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Abstract

During operation, ground-based laser interferometer gravitational-wave detectors experience uncontrollable displacements in their internal optical systems due to various noise sources. To improve detector sensitivity and enable detection of weak gravitational-wave signals, length sensing and control systems must be employed to ensure that multiple optical cavities within the detector remain in resonance, upon which readout schemes for gravitational-wave signals are designed. Starting from the configuration of laser interferometer gravitational-wave detectors, this paper introduces the fundamental principles and parameter design criteria of length feedback control, and provides a detailed analysis of the working principles and applications of length sensing and control systems in gravitational-wave detectors, incorporating specific parameters from the currently operational Advanced LIGO, Advanced Virgo, and KAGRA control systems.

Full Text

Preamble

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Length Sensing and Control of Ground-based Laser Interferometric Gravitational Wave Detectors

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Abstract: During operation, ground-based laser interferometric gravitational wave detectors experience uncontrollable displacements in their internal optical systems due to various noise sources. To improve detector sensitivity and enable detection of faint gravitational wave signals, a length sensing and control system is essential. This system ensures that multiple optical cavities within the detector remain in resonance, thereby facilitating the design of gravitational wave signal readout schemes. Beginning with the configuration of laser interferometric gravitational wave detectors, this paper introduces the fundamental principles of length feedback control and parameter design criteria. Drawing on specific parameters from the currently operational Advanced LIGO, Advanced Virgo, and KAGRA control systems, we provide a detailed analysis of the working principles and applications of length sensing and control systems in gravitational wave detectors.

Keywords: feedback control systems; length sensing and control; laser interferometers; gravitational wave detectors

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1 Introduction

In 1915, Einstein’ s general theory of relativity profoundly revealed the nature of gravity and predicted the existence of gravitational waves, laying the theoretical foundation for gravitational wave astronomy. Since the invention of the laser in the 1960s, researchers have proposed using laser interferometry to measure gravitational waves. During the 1990s, kilometer-scale large laser interferometric gravitational wave detectors were successively constructed worldwide and became operational in the early 21st century. These first-generation detectors included LIGO (Laser Interferometer Gravitational-wave Observatory) in the United States, Virgo (a collaboration between Italy and France), GEO600 (a joint German-British project), and TAMA300 in Japan. Following years of construction and upgrades, second-generation laser interferometric gravitational wave detectors were commissioned: Advanced LIGO, Advanced Virgo, and KAGRA in Japan, all achieving higher gravitational wave detection sensitivity across the full frequency band. In September 2015, Advanced LIGO made the first direct detection of gravitational waves from a binary black hole merger, a milestone that opened a new observational window to the universe. In 2017, scientists not only detected gravitational waves from a binary neutron star system but also identified its electromagnetic counterpart, ushering in a new era of multi-messenger astronomy. Since then, the LIGO and Virgo collaborations have detected numerous gravitational wave signals from binary black holes, binary neutron stars, and other astrophysical sources.

Currently, third-generation gravitational wave detectors such as the Einstein Telescope (ET) in Europe and Cosmic Explorer (CE) in the United States are in the planning stages.

Ground-based gravitational wave detectors operate on the principle of Michelson interferometry, measuring length changes in the two arms to detect gravitational wave signals. However, gravitational wave signals are extremely weak; even strong signals produce length changes of only ~10^{-19} m in kilometer-scale detectors. Consequently, mirror displacements of even minute amplitude can affect detector precision and potentially mask gravitational wave signals. To further enhance sensitivity, modern gravitational wave detectors employ a dual-recycled Fabry-Perot interferometer configuration comprising a power recycling cavity, a signal recycling cavity, and two arm cavities. For each resonant cavity to function as designed, the cavity lengths must satisfy specific parameters and maintain the required resonance conditions. Nevertheless, detectors are subject to various noise sources, including seismic noise from earthquakes, tectonic movements, storms, tides, and human activity. These noise sources cause micron-level displacements of optical components at low frequencies, leading to rapid and unpredictable variations in the interferometer' s optical fields.

Therefore, to achieve the designed performance, gravitational wave detectors require active control systems to precisely position the mirrors forming the resonant cavities before operation, transitioning the interferometer from an uncontrolled, non-resonant state to a stable, controlled global state. Specifically, it is necessary to ensure that all resonant cavities within the interferometer remain in resonance and that the interferometer' s output port is positioned at the dark fringe—the "operating point" where the detector achieves optimal sensitivity for gravitational wave signals. During operation, real-time feedback control of each optical element is also required to maintain the detector at this operating point (i.e., "locked") capable of detecting faint gravitational wave signals.

This objective is accomplished through the length sensing and control (LSC) subsystem. The LSC subsystem comprises sensors that detect length offsets and actuators that make fine adjustments to the mirrors, working together through feedback control loops to achieve extremely precise mirror position control.

