] Z. G. Ge, Z. X. Zhao, H. H. Xia et al., The updated version of chinese evaluated nuclear data library (CENDL-3.1). *J. Korean Phys. Soc.* 59, 1052-1056 (2011). <https://doi.org/10.3938/jkps.59.1052>
- [15] A. J. M. Plompen, O. Cabellos, C. De Saint et al., The joint evaluated fission and fusion nuclear data library, JEFF-3.3. *Eur. Phys. J. A* 56, 181 (2020). <https://doi.org/10.1140/epja/s10050-020-00141-9>
- [16] Experimental nuclear reaction data(EXFOR), <https://www-nds.iaea.org/exfor/>
- [17] U. Brosa, S. Grossmann, A. Müller, Nuclear scission. *Phys. Rep.* 197, 167-262 (1990). [https://doi.org/10.1016/0370-1573\(90\)90114-H](https://doi.org/10.1016/0370-1573(90)90114-H)
- [18] K. H. Schmidt, B. Jurado, C. Amouroux et al., General description of fission observables: GEF model code. *Nucl. Data Sheets* 131, 107-221 (2016). <https://doi.org/10.1016/j.nds.2015.12.009>
- [19] H. F. Zhang, L. H. Wang, J. P. Yin et al., Performance of the levenbergmarquardt neural network approach in nuclear mass prediction. *J. Phys. G:Nucl. Part. Phys.* 44, 045110 (2017). <https://doi.org/10.1088/1361-6471/aa5d78>
- [20] A. Boehnlein, M. Diefenthaler, N. Sato et al., Colloquium: Machine learning in nuclear physics. *Rev. Mod. Phys.* 94, 031003 (2022). <https://doi.org/10.1103/RevModPhys.94.031003>
- [21] P. Vicente-Valdez, L. Bernstein, M. Fratoni, Nuclear learning augmented by machine learning. *Annals of Nuclear Energy* 163, 108596 (2021). <https://doi.org/https://doi.org/10.1016/j.anucene.2021.108596>
- [22] R. Utama, J. Piekarewicz, H. B. Prosper, Nuclear mass predictions for the crustal composition of neutron stars: A bayesian neural network approach. *Phys. Rev. C* 93, 014311 (2016). <https://doi.org/10.1103/PhysRevC.93.014311>
- [23] R. Utama, J. Piekarewicz, Refining mass formulas for astrophysical applications: A bayesian approach. *Phys. Rev. C* 96, 044308 (2017). <https://doi.org/10.1103/PhysRevC.96.044308>
- [24] Z. M. Niu, H. Z. Liang, Nuclear mass predictions based on bayesian neural network approach with pairing and shell effects. *Phys. Lett. B* 778, 48-53 (2018). <https://doi.org/10.1016/j.physletb.2018.01.002>
- [25] Z. M. Niu, J. Y. Fang, Y. F. Niu, Comparative study of radial basis function and bayesian neural network approaches in nuclear mass predictions. *Phys. Rev. C* 100, 054311 (2019). <https://doi.org/10.1103/PhysRevC.100.054311>

- [26] Z. M. Niu, H. Z. Liang, Nuclear mass predictions with machine learning reaching the accuracy required by r-process studies. *Phys. Rev. C* 106, L021303 (2022). <https://doi.org/10.1103/PhysRevC.106.L021303>
- [27] Y. F. Liu, C. Su, J. Liu et al., Improved naive bayesian probability classifier in predictions of nuclear mass. *Phys. Rev. C* 104, 014315 (2021). <https://doi.org/10.1103/PhysRevC.104.014315>
- [28] Z. X. Yang, X. H. Fan, P. Yin et al., Taming nucleon density distributions with deep neural network. *Phys. Lett. B* 823, 136650 (2021). <https://doi.org/10.1016/j.physletb.2021.136650>
- [29] X. H. Wu, P. W. Zhao, Predicting nuclear masses with the kernel ridge regression. *Phys. Rev. C* 101, 051301 (2020). <https://doi.org/10.1103/PhysRevC.101.051301>
- [30] X. C. Ming, H. F. Zhang, R. R. Xu et al., Nuclear mass based on the multi-task learning neural network method. *Nucl. Sci. Tech.* 33, 48 (2022). <https://doi.org/10.1007/s41365-022-01031-z>
- [31] D. Wu, C. L. Bai, H. Sagawa et al., Calculation of nuclear charge radii with a feed-forward neural network. *Phys. Rev. C* 102, 054323 (2020). <https://doi.org/10.1103/PhysRevC.102.054323>
- [32] S. Akkoyun, T. Bayram, S. O. Kara et al., An artificial neural network application on nuclear charge radii. *J. Phys. G:Nucl. Part. Phys.* 40, 055106 (2013). <https://doi.org/10.1088/0954-3899/40/5/055106>
