Theoretical Derivation and MCNP5-Based Comparative Study of Neutron Flux Density Distribution in Annular Reactors

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Abstract

To bridge the theoretical gap in the analysis of neutron flux density distributions within annular reactors, this study derives closed-form solutions for the neutron flux density under critical conditions based on the monoenergetic neutron diffusion equation and validates them using MCNP5 simulations. The study categorizes annular reactors into three types according to inner radius: Type A ($a > 27\,\mathrm{cm}$), where the inner neutron flux density approaches zero, is modeled using zero-flux boundary conditions at both inner and outer radii; Type B ($a < 7\,\mathrm{cm}$), resembling cylindrical reactor characteristics, assumes zero neutron current at the core axis; Type C (7–27 cm), representing a transitional state, introduces a novel hybrid-weight model. A homogeneous annular core model was established in MCNP5 (referencing a typical reactor outer diameter of $b = 1.61\,\mathrm{m}$) to simulate neutron flux density distributions under critical conditions and compare them with the theoretical results after normalization. Results show that: For Type A, the average relative error between theoretical and simulated flux in the active region ($r = a \to b$) is 23.77%, indicating strong agreement; For Type B, the theoretical solution matches the zeroth-order Bessel distribution of cylindrical reactors, with an average error of 23.54%. However, for inner diameters greater than 5 cm, errors increase significantly; For Type C, the hybrid-weight model (with weights obtained via cubic polynomial fitting, $R^2 = 0.861 - 0.875$) effectively captures transitional flux behavior. In the 8-12 cm range, the relative error is lowest (10.77%–15.19%). Eigenvalue analysis further reveals a monotonic relationship between $Br \times b$ and the radius ratio a/b. When $a/b > 0.6$, neutron leakage increases sharply. This study fills a theoretical gap in modeling neutron flux density distributions in annular reactors and provides a robust foundation for geometry-based optimization in reactor core design.

Full Text

Preamble

Theoretical Derivation and MCNP5-Based Comparative Study of Neutron Flux Density Distribution in Annular Reactors

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To bridge the theoretical gap in analyzing neutron flux density distributions within annular reactors, this study derives closed-form solutions for the neutron flux density under critical conditions based on the monoenergetic neutron diffusion equation and validates them using MCNP5 simulations. Annular reactors are categorized into three types according to inner radius: Type A ($a > 27$ cm), where the inner neutron flux density approaches zero, is modeled using zero-flux boundary conditions at both inner and outer radii; Type B ($a < 7$ cm), resembling cylindrical reactor characteristics, assumes zero neutron current at the core axis; and Type C (7–27 cm), representing a transitional state, introduces a novel hybrid-weight model. A homogeneous annular core model was established in MCNP5 (referencing a typical reactor outer diameter of $b = 1.61$ m) to simulate neutron flux density distributions under critical conditions and compare them with normalized theoretical results.

Results show that for Type A, the average relative error between theoretical and simulated flux in the active region ($r = a \rightarrow b$) is 23.77%, indicating strong agreement. For Type B, the theoretical solution matches the zeroth-order Bessel distribution of cylindrical reactors, with an average error of 23.54%; however, for inner diameters greater than 5 cm, errors increase significantly. For Type C, the hybrid-weight model (with weights obtained via cubic polynomial fitting, $R^2 = 0.861$–0.875) effectively captures transitional flux behavior, with the lowest relative error (10.77%–15.19%) in the 8–12 cm range. Eigenvalue analysis further reveals a monotonic relationship between $Br \times b$ and the radius ratio a/b, with neutron leakage increasing sharply when $a/b > 0.6$.

This study fills a theoretical gap in modeling neutron flux density distributions in annular reactors and provides a robust foundation for geometry-based optimization in reactor core design.