Section 2 covers the fundamentals and control principles of gravitational wave detector length control systems and describes the interferometer' s operating states during gravitational wave detection. Section 3 introduces gravitational wave signal readout schemes and analyzes the design methodology for interferometer sensing and control systems using Advanced LIGO, Advanced Virgo, and KAGRA as examples. Section 4 examines the characteristic parameters and technical details of length sensing and control systems using Advanced LIGO' s specific technical parameters as a case study. Finally, we provide a summary and outlook.

2 Overview of Length Sensing and Control Systems

Ground-based laser interferometric gravitational wave detectors contain multiple optical resonant cavities composed of suspended test masses and optical components. Laser light can only resonate in the interferometer when all optical components are precisely maintained at their designed positions. However, when the interferometer is uncontrolled, each optical component is affected by seismic noise, producing micron-level mirror displacements at low frequencies. This causes the interferometer to jump between multiple interference fringes, making it nearly impossible to extract gravitational wave signals. Consequently, precise length control is essential during detector operation.

The role of the length sensing and control (LSC) subsystem is to maintain optical resonance in each detector cavity, minimize noise interference, bring the detector to a state capable of detecting gravitational wave signals, and provide gravitational wave readout signals. The LSC subsystem comprises photodetectors, demodulation electronics, and filters. Its design includes developing interferometer sensing schemes and calculating parameters such as modulation frequencies and macroscopic cavity lengths. During detector operation, the LSC subsystem employs PDH techniques to feedback-control each length degree of freedom, transitioning the interferometer from an uncontrolled state to a globally controlled operating point and fine-tuning detector parameters to match target gravitational wave sources. The three processes required to achieve these objectives—lock acquisition, transition mode, and science mode—are described in Section 2.3.

2.1 Interferometer Configuration and Length Degrees of Freedom

As early as 1963, researchers proposed using Michelson interferometry to detect faint gravitational wave signals. To improve the response of ground-based laser interferometric gravitational wave detectors, a resonant cavity was added to each arm, forming a Fabry-Perot interferometer. As shown in Figure 1a [Figure 1: see original paper], this configuration contains three length degrees of freedom: Michelson differential length (MICH), differential arm length (DARM), and common arm length (CARM). These are expressed as:

$$\begin{aligned}\mathrm{MICH} &= l_x - l_y \\ \mathrm{DARM} &= L_x - L_y \\ \mathrm{CARM} &= L_x + L_y\end{aligned}$$

When a gravitational wave signal impinges perpendicularly, it stretches space in one direction while compressing it in the perpendicular direction. Consequently, the gravitational wave increases the length of one arm while decreasing the other. This characteristic directly corresponds to the DARM degree of freedom. In other words, gravitational wave signals couple to the DARM motion mode. Therefore, DARM is considered the most important length degree of freedom in gravitational wave detectors and is designated the primary degree of freedom, while the others are called auxiliary degrees of freedom.

In 1988, Meers proposed placing partially reflective mirrors at the laser input and output ports, called the power recycling mirror and signal recycling mirror, respectively. The first-generation LIGO built in the 1990s employed a power-recycled Fabry-Perot interferometer configuration. Second-generation detectors such as Advanced LIGO added a signal recycling system, creating the dual-recycled Fabry-Perot interferometer configuration shown in Figure 1b. The power recycling cavity increases the light power entering the interferometer, thereby enhancing the detector' s response to gravitational wave signals and helping to reduce laser noise. The signal recycling cavity reflects light carrying gravitational wave signals back into the interferometer, where it interferes constructively with light still circulating in the interferometer. This reduces the detector' s bandwidth while amplifying low-frequency gravitational wave signals. By adjusting the signal recycling cavity' s resonance condition, optimal sensitivity can be achieved at specific frequencies. The dual-recycling system further increases detector sensitivity but adds two length degrees of freedom that must be controlled: power recycling cavity length (PRCL) and signal recycling cavity length (SRCL), expressed as:

$$\mathrm{PRCL} = l_p + \frac{l_x + l_y}{2}$$
$$\mathrm{SRCL} = l_s + \frac{l_x + l_y}{2}$$

Today, most laser interferometric gravitational wave detectors worldwide employ or will employ the dual-recycled Fabry-Perot interferometer configuration with five length degrees of freedom requiring control. By placing photodetectors at different interferometer locations, the length sensing and control system obtains multiple readout ports, each providing error signals reflecting offsets in various length degrees of freedom. While the number and position of readout ports differ among detectors, they generally include the reflection port (REFL), picking-off port (POP), and anti-symmetric port (AS). Their positions are shown in Figure 1, corresponding to the reflected light field from the power recycling mirror, the circulating light field in the power recycling cavity, and the transmitted light field from the signal recycling mirror, respectively. Due to the interferometer' s characteristics, changes in CARM and DARM length degrees of freedom alter the intensity of reflected and transmitted light at the beam splitter. Therefore, in control scheme design, signals from the reflection or picking-off ports are typically used to control CARM, while signals from the anti-symmetric port control DARM. Additionally, transmitted light from the two arm cavities is also used to obtain sensing signals.

