# Interpolation Based Initial Image for Fast Fractal Decoding

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## Abstract

Fractal image decoding has been effectively accelerated by using the range-averaged image (RAI) as the initial image in decoding process, and only one iteration is needed to obtain the decoded image quality with acceptable quality. To further improve the decoded image quality while maintaining real-time decoding and acceptable decoded image quality, an interpolation based initial image (IBII) was proposed in this study. First, the main drawback of RAI was its obvious block artifact. To make RAI appear to be closer to natural images, IBII was proposed to make the initial image appear smoother and can better approximate the original image than RAI. Then, higher decoded image quality can be obtained with one iteration under specific decoding strategy. Experimental results show that the IBII based method can improvethe decoded image quality by 0.56-1.41dB in peak signal-to-noise ratio (PSNR) and by 0.0061-0.0173 in mean structural similarity (MSSIM).

## Full Text

## Preamble

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**Abstract:** Fractal image decoding has been effectively accelerated by using the range-averaged image (RAI) as the initial image in the decoding process, requiring only one iteration to obtain decoded images of acceptable quality. To further improve decoded image quality while maintaining real-time decoding capabilities, this study proposes an interpolation-based initial image (IBII). First, we address the main drawback of RAI—its obvious block artifacts. To make the initial image appear more like natural images, IBII was designed to produce a smoother initial image that better approximates the original image than RAI, thereby yielding higher decoded image quality with a single iteration under specific decoding strategies. Experimental results demonstrate that the IBII-based method improves decoded image quality by 0.56–1.41 dB in peak signal-to-noise ratio (PSNR) and by 0.0061–0.0173 in mean structural similarity (MSSIM).

**Keywords:** Fractal image coding; Fast fractal decoding; Initial image

## Introduction

Fractal image coding (FIC) represents a successful application of fractal theory to image compression. The core concept of FIC was pioneered by Barnsley, and Jacquin subsequently proposed the first block-based fractal coding algorithm [1]. Unlike traditional compression techniques such as discrete cosine transform and wavelet methods that achieve compression by removing high-frequency components in the frequency domain, FIC compresses images by exploiting self-similarity redundancy among different local regions [2]. Beyond image compression, FIC has found increasing application in other domains in recent years, including image denoising [3-6], image encryption [7-9], and human pose estimation [10, 11].

To make FIC more practical, researchers have proposed fast fractal encoding algorithms that complete the encoding process as quickly as possible while preserving decoded image quality [12, 13]. Others have combined FIC with complementary compression techniques to improve overall performance [14, 15]. At the decoding stage, although fractal image decoding can converge in a small number of iterations, accelerating the decoding process remains valuable for real-time applications such as video surveillance. To shorten the fractal decoding process, some researchers employ appropriate initial images, such as the range-averaged image (RAI), which approximates the input image well and enables rapid convergence to the final decoded image under specified iteration strategies [16–18]. Other researchers have developed improved iteration strategies that effectively reduce iteration errors relative to the final decoded image, thereby accelerating the decoding process [19].

In previous RAI-based decoding approaches, the range blocks in the initial image are directly replaced with their respective averages, bringing the initial image much closer to the input image than a conventional blank image. This enables real-time decoding with only one iteration while producing decoded images of acceptable quality for applications that do not demand high fidelity, such as video surveillance.

Nevertheless, room remains to improve decoded image quality within a single iteration. This study proposes an interpolation-based initial image (IBII) that better approximates the input image than RAI, thereby improving decoded image quality while maintaining real-time decoding and acceptable quality standards. The contributions of this study are twofold: (1) We define IBII to provide a superior initial image that more closely approximates the input image than RAI, enabling IBII-based fractal decoding to achieve higher decoded image quality with one iteration compared to the RAI-based method. (2) The IBII-based method can be combined with other fast fractal decoding techniques to achieve real-time decoding while obtaining higher decoded image quality.

This paper is organized as follows: Section 2 reviews conventional fractal image coding and related work. Section 3 describes the definition of IBII. Sections 4 and 5 present the proposed algorithm and experimental results, respectively. Section 6 provides the conclusion.

## 2.1 Traditional Fractal Image Coding

In the fractal encoding scheme, an input image of size $M \times N$ is partitioned into two types of blocks: range blocks and domain blocks. The range blocks constitute a set of non-overlapping image blocks of size $B \times B$, denoted as $R_i$, where $i = 1, 2, \dots, \text{NumR}$ and NumR represents the total number of range blocks. The domain blocks form a set of overlapping image blocks of size $L \times L$, denoted as $D_j$, where $j = 1, 2, \dots, \text{NumD}$ and NumD denotes the total number of domain blocks. Domain blocks are obtained by sliding an $L \times L$ window across the image to construct the domain block pool, with $L > B$. The domain block size is typically set to four times that of the range block, i.e., $L = 2B$. Furthermore, domain blocks are scaled to the size of range blocks and extended by eight isometric transformations to facilitate subsequent block matching. For each range block $R_i$, every domain block in the extended domain pool is compared with $R_i$ through intensity contrast and offset parameters determined by minimizing the following function:

$$\|R_i - (\alpha_{ij} \cdot D_j + \beta_{ij} \cdot I)\|^2$$

where $I$ denotes a unit vector of dimension NumR, and $\alpha_{ij}$ and $\beta_{ij}$ represent the contrast and offset parameters, respectively. These parameters can be computed using the least squares method as:

$$\alpha_{ij} = \frac{\langle R_i - \bar{R}_i, D_j - \bar{D}_j \rangle}{\|D_j - \bar{D}_j\|^2}, \quad \beta_{ij} = \bar{R}_i - \alpha_{ij}\bar{D}_j$$

where $\bar{R}_i$ and $\bar{D}_j$ are the averages of $R_i$ and $D_j$, respectively, and $\langle \cdot, \cdot \rangle$ denotes the inner product. In the decoding process, any $M \times N$ image can be selected as the initial image, and the decoding process converges to the final decoded image after several iterations.

## 2.2 Fast Fractal Image Decoding

Fractal image decoding can be accelerated through two primary approaches: (1) Adopting appropriate initial images, and (2) Employing better iteration strategies. Regarding initial images, the approximate RAI, which closely approximates the input image, can effectively accelerate decoding [16]. The exact RAI is obtained by replacing the range blocks of the initial image with their respective averages as:

$$\mathrm{RAI} = \sum_{i=1}^{\mathrm{NumR}} \bar{R}_i \cdot \chi_{R_i}$$

where $\chi_{R_i}$ is the characteristic function for range block $R_i$. The decoding speed can be further improved by using RAI as the initial image under specific iteration strategies [17]. To our knowledge, no better initial image for fast fractal decoding has been proposed in recent years. Additionally, the collage image established during the encoding process can effectively shorten decoding for fractal image denoising and magnification rather than compression [18].

Regarding iteration strategies, one-buffer-decoding (OBD) performs mapping operations from a single buffer to itself, unlike conventional decoding algorithms that use two buffers. This allows reconstructed range blocks to benefit from previously reconstructed blocks, enabling each iteration to better approximate the final decoded image and effectively accelerating the decoding process [19].

In this study, we measure decoded image quality using peak signal-to-noise ratio (PSNR), defined as:

$$\mathrm{PSNR} = 10 \log_{10} \left( \frac{\mathrm{MAX}_I^2}{\mathrm{MSE}} \right)$$

where $\mathrm{MAX}_I$ is the maximum possible pixel value and MSE is the mean squared error between the input and decoded images. Previous work combining RAI with OBD effectively enhanced decoding convergence, achieving acceptable decoded image quality with only one iteration. For example, the RAI for the Peppers image in Fig. 1 achieves 23.15 dB PSNR with obvious block artifacts. After one iteration, the decoded image quality improves significantly, reaching 29.08 dB PSNR. This demonstrates that existing RAI-based methods can provide acceptable decoded image quality with a single iteration, simultaneously achieving real-time decoding and acceptable quality. This study aims to further improve decoded image quality while maintaining these benefits.

[Figure 1: see original paper] Illustration of RAI and its first iteration image.

## 3. Interpolation Based Initial Image

[Figure 2: see original paper] Illustration of the interpolation operations of IBII.

Since the intensities of most natural images change smoothly, we introduce transition regions between different range blocks to create a more natural appearance. This study uses four neighboring $4 \times 4$ range blocks (as shown in Fig. 2) to illustrate how IBII is constructed as the initial image for decoding. The four range blocks are replaced with a series of $2 \times 2$ blocks divided into three categories:

(1) **Average Blocks (ABs)**, denoted as $AB_l$ where $l = 1, 2, 3, 4$, are located at the center of range blocks and represented by green dashed blocks. ABs are obtained by replacing the central $2 \times 2$ blocks with the average of the range block:

$$AB_l = \bar{R}_l$$

(2) **Transition Blocks (TBs)**, denoted as $TB_{pq}$ where $p, q = 1, 2, 3$, are located between two neighboring ABs and represented by red dashed blocks. TBs are obtained by replacing the $2 \times 2$ blocks between two neighboring ABs with their average:

$$TB_{pq} = \frac{AB_p + AB_q}{2}$$

(3) **Central Blocks (CBs)** are located at the center of four neighboring ABs and represented by blue dashed blocks. CBs are obtained by replacing the $2 \times 2$ blocks with the average of the four surrounding ABs:

$$CB = \frac{AB_1 + AB_2 + AB_3 + AB_4}{4}$$

[Figure 3: see original paper] Illustration of IBII and its first iteration image.