Keywords: neutron flux density distribution, MCNP5 simulation, annular reactor, theoretical analytical solution, geometric curvature

Introduction

Annular reactor configurations have attracted increasing interest because recent advances in artificial intelligence and additive manufacturing enable more flexible geometric designs and data-driven optimization of neutron flux distributions [1]. Traditional reactor performance shaping relied primarily on axial assembly loading patterns and radial fuel rearrangement to smooth temperature and power profiles, whereas emerging AI-based methods can directly influence flux and thermal metrics through geometry control [1, 2]. Previous studies have demonstrated that AI-optimized geometries can significantly improve key performance indicators such as the temperature peak factor, with reported gains up to threefold [1, 3]. Machine learning–assisted multi-physics surrogates and large-scale evaluation of candidate geometries on high-performance computing platforms have opened new avenues for reactor concepts with nonstandard shapes [4, 5].

Compared with conventional rectangular, cylindrical, or spherical cores, annular reactors—characterized by a ring-like active region surrounding a central void—offer distinct neutronic and thermal advantages. They can enhance fuel cooling, reduce neutron flux density in the core center, and smooth power distributions, thereby improving both economic and safety performance [6, 7]. The annular form has also shown promise in applications such as submarine propulsion and small modular reactors (SMRs) due to its compactness and flux-shaping flexibility [6, 8]. Owing to these neutronic features and engineering potentials, annular geometries have become a focal direction in advanced reactor design [9, 10].

Neutron flux density is a fundamental quantity in reactor physics, directly influencing power distribution, fuel management, radiation shielding, and reactor control operations including startup, shutdown, and power regulation [11–13]. Despite the practical relevance of annular cores, a theoretical gap remains in analytically characterizing neutron flux behavior in annular geometries. This deficiency limits the availability of closed-form guidance for design optimization and safety assessment in such systems [14].

Monte Carlo methods, and MCNP5 in particular, have emerged as a widely accepted benchmark for neutron transport and flux calculations. They have been extensively validated across diverse reactor applications, including the estimation of delayed neutron fractions in molten salt systems and high-fidelity flux modeling in fast reactors [15–21]. The reliability of MCNP5 in neutron flux calculations has been demonstrated in shielding evaluation and core physics parameter studies [22–24]. Nevertheless, the interplay between complex annular geometry and neutron transport, especially the effect of geometric curvature on flux distributions, is not yet fully elucidated, posing challenges for both theoretical modeling and numerical simulation in these configurations [25, 26].

Cross-platform and multi-software verification has become a standard strategy to bolster computational credibility in nuclear engineering [25–29]. Coupled approaches—such as integrating MCNP5 with auxiliary tools for related multiphysics reconstruction—and adjoint-weighted formulations for neutron dynamics have demonstrated the feasibility of bridging analytical diffusion models with Monte Carlo benchmarks in complex geometries like annular cores [27, 30]. Brian C. Kiedrowski proposed an adjoint-weighted calculation method for neutronic dynamic parameters, which further verified the feasibility of coupled analysis between multi-group diffusion models and Monte Carlo methods in complex geometries (such as annular cores) [31]. These methodologies motivate and support the systematic comparison carried out in this work, ensuring that the derived analytical solutions and MCNP5 results are mutually consistent and engineering-relevant [28, 29].

This study aims to fill this gap by deriving analytical expressions for neutron flux density in annular reactors under critical conditions and validating them through MCNP5 simulations. The specific objectives are as follows:

1. Derive theoretical solutions for neutron flux density in annular reactors under critical conditions based on the monoenergetic neutron diffusion equation;
2. Construct MCNP5 models to simulate the neutron flux density distribution in annular reactor cores;
3. Compare the analytical results with simulation outputs to assess their consistency and applicable range.

II. Theoretical Model and Derivation of Neutron Flux Density Distribution

Let the outer radius, inner radius, and height of the annular reactor core be denoted b, a, and H, respectively. All values include extrapolated lengths, as shown in Fig. 1 [Figure 1: see original paper].