2.2 Introduction to Linear Feedback Control Systems

To ensure the detector remains in a state suitable for gravitational wave detection during operation, feedback control systems must be employed to control each length degree of freedom and satisfy resonance conditions. A typical control loop is shown in Figure 2 [Figure 2: see original paper].

A disturbance signal is injected into the plant, whose output is received by a sensor. A filter then shapes this signal before applying it to an actuator. Each component can be approximated as a linear time-invariant (LTI) system. For each LTI subsystem, the relationship between output and input is called a transfer function. For example, in the frequency domain, we can express the filter' s transfer function as $F = \text{output/input}$. Therefore, the cascaded transfer function around the control loop, known as the open-loop gain, is denoted by G and can be written as:

$$G = P \times S \times F \times A$$

where P, S, F, and A are transfer functions corresponding to the plant, sensor, filter, and actuator, respectively. When a disturbance is introduced, it superimposes with itself after circulating once through the loop. As shown in Figure 2, injecting a disturbance into the plant results in an actual signal entering the plant of:

$$\text{Closed-loop gain} = 1 + G$$

The closed-loop gain can be viewed as the transfer function from outside to inside the system. When a signal is injected into the loop, its amplitude is immediately multiplied by the closed-loop gain, becoming $1/(1+G)$ times the original signal. Therefore, by adjusting the open-loop gain G, different levels of signal suppression can be achieved. For systems with large open-loop gain ($G \gg 1$), external input signals are strongly suppressed. This characteristic can be exploited in gravitational wave detector control loop design to reduce noise effects.

In practice, LTI systems cannot respond infinitely fast and have finite response times. Specifically, for feedback control loops in gravitational wave detectors, the open-loop gain G decreases at high frequencies, establishing a maximum useful frequency. Generally, this frequency is defined as the unity gain frequency (UGF), where $G = 1$. The UGF represents the servo loop' s bandwidth—the highest frequency at which the control loop can effectively operate.

For a single Fabry-Perot cavity, PDH (Pound-Drever-Hall) technology is required for precise cavity length control. The positions of the plant, sensor, filter, and actuator in this system are shown in Figure 3 [Figure 3: see original paper].

An electro-optic modulator (EOM) is inserted into the optical path, driven by a signal generator called the local oscillator. The local oscillator' s signal drives the EOM at a modulation frequency, typically in the radio frequency (RF) band. When added to the optical path, this creates a pair of sidebands on either side of the incident laser frequency. By appropriately setting the modulation frequency, the two sidebands can be placed far from the cavity resonance, meaning most sideband frequency light is reflected. Qualitatively, the phases of the two sidebands are essentially unaffected by cavity length changes. In contrast, the incident laser (the carrier) resonates in the arm cavity, and its phase variations carry information about cavity length changes. By comparing these differences, the cavity length offset can be determined.

Specifically, the sensor system consists of a photodetector and a mixer, which convert the optical signal to an electrical signal and then mix it with the local oscillator signal, respectively. The demodulation phase in phase with the signal generator is called the I-phase, while the orthogonal direction is called the Q-phase. After processing by a filter, the sensor system’ s output yields an error signal representing both the magnitude and direction of cavity length adjustments.

The error signal shape is shown in Figure 4 [Figure 4: see original paper], displaying a bipolar signal crossing zero. For a single Fabry-Perot cavity system, the zero-crossing point of the error signal is the operating point, corresponding to the cavity length precisely satisfying the resonance condition. The control system’ s function is to maintain the cavity at this operating point. In a small region near the operating point, the error signal varies linearly with mirror displacement. The width of this linear region is determined by the cavity’ s finesse. Within this range, the error signal is proportional to the system’ s deviation from its operating point, and the tangent slope dE/dx at the operating point reflects the control system gain. Finally, the error signal is fed back to actuators connected to the optical components, typically using piezoelectric ceramics for fine cavity length adjustments.

This control system has only one input (cavity length variation) and one output (error signal obtained through PD demodulation), making it a single-input single-output (SISO) system.

However, laser interferometric gravitational wave detectors have multiple readout ports and length degrees of freedom that must be controlled. When one degree of freedom changes length, the voltage at all photodetectors in all readout ports changes. Furthermore, demodulation frequency and phase during readout also affect signal magnitude. Unlike SISO systems, this situation is called a multiple-input multiple-output (MIMO) system.