- [33] R. Utama, W. C. Chen, J. Piekarewicz, Nuclear charge radii: density functional theory meets bayesian neural networks. *J. Phys. G:Nucl. Part. Phys.* 43, 114002 (2016). <https://doi.org/10.1088/0954-3899/43/11/114002>
- [34] Y. F. Ma, C. Su, J. Liu et al., Predictions of nuclear charge radii and physical interpretations based on the naive bayesian probability classifier. *Phys. Rev. C* 101, 014304 (2020). <https://doi.org/10.1103/PhysRevC.101.014304>
- [35] Y. Y. Cao, J. Y. Guo, B. Zhou, Predictions of nuclear charge radii based on the convolutional neural network. *Nucl. Sci. Tech.* 34, 152 (2023). <https://doi.org/10.1007/s41365-023-01308-x>
- [36] C. Q. Li, C. N. Tong, H. J. Du et al., Deep learning approach to nuclear masses and α -decay half-lives. *Phys. Rev. C* 105, 064306 (2022). <https://doi.org/10.1103/PhysRevC.105.064306>
- [37] Z. M. Niu, H. Z. Liang, B. H. Sun et al., Predictions of nuclear β -decay half-lives with machine learning and their impact on r-process nucleosynthesis. *Phys. Rev. C* 99, 064307 (2019). <https://doi.org/10.1103/PhysRevC.99.064307>
- [38] P. Li, Z. M. Niu, Y. F. Niu, Selection of abnormal trends in nuclear β -decay half-lives by neural network and exploration of the physical mechanisms. *Nucl. Sci. Tech.* 36, 50 (2025). <https://doi.org/10.1007/s41365-025-01663-x>
- [39] D. Liu, A. N. A, Z. Z. Qin et al., Neural network study of the nuclear ground-state spin distribution within a random interaction ensemble. *Nucl. Sci. Tech.* 35, 64 (2024). <https://doi.org/10.1007/s41365-024-01424-2>
- [40] Z. A. Wang, J. C. Pei, Y. Liu et al., Bayesian evaluation of incomplete fission yields. *Phys. Rev. Lett.* 123, 122501 (2019). <https://doi.org/10.1103/PhysRevLett.123.122501>
- [41] A. Lovell, A. Mohan, P. Talou et al., Constraining fission yields using machine learning. *EPJ Web Conf.* 211, 04006 (2019). <https://doi.org/10.1051/epjconf/201921104006>
- [42] Q. F. Song, L. Zhu, H. Guo et al., Verification of neutron-induced

- fission product yields evaluated by a tensor decomposition model in transport-burnup simulations. Nuclear Science and Techniques 34, 32 (2023). <https://doi.org/10.1007/s41365-023-00000-0>
- [43] Q. F. Song, L. Zhu, B. S. Cai et al., Image processing of isotope yield in neutron-induced fission. Phys. Rev. C 107, 044609 (2023). <https://doi.org/10.1103/PhysRevC.107.044609>
- [44] L. Tong, R. N. He, S. W. Yan, Prediction of neutron-induced fission product yields by a straightforward k-nearest-neighbor algorithm. Phys. Rev. C 104, 064617 (2021). <https://doi.org/10.1103/PhysRevC.104.064617>
- [45] D. Peng, H. L. Wei, X. X. Chen et al., Bayesian evaluation of residual production cross sections in proton-induced nuclear spallation reactions. J. Phys. G:Nucl. Part. Phys. 49 (2022). <https://doi.org/10.1088/1361-6471/AC7069>
- [46] D. Peng, H. L. Wei, J. Pu et al., Bayesian neural network prediction methods for fragment cross sections in proton-induced spallation reactions. Sci. Sin. Phys. Mech. Astron. 52, 252012-(2022). <https://doi.org/10.1360/SSPMA-2021-0298>
- [47] C. W. Ma, D. Peng, H. L. Wei et al., A bayesian-neural-network prediction for fragment production in proton induced spallation reaction. Chin. Phys. C 44, 124107 (2020). <https://doi.org/10.1088/1674-1137/abb657>
- [48] C. W. Ma, D. Peng, H. L. Wei et al., Isotopic cross-sections in proton induced spallation reactions based on the bayesian neural network method. Chin. Phys. C 44, 014104 (2020). <https://doi.org/10.1088/1674-1137/44/1/014104>
- [49] C. W. Ma, X. B. Wei, X. X. Chen et al., Precise machine learning models for fragment production in projectile fragmentation reactions using bayesian neural networks. Chin. Phys. C 46, 074104 (2022). <https://doi.org/10.1088/1674-1137/ac5efb>
- [50] C. W. Ma, X. X. Chen, X. B. Wei et al., Systematic behavior of fragments in bayesian neural network models for projectile fragmentation reactions. Phys. Rev. C 108, 044606 (2023). <https://doi.org/10.1103/PhysRevC.108.044606>