Fig. 1 [Figure 1: see original paper]. Schematic of an annular reactor core

Under critical conditions, the spatial distribution of neutron flux density in the reactor satisfies the Helmholtz equation:

$$\nabla^2\phi(r) + B_g^2\phi(r) = 0$$

where (n/(cm²·s)) is the neutron flux density, and B_g^2 (cm ²) is the geometric buckling. Under cylindrical symmetry, and setting the origin at the central axis of the cylinder, the flux depends only on the radial and axial variables r and z, simplifying Eq. (1) to:

$$\frac{\partial^2\phi(r,z)}{\partial r^2} + \frac{1}{r}\frac{\partial\phi(r,z)}{\partial r} + \frac{\partial^2\phi(r,z)}{\partial z^2} + B_g^2\phi(r,z) = 0$$

Assuming separable variables, the flux function is expressed as:

$$\phi(r,z) = \varphi(r)Z(z)$$

Substituting into Eq. (2), we obtain:

$$\varphi(r) = C_1 J_0(B_r r) + C_2 Y_0(B_r r)$$
$$Z(z) = F\cos(B_z z)$$
$$B_g^2 = B_r^2 + B_z^2$$

where $Z(z)$ describes the axial neutron flux density distribution, with axial buckling defined as $B_z^2 = (\pi/H)^2$. Thus, the general solution for the radial flux component is:

$$\varphi(r) = C_1 J_0(B_r r) + C_2 Y_0(B_r r)$$

Therefore, solving $\phi(r,z)$ reduces to solving $\varphi(r)$ and determining the eigenvalue B_r.

Since this work focuses on flux distributions within the actual core dimensions, the extrapolated boundary is treated as coinciding with the physical boundary. Unlike cylindrical cores—where the flux is zero at the extrapolated outer boundary—annular reactors exhibit unique geometric boundary conditions:

- At the outer radius r = b, the flux is zero;
- At the inner radius r = a, three scenarios may arise:

1. Large a: No neutrons exist in the central void, so the inner boundary condition is $\phi(a) = 0$;
2. Small a: Significant neutron presence exists within the central void, so the inner boundary is treated as a virtual boundary and the condition becomes $\partial\phi/\partial r|_{r=0} = 0$;
3. Intermediate a: A transitional state where flux is neither zero nor maximal, requiring a weighted boundary condition.

According to diffusion theory, when the material thickness exceeds approximately three times the neutron diffusion length, most neutrons scatter back before reaching the boundary, leading to significantly reduced leakage [33]. Taking light water' s diffusion length (2–3 cm) as a reference, the core is classified by inner radius as:

- Type B: a < 7 cm (approx. 3 diffusion lengths);
- Type C: 7 a 27 cm;
- Type A: a > 27 cm.

In subsequent chapters, the solution of $\varphi(r)$ and comparison with MCNP5 simulations will be analyzed for each type: A, B, and C.

The boundary conditions are:

$$\begin{cases} \phi(a,z) = 0 \\ \phi(b,z) = 0 \end{cases}$$

Substituting the boundary conditions into Eq. (5), we obtain the following system of equations:

$$\begin{cases} \phi(a,z) = C_1 J_0(B_r a) + C_2 Y_0(B_r a) = 0 \\ \phi(b,z) = C_1 J_0(B_r b) + C_2 Y_0(B_r b) = 0 \end{cases}$$

III. MCNP5 Modeling Method

To validate the rationality of the theoretical solution for neutron flux density distribution in annular reactor cores, this study employs MCNP5 to construct a homogeneous annular reactor core model with uniformly mixed fuel and moderator materials. The core design references a typical third-generation reactor configuration. Fuel assemblies are arranged in a square lattice, and the moderator is borated water. The water-to-fuel volume ratio is maintained consistent with that of conventional PWRs. By adjusting the boron concentration, the model achieves a critical state. The corresponding neutron flux density distribution was then simulated under critical conditions.

Based on the previously defined classification of A, B, and C types, the annular core models established in MCNP5 are illustrated in Fig. 2 [Figure 2: see original paper].