For MIMO systems, the relationship between inputs and outputs cannot be described by simple transfer functions but requires a matrix to describe the relationship between length variations $\mathbf{L}$ and readout voltages $\mathbf{P}$, known as the sensing matrix $\mathbf{M}$:

$$\mathbf{P} = \mathbf{M} \cdot \mathbf{L}$$

Each element in the sensing matrix is the derivative of the output signal with respect to optical component displacement, numerically equal to the slope of the error signal near the operating point. This represents the sensitivity of readout voltage to length changes, with units of $\mathrm{W \cdot m^{-1}}$. The presence of power and signal recycling cavities creates strong coupling between length degrees of freedom. The recycling cavity states significantly affect the circulating light power in arm cavities, while arm cavity states determine the interferometer' s output signal. Consequently, the sensing matrix is typically non-diagonal, requiring careful selection of appropriate error signals to control each length degree of freedom and ensure that weakly sensed degrees of freedom (such as SRCL) are not overwhelmed by sensing noise from other degrees of freedom (such as CARM). Sato et al. developed a control scheme that diagonalizes the sensing matrix, reducing detector noise effects and increasing control robustness.

By comparing the orders of magnitude of sensing matrix elements, optimal readout ports and error signal demodulation frequencies can be selected to control each length degree of freedom. This creates multiple control loops, each with corresponding gain. In addition to the main length control loops, auxiliary loops are required to minimize additional sensing noise introduced by the control system itself.

2.3 Interferometer Operating States

The LSC subsystem' s task is to transition the interferometer from an unlocked state to a globally controlled state. It uses PDH techniques to feedback-control each degree of freedom, maintaining the detector at the operating point and minimizing noise interference. Additionally, interferometer length parameters must be fine-tuned for different gravitational wave sources to achieve optimal sensitivity in specific frequency bands. Achieving these objectives requires three processes:

(1) **Lock Acquisition**: This process aims to transition each length degree of freedom from globally uncontrolled to globally controlled and bring them to the operating point. When the interferometer is uncontrolled, mirrors can move freely with displacements spanning multiple laser wavelengths. Due to multiple coupled length degrees of freedom, the system response is highly nonlinear before reaching the operating point. When using PDH feedback control, each degree of freedom' s error signal varies linearly with length offset only in a small region near the operating point. The linear region widths for DARM and CARM are determined by arm cavity finesse, while those for PRCL and SRCL are determined by power and signal recycling cavity finesse, respectively. Evans first proposed the interferometer lock acquisition process in 2002, solving the critical problem of bringing each gravitational wave detector length degree of freedom to the operating point. Due to the complexity of second-generation detector lock acquisition, Advanced LIGO employs techniques such as triple demodulation and green-light auxiliary locking during lock acquisition, while Advanced Virgo misaligns the power recycling cavity, first locks the two arm cavities, then

treats the entire system as a simple Michelson interferometer for control, and finally locks the two recycling cavities.

(2) **Transition Mode**: In this stage, detector parameters such as incident laser power are adjusted from configurations optimal for lock acquisition to modes suitable for gravitational wave data collection.

(3) **Science Mode**: Gravitational wave signals from different types of sources at different distances vary in frequency and intensity. In this stage, detector parameters such as signal recycling cavity detuning angle are fine-tuned to achieve optimal sensitivity at specific frequencies, matching detector configuration to scientific objectives.

3 Design of Length Sensing and Control Schemes

In addition to stabilizing interferometer length degrees of freedom during operation, the LSC subsystem for ground-based laser interferometric gravitational wave detectors is responsible for developing interferometer sensing and control schemes, specifically determining modulation frequencies (the local oscillator frequencies driving electro-optic modulators in PDH techniques). Furthermore, since gravitational wave signals interact with detectors by changing cavity lengths, the LSC subsystem also provides gravitational wave signal readout schemes. This chapter addresses both aspects.

3.1 Gravitational Wave Signal Readout Scheme

Typically, gravitational wave detectors operate at the dark fringe, where output port light interferes destructively through control of the two arm lengths. Gravitational wave signals interact with the interferometer by increasing one arm’ s length while decreasing the other. This length change, coupled to the DARM mode, phase-modulates the carrier light in the arm cavities, generating gravitational wave signal sidebands that transmit to the anti-symmetric port. The absolute frequency of the generated gravitational wave sidebands is:

$$f_s = f_c \pm f_g$$

where f_s is the signal frequency at the output port, f_c is the carrier frequency, and f_g is the gravitational wave signal frequency, typically in the audio band. Since f_s is on the order of 10^{14} Hz, direct detection is impossible. A local oscillator must be provided to beat with the gravitational wave sidebands, enabling readout of the lower-frequency gravitational wave signal f_g. Three schemes exist for providing a stable local oscillator at the readout port: heterodyne readout, homodyne readout, and DC readout.