- [51] Y. Y. Li, F. Zhang, J. Su, Improvement of the bayesian neural network to study the photoneutron yield cross sections. Nucl. Sci. Tech. 33, 135 (2022). <https://doi.org/10.1007/s41365-022-01131-w>
- [52] X. B. Wei, H. L. Wei, C. W. Ma et al., Predictions from several models for the cross sections of light neutron-rich isotopes by Qg systematics in 40Ar projectile-fragmentation reactions. Phys. Rev. C 111, 034607 (2025). <https://doi.org/10.1103/PhysRevC.111.034607>
- [53] Q. K. Sun, Y. Zhang, Z. R. Hao et al., Enhancing reliability in photonuclear cross-section fitting with bayesian neural networks. Nucl. Sci. Tech. 36, 52 (2025). <https://doi.org/10.1007/s41365-024-01611-1>
- [54] X. B. Wei, H. L. Wei, Y. T. Wang et al., Multiple-models predictions for drip line nuclides in projectile fragmentation of 40,48Ca, 58,64Ni, and 78,86Kr at 140 MeV/u. Nucl. Sci. Tech. 33, 155 (2022). <https://doi.org/10.1007/s41365-022-01137-4>
- [55] C. Y. Qiao, J. C. Pei, Z. A. Wang et al., Study of energy dependence of neutron-induced fission yield of 235U with bayesian machine learning. At. Energy Sci. Technol. 56, 937-943 (2022). <https://doi.org/10.7538/yzk.2022.youxian.0198>

- [56] D. R. Phillips, R. J. Furnstahl, U. Heinz et al., Get on the band wagon: a bayesian framework for quantifying model uncertainties in nuclear dynamics. *J. Phys. G:Nucl. Part. Phys.* 48, 072001 (2021). <https://doi.org/10.1088/1361-6471/abf1df>
- [57] V. Kejzlar, L. Neufcourt, W. Nazarewicz et al., Statistical aspects of nuclear mass models. *J. Phys. G:Nucl. Part. Phys.* 47, 094001 (2020). <https://doi.org/10.1088/1361-6471/ab907c>
- [58] Z. A. Wang, J. C. Pei, Y. J. Chen et al., Bayesian approach to heterogeneous data fusion of imperfect fission yields for augmented evaluations. *Phys. Rev. C* 106, L021304 (2022). <https://doi.org/10.1103/PhysRevC.106.L021304>
- [59] J. Y. Yi, C. Y. Qiao, J. C. Pei et al., Bayesian machine learning for the uncertainty evaluation of nuclear fission product yields. *Sci. Sin. Phys. Mech. Astron.* 52, 123-129 (2022). <https://doi.org/10.1360/SSPMA-2021-0340>
- [60] Z. A. Wang, J. C. Pei, Optimizing multilayer bayesian neural networks for evaluation of fission yields. *Phys. Rev. C* 104, 064608 (2021). <https://doi.org/10.1103/PhysRevC.104.064608>
- [61] X. W. Jia, J. Willard, A. Karpatsne et al., Physics-guided machine learning for scientific discovery: an application in simulating lake temperature profiles. *ACM/IMS Trans. Data Sci.* 2, Article 20 (2021). <https://doi.org/10.1145/3447814>
- [62] G. E. Karniadakis, I. G. Kevrekidis, L. Lu et al., Physics-informed machine learning. *Nat. Rev. Phys.* 3, 422-440 (2021). <https://doi.org/10.1038/s42254-021-00314-5>
- [63] D. R. Phillips, R. J. Furnstahl, U. Heinz et al., Get on the band wagon: a bayesian framework for quantifying model uncertainties in nuclear dynamics. *J. Phys. G:Nucl. Part. Phys.* 48, 072001 (2021). <https://doi.org/10.1088/1361-6471/abf1df>
- [64] D. Neudecker, O. Cabellos, A. R. Clark et al., Informing nuclear physics via machine learning methods with differential and integral experiments. *Phys. Rev. C* 104, 034611 (2021). <https://doi.org/10.1103/PhysRevC.104.034611>
- [65] R. M. Neal, *Bayesian Learning for Neural Networks*, Springer, New York, 2012. <https://doi.org/10.1007/978-1-4612-0745-0>
- [66] L. Neufcourt, Y. C. Cao, W. Nazarewicz, Bayesian approach to model-based extrapolation of nuclear observables. *Phys. Rev. C* 98, 034318 (2018). <https://doi.org/10.1103/PhysRevC.98.034318>
- [67] C. Y. Qiao, J. C. Pei, Z. A. Wang et al., Bayesian evaluation of charge yields of fission fragments of ^{239}U . *Phys. Rev. C* 103, 034621 (2021). <https://doi.org/10.1103/PhysRevC.103.034621>

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