Fig. 2 [Figure 2: see original paper]. MCNP5 reactor model

IV. Comparison Between Theoretical Derivation of Neutron Flux Density and MCNP5 Simulation

A. Comparative Analysis of Type A Theoretical Results vs. Simulation Results

In Type A, where the annular reactor inner radius a > 27 cm, the value significantly exceeds three times the thermal neutron diffusion length. As a result, the neutron flux density approaches zero within the central region. Therefore, the boundary conditions for solving Eq. (5) are defined such that the neutron flux density vanishes at both the inner and outer radii of the core:

Solving system Eq. (7), the general solution becomes:

$$\phi(r,z) = C\left[J_0(B_r r) - \frac{J_0(B_r a)}{Y_0(B_r b)} Y_0(B_r r)\right]$$

where the constant C is determined by the total reactor power.

To determine the eigenvalue B_r, substitute r = b into Eq. (8), which leads to the transcendental equation:

$$J_0(B_r b)Y_0(B_r a) - J_0(B_r a)Y_0(B_r b) = 0$$

1. Numerical Solution of Eigenvalue Br Since Eq. (9) does not yield a closed-form analytical solution for the eigenvalue B_r, a numerical approach is adopted in this study. The Bessel function library in MATLAB is used, and a tolerance of 10^{-8} is set. The bisection method is applied to iteratively solve for B_r. The computed eigenvalues for various combinations of inner radius a and outer radius b are summarized in Table 1 .

From Table 1, it can be observed that the eigenvalue B_r increases with increasing inner radius a and decreases with increasing outer radius b. The maximum B_r value appears in the configuration with the largest a and smallest b, while the minimum appears in the opposite configuration. When b is held constant, the growth of B_r with respect to a accelerates.

Fig. 3 [Figure 3: see original paper]. Distribution of eigenvalue Br

To clarify the relationship between B_r and geometry, the product $B_r \times b$ is plotted on the vertical axis, and the radius ratio a/b on the horizontal axis. The trends for various outer radii are shown in Fig. 4 [Figure 4: see original paper]. The results indicate that $B_r \times b$ increases monotonically with increasing a/b, regardless of the value of b. The plotted curve is highly consistent with the curve obtained via the graphical method in reference [32], thereby verifying the accuracy of the numerical solution. Therefore, once the inner and outer radii of an annular reactor are known, the corresponding eigenvalue B_r can also be identified using Fig. 4 [Figure 4: see original paper].

Fig. 4 [Figure 4: see original paper]. Variation of Br × b with the radius ratio a/b

2. Results Under Type A geometric conditions, using a conventional outer core radius of b = 161 cm as a reference, the theoretical neutron flux density solutions for various inner radii a were normalized and compared against MCNP5 simulation results, as shown in Fig. 5 [Figure 5: see original paper].

As illustrated in Fig. 5, when the inner radius of the annular core exceeds 27 cm, the MCNP5 simulation shows that neutron flux density in the inner annular region (from center to radius a) tends to approach zero. As the radius a increases, the region with near-zero flux expands. The theoretical solution also predicts zero flux in this region. Between radius a and outer radius b, the theoretical and simulated flux distributions match well, both showing that the neutron flux density increases initially, peaks in the middle region, and then decreases toward the outer boundary.

Fig. 5 [Figure 5: see original paper]. Comparison of normalized neutron flux density between theoretical and MCNP5 results under different inner radii for Type A

An error analysis of the normalized neutron flux density is presented in Table 2 . The average relative error between the Type A theoretical solution and MCNP5 simulation is 23.77%, indicating good agreement between the two.