Heterodyne readout was primarily used in first-generation detectors. This technique intentionally introduces an asymmetry in the MICH degree of freedom, making the distances from the beam splitter to the two arm cavities unequal.

This asymmetry magnitude is called the Schnupp asymmetry, $l_{\mathrm{sch}} = l_x - l_y$. Its function is to allow RF sidebands to transmit to the anti-symmetric port while the carrier remains at the dark fringe, serving as the local oscillator. The RF sidebands then beat with gravitational wave signal sidebands and are demodulated to obtain f_g.

Homodyne readout uses a beam splitter to divert a small portion of carrier light directly to the output port as the local oscillator without entering the interferometer. Since the local oscillator and signal traverse different optical paths, extremely high demands are placed on carrier light collimation and stability for frequency reference. The local oscillator path must be in a vacuum system with active vibration isolation. Consequently, homodyne readout imposes stringent hardware requirements and is difficult to implement in practice.

DC readout is a special case of homodyne readout that more easily integrates with current gravitational wave detector hardware. In DC readout, a small offset is introduced in the arm cavity lengths—a dark fringe offset that slightly moves the interferometer's operating point away from the dark fringe, allowing a certain amount of carrier light to reach the output port as the local oscillator. In DC readout, the local oscillator and gravitational wave signal share the same optical system, ensuring optimal spatial mode-matching. Additionally, DC readout offers higher signal-to-noise ratio, fewer beat fields at the readout port (reducing noise), and a simpler system compared to heterodyne readout. Due to these advantages, most second-generation detectors employ DC readout. Note that in DC readout, the MICH degree of freedom also incorporates Schnupp asymmetry, but its primary role is to create different reflectivities for the beam splitter at different RF sideband frequencies, which is crucial for controlling the SRCL degree of freedom, as discussed in Chapter 3.

3.2 Modulation Frequency and Recycling Cavity Lengths

Laser interferometric gravitational wave detectors are complex MIMO systems. To simultaneously sense and control multiple length degrees of freedom, the lengths of various cavities and RF sideband modulation frequencies must be carefully designed. These RF sidebands beat with the carrier or other sidebands, producing beat signals containing motion information from various length degrees of freedom, typically coupled together. Therefore, for detectors with multiple cavities, most second-generation ground-based laser interferometric gravitational wave detectors use multiple modulation frequencies and arrange them to resonate in different interferometer cavities to maximize independent extraction of each degree of freedom' s information.

Figure 5 [Figure 5: see original paper] illustrates a typical gravitational wave detector configuration. The laser first enters a triangular resonant cavity called the input mode cleaner (IMC), which primarily removes non-fundamental spatial modes from the incident beam. Sidebands must resonate in the IMC to enter subsequent optical systems. Importantly, all RF sidebands are non-resonant in the interferometer' s two arm cavities. Therefore, another factor must be considered when calculating modulation frequencies: they must avoid arm cavity resonance frequencies. The resonance condition for a cavity is that its length must be an integer multiple of half the laser wavelength, i.e., $2L = n\lambda$. The concept of free spectral range (FSR) is useful here. The free spectral range is defined as $\Delta f_{\mathrm{FSR}} \equiv c/2L$, where c is the speed of light, representing the frequency spacing between adjacent resonant modes in a cavity of length L. Therefore, sideband frequencies that can resonate in a cavity must be integer multiples of that cavity' s free spectral range. In summary, all sideband frequencies used for optical system control must satisfy:

$$f_{\mathrm{mod}} = n\frac{c}{2L_{\mathrm{mc}}}; \quad f_{\mathrm{mod}} \neq k\frac{c}{2L_{\mathrm{arm}}}$$

where n and k are positive integers, f_{mod} is the modulation frequency, L_{mc} is the mode cleaner cavity length, and L_{arm} is the arm cavity length.

After passing through the IMC, the beam enters the power recycling cavity (PRC). Modulation frequencies are divided into two types based on whether they resonate in the PRC.

(1) **Resonant modulation frequencies**: Frequencies that resonate in the power recycling cavity but not in the arm cavities. Second-generation detectors typically use two resonant modulation frequencies simultaneously. These two frequencies work together to control arm cavity lengths and align the beam splitter (BS). Resonant modulation frequencies impose constraints on the power recycling cavity length:

$$L_{\mathrm{prc}} = N\frac{c}{2f_{\mathrm{mod}}}$$

where N is an integer (0, 1, 2, ⋯). During detector design, parameter N can be selected based on practical considerations such as vacuum system length.