For the Peppers image, the corresponding IBII is shown in Fig. 3(a). Compared with RAI in Fig. 1(a), IBII appears smoother and closer to the input image. The PSNR of IBII increases from 23.15 dB to 24.90 dB, confirming that IBII better approximates the input image than RAI. After one decoding iteration, the PSNR improves from 29.08 dB (RAI) to 30.42 dB (IBII), demonstrating that IBII provides higher decoded image quality than RAI with a single iteration.

Regarding computational complexity, division by 2 and 4 in the TB and CB calculations can be implemented via 1-bit and 2-bit right shifts, respectively. Constructing one TB requires 4 additions, while one CB requires 3 additions. Since each range block contains one AB, two TBs, and one CB, each range block in IBII requires 11 additions total. This computational overhead is negligible compared to the operations in each decoding iteration.

## 4.1 Experimental Setup

In our experiments, six $256 \times 256$ images—Peppers, House, Lake, Baboon, Boat, and Airplane—are used as test images. The range block size is set to $4 \times 4$, with a sliding step of 8. We evaluate the proposed method using Jacquin' s method and two state-of-the-art fast fractal coding methods: Chaurasia' s [12] and Gupta' s methods [13]. Both RAI and IBII are combined with OBD for fractal image decoding. In addition to PSNR, we adopt structural similarity (SSIM) to measure decoded image quality [20]:

$$\mathrm{SSIM}(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

where $x$ and $y$ represent corresponding local image patches in the reference and distorted images, respectively; $\mu_x$ and $\mu_y$ are their mean intensities; $\sigma_x$ and $\sigma_y$ are their standard deviations; and $\sigma_{xy}$ is the covariance between them. The constants are $C_1 = 0.01L^2$ and $C_2 = 0.03L^2$, where $L$ is the dynamic range of pixel values. For the entire image, we use mean SSIM (MSSIM) to evaluate overall quality:

$$\mathrm{MSSIM} = \frac{1}{M}\sum_{k=1}^{M} \mathrm{SSIM}(x_k, y_k)$$

where $M$ is the number of local windows, and $x_k$ and $y_k$ represent the $k$-th local window in the reference and distorted images, respectively.

[Figure 4: see original paper] Six test images.

## 4.2 Experimental Procedures

To compare the performance of the IBII-based method with the RAI-based method, the experimental procedures are designed as follows:

**Step 1:** Encode the input image.

**Step 2:** Compute ABs, TBs, and CBs using the equations above to obtain IBII. Then, use IBII as the initial image, perform one iteration of fractal decoding with OBD, and compute the PSNR and MSSIM of the decoded image.

**Step 3:** Use RAI as the initial image, perform one iteration of fractal decoding with OBD, and compute the PSNR and MSSIM of the decoded image.

**Step 4:** Compare the PSNR and MSSIM values of the decoded images from the RAI-based and IBII-based methods.

### 4.3 Experimental Results

As shown in Table 1 , compared with RAI, IBII improves the average PSNR from 22.28 dB to 23.26 dB and the average MSSIM from 0.6102 to 0.6613, confirming that IBII better approximates the input image. Furthermore, all PSNR values in Tables 2, 3, and 4 exceed 25 dB, with IBII providing higher PSNR and MSSIM for all test images.

For Jacquin's method (Table 2 ), the IBII-based method improves average PSNR from 27.28 dB to 28.04 dB and average MSSIM from 0.8598 to 0.8729 compared to the RAI-based method. For Chaurasia' s method (Table 3 ), the IBII-based method improves average PSNR from 27.04 dB to 27.78 dB and average MSSIM from 0.8522 to 0.8664. For Gupta' s method (Table 4 ), the IBII-based method improves average PSNR from 27.18 dB to 27.92 dB and average MSSIM from 0.8536 to 0.8666.

Across all test images and coding methods, the decoded image quality improves by 0.56–1.41 dB in PSNR and by 0.0061–0.0173 in MSSIM. These results demonstrate that IBII better approximates the input image than RAI and achieves higher decoded image quality with one iteration for various fractal coding methods. In summary, the proposed IBII-based method effectively improves decoded image quality while maintaining both acceptable quality and real-time decoding.

Performance comparison between RAI and IBII.

Performance comparison between previous and proposed methods for Jacquin' s method [2].

Performance comparison between previous and proposed methods for Chaurasia' s method [12].

Performance comparison between previous and proposed methods for Gupta' s method [13].

## 5. Conclusion

This paper aims to improve decoded image quality while maintaining real-time decoding and acceptable quality standards in the fractal decoding process. Previous RAI-based fractal image decoding achieved acceptable decoded image quality with only one iteration. To further enhance quality, the proposed IBII better approximates the original image, enabling higher decoded image quality with one iteration under specific decoding strategies. Additionally, IBII can be combined with other fast fractal decoding methods to achieve real-time decoding and enhance their utility in practical applications where extremely high decoded image quality is not strictly required.

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*Note: Figure translations are in progress. See original paper for figures.*