Table 2. Mean relative error between Type A theoretical solution and MCNP5 simulation results

a(cm)	Average MRE
30	32.49%
40	21.06%
50	22.517%
60	21.16%
70	21.6012%
Overall	**23.77%**

B. Comparison Between Type B Theoretical Results and Simulation Outcomes

1. Theoretical Derivation of Neutron Flux Density Distribution When the inner radius a of the annular reactor core is sufficiently small (a < 7 cm), a large number of neutrons leak into the core center region with radius zero. In this case, the boundary condition that the neutron flux density is zero at radius a no longer holds. A new boundary condition must be applied, assuming the neutron flux density gradient at the core axis is zero, and the neutron flux density is zero at the outer radius b. The new boundary conditions are:

$$\begin{cases} C_1 J_0(B_r b) + C_2 Y_0(B_r b) = 0 \\ J(0,z) = -\lambda_r \frac{\partial \phi}{\partial r}\Big|_{r=0} = 0 \Rightarrow \frac{\partial \phi}{\partial r}(0,z) = 0 \end{cases}$$

Substituting Eq. (10) into the neutron flux density distribution Eq. (5), and the gradient of the neutron flux density is obtained as:

$$\frac{\partial}{\partial r}\left[C_1 J_0(B_r r) + C_2 Y_0(B_r r)\right]$$

Using the derivatives of the Bessel functions:

$$\begin{cases} \frac{d}{dr} J_0(B_r r) = -B_r J_1(B_r r) \\ \frac{d}{dr} Y_0(B_r r) = -B_r Y_1(B_r r) \end{cases}$$

Substituting Eq. (11) into the expression yields:

$$\left.\frac{\partial \phi}{\partial r}\right|_{r=0} = -B_r(C_1 J_1(0) + C_2 Y_1(0)) = 0$$

To satisfy Eq. (13), it is required that $C_2 = 0$. Substituting $C_2 = 0$ into Eq. (5), we get:

$$C_1 J_0(B_r b) = 0$$

For a non-trivial solution:

$$J_0(B_r b) = 0$$

This gives the smallest eigenvalue:

$$B_r b = 2.405$$

Substituting $C_2 = 0$ and Eq. (16) into equation (1-5), the theoretical solution for neutron flux density in Type B becomes:

$$\phi(r, z) = C_1 J_0\left(\frac{2.405}{b} r\right)$$

2. Comparison of Theoretical and Simulation Results Taking b = 161 cm as a reference, the neutron flux density under different inner radii a for the Type B configuration was compared between the theoretical solution and MCNP5 simulation results after normalization (Fig. 6 [Figure 6: see original paper]).

The theoretical results follow a zero-order Bessel distribution. The simulation results show that the neutron flux density reaches its maximum at the core axis and gradually decreases with increasing radial distance, becoming zero at the outer radius b. The overall trend aligns with the theoretical solution.

According to the error analysis (Table 3), the average relative error between the Type B theoretical solution and simulation results is 23.54%. However, when the inner radius a > 5 cm, the error increases significantly (e.g., 28.59% when a = 5 cm), indicating that the traditional cylindrical core model is only suitable for annular cores with very small inner radii.

Fig. 6 [Figure 6: see original paper]. Comparison between the theoretical solution and MCNP5 simulation results of neutron flux density for Type B configurations with different inner radii

Table 3. Average relative error between Type B theoretical and simulation results

a(cm)	Average MRE
1	15.66%
2	24.98%
3	22.06%
4	34.41%
5	28.59%
6	15.56%
Overall	**23.54%**

C. Comparing Type C Theoretical Predictions with Simulation Results

For Type C annular cores, the inner radius a ranges from 7–27 cm, representing a transitional state between Type A and Type B. In this regime, the boundary conditions no longer satisfy $\phi(r = a) = 0$ nor $J(r = 0) = 0$, making direct analytical solutions infeasible. This study combines the theoretical neutron flux density distributions of Type A and Type B to establish a transitional model describing the flux behavior of Type C cores:

$$\begin{cases} \phi_A(r) = C\left[J_0(B_r r) - \frac{J_0(B_r a)}{Y_0(B_r b)} Y_0(B_r r)\right] \\ \phi_B(r) = C' J_0(r) \\ \phi_C(r) = \lambda_1 \phi_A(r) + \lambda_2 \phi_B(r) \end{cases}$$

where $\phi_A(r)$, $\phi_B(r)$, and $\phi_C(r)$ represent the neutron flux density distributions for Type A, Type B, and Type C, respectively, while λ_1 and λ_2 are the weighting coefficients.