(2) **Non-resonant modulation frequency**: A frequency that does not resonate in any optical system except the IMC. Typically, its frequency offset from resonant frequencies is set to several times the cavity linewidth to ensure it does not resonate in the power recycling cavity. It is almost completely reflected by the power recycling mirror, producing a "reflection phase shift" of zero. Due to this property, it is nearly unaffected by subsequent optical systems, providing a stable phase reference and primarily serving to control the signal recycling mirror position.

When designing modulation frequencies for gravitational wave detectors, hardware limitations must also be considered. Generally, modulation frequencies should be below 50 MHz, primarily limited by photodetectors—at higher frequencies, no photodetectors with sufficient response speed and aperture are available for readout.

Using the currently operational Advanced LIGO, Advanced Virgo, and KAGRA detectors as examples, we can analyze gravitational wave detector control system design principles based on their technical parameters.

Advanced LIGO' s input mode cleaner has a length of $L_{\mathrm{mc}} = 32.9$ m. According to the equation above, modulation frequencies should be integer multiples of 4.54 MHz. The sensing and control scheme uses two RF modulation frequencies: $f_1 = 9$ MHz and $f_2 = 45$ MHz, corresponding to 2 and 10 times the input mode cleaner' s free spectral range, respectively. Using $N = 3$ in the equation yields $L_{\mathrm{prc}} = 57$ m. Sidebands at both frequencies can resonate in the power recycling cavity. For the signal recycling cavity (SRC), the 45 MHz sideband is near resonance while the 9 MHz sideband is non-resonant. Therefore, the two modulation frequencies must satisfy:

$$L_{\mathrm{src}} = M\frac{c}{2f_1}; \quad L_{\mathrm{src}} \neq K\frac{c}{2f_2}$$

where L_{src} is the signal recycling cavity length and M and K are integers. Additionally, neither sideband resonates in the arm cavities.

Advanced Virgo' s configuration is similar to Initial LIGO, with a partially reflective mirror added at the input port, resulting in four primary length degrees of freedom: DARM, CARM, MICH, and PRCL. Advanced Virgo' s input mode cleaner has $L_{\mathrm{mc}} = 143.4$ m and power recycling cavity length $L_{\mathrm{prc}} = 11.95$ m. To control four length degrees of freedom, Advanced Virgo' s LSC system employs three RF modulation frequencies: $f_1 = 6.27$ MHz, $f_2 = 9f_1 = 56.43$ MHz, and $f_3 = 8.36$ MHz. The relationships between these three modulation frequencies and the free spectral ranges of the input mode cleaner and power recycling cavity are shown in Table 1 .

Table 1 shows that all modulation frequencies are integer multiples of the input mode cleaner' s free spectral range. Frequencies f_1 and f_2 are resonant modulation frequencies used to control arm cavities. To reach subsequent optical systems, sidebands at f_1 and f_2 must resonate in the power recycling cavity, requiring their frequencies to satisfy the PRC resonance condition. Frequency f_3 is a non-resonant modulation frequency that does not resonate in any main interferometer cavity and is used exclusively to control the PRCL degree of freedom.

The KAGRA gravitational wave detector, located underground in the Kamioka mine in Japan, is a cryogenic laser interferometric gravitational wave detector. The main interferometer contains four cryogenic mirrors cooled to approximately 20 K to reduce thermal noise, forming two 3 km arm cavities. KAGRA has five length degrees of freedom: DARM, CARM, MICH, PRCL, and SRCL. KAGRA' s LSC scheme uses three modulation frequencies: $f_1 = 16.875$ MHz, $f_2 = 45.0$ MHz, and $f_3 = 56.26$ MHz. Their resonance conditions in the input mode cleaner are shown in Table 2 .

In Table 2, f_1 and f_2 are resonant modulation frequency sidebands obtained through phase modulation of the main laser. Compared to Advanced Virgo, KAGRA' s optical configuration adds a signal recycling cavity. As shown in Table 2, by setting the Schnupp asymmetry, only the f_1 sideband resonates in the signal recycling cavity among the two resonant sidebands, thus carrying information about both PRCL and SRCL degrees of freedom. Conversely, the f_2 sideband is only sensitive to power recycling cavity length changes. Frequency f_3 is a non-resonant modulation frequency sideband that does not resonate anywhere in the interferometer except the input mode cleaner, providing a stable local oscillator field for the carrier and other RF sidebands during lock acquisition. By having the three modulation frequency sidebands resonate at different interferometer locations, independent control of each degree of freedom can be maximally achieved experimentally.

Based on Advanced LIGO, Advanced Virgo, and KAGRA, ground-based laser interferometric gravitational wave detectors contain multiple length degrees of freedom and typically use two resonant modulation frequencies that work together to control length degrees of freedom. Building upon this, non-resonant modulation frequency sidebands are added to increase lock acquisition robustness and reduce coupling between multiple degrees of freedom.