1. Determination of Weighting Factors 1 and 2 Taking the MCNP5 simulation results at different inner radii a as reference, the weighting factors λ_1 and λ_2 in $\phi_C(r)$ were determined by adjusting their ratio. The obtained values of the weighting factors are listed in Table 4 .

An exponential fitting was performed on the data using Origin software (Fig. 7 [Figure 7: see original paper]). The coefficient of determination for the fitting curve was $R^2 = 0.773$, indicating that the model adequately captures the underlying variation pattern.

Table 4. Values of weighting factors 1 and 2 for different inner radii

a(cm)	b(cm)	1	2
8	161	0.85	0.15

a(cm)	b(cm)	1	2
10	161	0.70	0.30
15	161	0.50	0.50
20	161	0.30	0.70
25	161	0.15	0.85

Fig. 7 [Figure 7: see original paper]. Results of Exponential Regression Fitting for Weighting Factors

To further improve accuracy, a third-order polynomial regression was applied. The fitted expressions are given as follows:

$$\begin{cases}\lambda_1 = -0.0016a^3 + 0.072a^2 - 0.81a + 2.68\\ \lambda_2 = 0.0016a^3 - 0.072a^2 + 0.81a - 1.68\end{cases}$$

The coefficients of determination were $R_1^2 = 0.861$, $R_2^2 = 0.875$, indicating improved fitting accuracy. Compared with exponential fitting, the third-order polynomial provides better performance and avoids overfitting, thus it is adopted for further analysis.

2. Comparison Between Theoretical and Simulated Results Taking b = 161 cm as a reference, the neutron flux density distributions of Type C with different inner radii a were normalized and compared between the theoretical solution and MCNP5 simulation results (Fig. 8 [Figure 8: see original paper]).

As observed in Fig. 8, the neutron flux density in the Type C annular core is neither zero nor maximal at the center, but instead exhibits a trend of first increasing from a non-zero value to a peak and then decreasing to zero at the boundary. The theoretical distribution determined using the weighting factors shows good agreement with the MCNP5 simulation results, effectively capturing the flux characteristics of the transitional region.

Error analysis (Table 5) indicates that the overall average relative error between the theoretical and simulated results for Type C is 35.87%. However, the relative error is lowest (10.77%-15.19%) within the inner radius range of 8-12 cm, suggesting that the theoretical model in this range offers higher accuracy and may be more suitable for engineering applications.

Fig. 8 [Figure 8: see original paper]. Comparison between the theoretical solution and MCNP5 simulation results of neutron flux density for Type C configurations with different inner radii

Table 5. Mean Relative Errors between Theoretical and Simulated Results for Type C

a(cm)	MRE
8	15.19%
10	10.77%
15	36.46%
20	36.01%
25	33.15%
Average	**35.87%**

V. Results and Discussion

A. Consistency Analysis Between Theoretical and Simulated Results

This study verifies the theoretical derivation by comparing it with MCNP5 simulations, revealing three representative neutron flux density distribution patterns in the annular reactor:

For Type A (a > 27 cm), the theoretical solution agrees well with the simulation results in the active region ($r = a \rightarrow b$), yielding a mean relative error of 23.77%, indicating good consistency. For Type B (a < 7 cm), the theoretical results are consistent with the Bessel-type distribution in cylindrical reactors, with a mean relative error of 23.54%. However, the error increases significantly when the inner radius exceeds 5 cm, suggesting that the conventional cylindrical reactor model is only applicable to annular cores with extremely small inner radii. For Type C (7-27 cm), the proposed hybrid-weight model successfully captures the transitional neutron flux density characteristics, and the relative error is lowest in the 8-12 cm range.

B. Model Innovation for the Transition State (Type C)

Targeting the transitional region of 7-27 cm, this study proposes a hybrid-weight model described as:

$$\phi_C(r) = \lambda_1 \phi_A(r) + \lambda_2 \phi_B(r)$$

Using third-order polynomial regression, the weighting coefficients were determined (R^2 = 0.861-0.875), successfully capturing the unique distribution behavior of neutron flux density characterized by a non-zero peak at the center followed by a decreasing trend toward the boundary. Although the overall average relative error reached 35.87%, the error in the 8-12 cm region was the lowest (10.77%-15.19%), suggesting this range as a priority for practical engineering applications. The primary sources of error stem from the localized limitations of diffusion theory at the boundaries and the incomplete representation of the nonlinear features in the weighting coefficients.

C. Geometric Curvature Size Effect

Analysis of the eigenvalue B_r reveals a key monotonic relationship: $B_r \times b$ increases with a/b (as shown in Fig. 4 [Figure 4: see original paper]), implying:

- When a/b > 0.3, B_r becomes increasingly sensitive to changes in geometric size.
- The minimum B_r appears at the limit case a → 0, b → ∞, i.e., $B_r = 2.405/b$.
- Practical design should avoid configurations with a/b > 0.6, where a sharp increase in B_r may significantly enhance neutron leakage.

D. Limitations and Future Prospects

This study simplifies the theoretical modeling of neutron flux density distribution in annular reactors by assuming single-energy neutron diffusion. The MCNP5 simulation was used to verify the applicability of traditional cylindrical reactor models under a reference condition of outer radius b = 161 cm. However, the accuracy and validity of the derived theoretical solutions under broader conditions remain questionable. Future work will focus on advancing multi-group transport theory, optimizing models through machine learning, and incorporating experimental validation in higher dimensions, with the goal of establishing a more accurate neutron flux density prediction system for annular reactors.

VI. Conclusions

This study addresses the theoretical gap in neutron flux density distributions within annular reactor cores. Analytical solutions for the critical-state flux profile were derived from the monoenergetic neutron diffusion equation and systematically validated against MCNP5 simulations. The main conclusions are as follows:

1. Three theoretical models were established to accurately characterize the neutron flux density distribution in annular reactors across different inner radius regimes. For inner radii a > 27 cm (Type A), the theoretical predictions agree well with MCNP5 simulations in the active zone (r = a to r = b), with an average relative error of 23.77%. As the inner radius increases, the near-zero flux phenomenon within the annular core aligns with the theoretical model, validating its applicability in large-radius annular reactor design. Additionally, the characteristic parameter B_r was introduced to quantify design metrics.

2. The applicability of traditional cylindrical reactor models to small-radius annular cores (a < 7 cm, Type B) was evaluated. When a < 7 cm, the neutron flux density distribution in the annular core closely follows the zero-order Bessel function distribution ($B_r = 2.405/b$). The simulation results show good agreement with theoretical predictions at the core center

(average relative error: 23.54%). However, when the inner radius exceeds 5 cm, the error increases significantly, indicating that cylindrical reactor models are only suitable for extremely small-radius annular designs.

3. A novel hybrid-weight model was proposed for the transition region (7-27 cm, Type C). This model effectively captures the unique neutron flux density distribution in this regime by combining the Type A and Type B theoretical solutions (Eq. 20) through a cubic polynomial weighting function ($R^2 = 0.861$-0.875). The model successfully describes the transitional behavior where the neutron flux density decreases after reaching a non-zero maximum near the core center. Notably, the 8-12 cm inner radius range exhibits the lowest errors (10.77%-15.19%), providing a priority reference for engineering applications.

4. A scaling law for the geometric ratio a/b was identified. The product $B_r \times b$ shows a monotonically increasing relationship with a/b (see Fig. 4 [Figure 4: see original paper]). Specifically:

   - When $a/b > 0.3$, the flux distribution becomes highly sensitive to geometry changes.
   - When $a/b > 0.6$, neutron leakage increases sharply.

   These findings offer quantitative guidance for optimizing the geometric design parameters of annular reactors.

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Declarations

Conflict of interest: The authors declare that they have no conflict of interest.

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Note: Figure translations are in progress. See original paper for figures.