4 Advanced LIGO Length Sensing and Control System

This chapter provides a detailed introduction to length sensing and control systems using Advanced LIGO as an example. Advanced LIGO' s optical configuration, shown in Figure 6 [Figure 6: see original paper], contains multiple length degrees of freedom. The LSC system must precisely control each degree of freedom and enable the interferometer to operate in different modes to achieve good gravitational wave signal sensitivity across a wide frequency band. By fine-tuning the signal recycling cavity length, sensitivity enhancement can be achieved for specific frequency bands—the "science mode" introduced in Chapter 2. Specifically, Advanced LIGO has three operating modes:

(1) **Mode 0**: The input laser power is 25 W. No signal recycling mirror is installed, resulting in four length degrees of freedom. The detector operates more quickly in this mode.

(2) **Mode 1**: Error signals for all degrees of freedom are more easily obtained. The signal recycling mirror is added and locked to resonance. The detector can operate at various input powers ranging from 25 to 125 W, further improving sensitivity.

(3) **Mode 2**: The detector uses maximum input laser power of 125 W. Additionally, the LSC system introduces a small offset in the signal recycling mirror position from resonance, placing the signal recycling cavity in a detuned state to achieve optimal sensitivity for specific scientific targets. For example, for binary neutron star gravitational wave sources within 200 Mpc, the optimal detuning angle is approximately 18°.

Table 3 shows Advanced LIGO’ s sensing matrix in Mode 1 operation. Due to coupling among the detector’ s five length degrees of freedom, Advanced LIGO’ s sensing matrix is not diagonalized. Therefore, appropriate signals must be selected from error signals at different demodulation frequencies and phases at various readout ports to control each length degree of freedom. The selection criterion is that for the same readout port, the error signal of the controlled degree of freedom should have the maximum slope compared to the remaining four degrees of freedom—i.e., the highest sensitivity. This minimizes crosstalk from length variations in other degrees of freedom and improves control system robustness.

The control loops used for control are highlighted in red and called primary matrix elements. Blue elements have large magnitudes comparable to primary matrix elements, representing the largest interference terms.

The main characteristics of Advanced LIGO’ s five length degree of freedom control loops in Mode 2 are shown in Table 4 . For the primary degree of freedom DARM, DC readout is employed with a dark fringe offset of approximately 12 pm, corresponding to about 0.1 W of carrier light power at the anti-symmetric port. Error signals for other length degrees of freedom are obtained by demodulating signals at the REFL and POP ports at frequencies f_1, f_2, $f_2 + f_1$, and $f_2 - f_1$.

For example, in error signals used to control the SRCL degree of freedom, M indicates demodulation at frequency $f_2 - f_1$ and P indicates demodulation at $f_2 + f_1$. This scheme of demodulating at the beat frequency of two modulation frequencies is called double demodulation. First developed on Caltech’ s 40 m gravitational wave detector prototype, this technique primarily reduces coupling between DARM, CARM, and the other three length degrees of freedom. The photodetector’ s photocurrent is mixed sequentially with local oscillator signals at frequencies f_1 and f_2, producing signals at $f_2 + f_1$ and $f_2 - f_1$. After low-pass filtering, these two frequency signals are summed to obtain signals suitable for controlling short degrees of freedom (MICH, SRCL, and PRCL). This is a heterodyne detection technique. Since neither RF sideband resonates in the arm cavities, they are unaffected by arm cavity motion, thus isolating the control signals from carrier phase offsets caused by arm cavity motion. By carefully selecting demodulation phases, MICH, PRCL, and SRCL degrees of freedom can be maximally decoupled.

Through control loop design, control of primary length degrees of freedom can be achieved. However, the length sensing and control system includes components such as photodetectors, demodulators, and piezoelectric ceramics that generate noise during operation. In other words, all control processes introduce additional uncontrollable displacements in optical components, adding noise across the detector’ s operating band—known as sensing noise. This noise couples into the primary DARM degree of freedom through known mechanisms, with intensity of approximately 9.5×10^{-21} $\mathrm{m} \cdot \mathrm{Hz}^{-1\ 2}$ at 100 Hz, sufficient to affect detector sensitivity.

To reduce sensing noise impact, correction paths are introduced. The primary goal of correction paths is to increase DARM control precision to reduce noise in gravitational wave signal readout. Since DARM corresponds to opposite-phase length changes in the two arm cavities, control signals must be sent to the interferometer’ s two end test masses (ETMX and ETMY) to cancel introduced sensing noise. The correction path consists of three control loops that send PRCL-, SRCL-, and MICH-sensing-noise-limited control signals to the two end test masses. The PRCL path has approximately 10% precision, while the latter two have about 1% precision. Through coordination between the LSC subsystem and other subsystems, Advanced LIGO can achieve sensitivity of 10^{-23} Hz$^{-1\ 2}$.

5 Summary and Outlook

Ground-based laser interferometric gravitational wave detectors measure gravitational wave signals by detecting length changes in the two arms. To improve detector sensitivity, arm cavities, power recycling cavities, and signal recycling cavities are introduced based on Michelson interferometry. However, because gravitational wave signals are extremely weak, environmental noise can mask the signals. Therefore, during detector operation, precise control of each optical component’ s position through length sensing and control systems is essential to maintain optical resonance in multiple cavities.

This paper detailed linear control systems, explaining the components and control principles of feedback control for cavity lengths using PDH techniques. It then introduced methods for reading out gravitational wave signals based on detector length variations. Using Advanced LIGO, Advanced Virgo, and KAGRA as examples, we analyzed calculation methods for parameters such as cavity lengths and modulation frequencies in length sensing and control schemes, and how multiple RF sidebands resonating in different interferometer sections solve the problem of coupling between multiple length degrees of freedom. Finally, through analysis of Advanced LIGO’ s technical details, we systematically introduced current gravitational wave detector length sensing and control systems, providing references for designing detector sensing and control schemes.

Since detectors read out gravitational wave signals by measuring length changes, length sensing and control system performance and parameters directly determine detector sensitivity. Currently, the LIGO research team has proposed the “Sensor Fusion” method, which reduces control system sensitivity to input noise by utilizing noise correlations between sensors. Additionally, the recently proposed “H-infinity” method has been tested on the KAGRA detector and confirmed to improve feedback control system performance. These advances enhance both sensing and control system stability and gravitational wave detector capability. In practice, customized control schemes must be designed for different detector configurations. For the currently planned “Advanced Virgo plus” upgrade project, research has already designed robust length sensing and control schemes based on its more complex optical configuration. In 2023, Zhang et al. proposed a new interferometer configuration using L-shaped arm cavities to enhance gravitational wave signals in the kilohertz band, with a matching sensing and control scheme already developed. In-depth research on length sensing and control system design is crucial for next-generation gravitational wave detector development and represents an essential requirement for China’ s independent development of next-generation laser interferometric gravitational wave detectors.

References

[1] Einstein A. Sitzungsberichte der Physikalisch-mathematischen Klasse. Berlin: Preuss Akad Wiss, 1915, 1915: 844

[2] Sigg D. Classical and Quantum Gravity, 2006, 23(8): S51

[3] Acernese F, Amico P, Arnaud N, et al. Classical and Quantum Gravity, 2003, 20(17): S609

[4] Willke B, Aufmuth P, Aulbert C, et al. Classical and Quantum Gravity, 2002, 19(7): 1377

[5] Takahashi R, TAMA collaboration. Classical and Quantum Gravity, 2004, 21(5): 403

[6] Harry G M. Classical and Quantum Gravity, 2010, 27(8): 084006

[7] Acernese F, Agathos M, Agatsuma K, et al. Classical and Quantum Gravity, 2014, 32(2): 024001

[8] Somiya K. Classical and Quantum Gravity, 2012, 29(12): 124007

[9] Abbott B P, Abbott R, Abbott T D, et al. Physical review letters, 2016, 116(6): 1102

[10] Abbott B P, Abbott R, Abbott T D, et al. Physical review letters, 2017, 119(16): 1101

[11] Punturo M, Abernathy M, Acernese F, et al. Classical and Quantum Gravity, 2010, 27(19): 4002

[12] Hall E D, Kuns K, Smith J R, et al. Physical Review D, 2021, 103(12): 2004

[13] Saulson P R. Fundamentals of Interferometric Gravitational Wave Detectors. USA: World Scientific Publishing Company, 1994: 62

[14] Abbott R, Adhikari R, Ballmer S, et al. https://dcc.ligo.org/ligo-t1000298/public, 2025

[15] Adhikari R, Ballmer S, Fritschel P. https://dcc-llo.ligo.org/public/0022/T070236/001/T070236-00.pdf,

[16] Allocca A, Bersanetti D, Casanueva D J, et al. Galaxies, 2020, 8(4): 85

[17] Meers B J. Physical Review D, 1988, 38(8): 2317

[18] Abramovici A, Althouse W E, Drever R W, et al. Science, 1992, 256: 325

[19] Drever R W, Hall J L, Kowalski F V, et al. Applied Physics B, 1983, 31: 97

[20] Bond C, Brown D, Freise A, et al. Living Reviews in Relativity, 2016, 19: 1

[21] Evans M. Dissertation. California: California Institute of Technology, 2002: 836

[22] Staley A, Martynov D, Abbott R, et al. Classical and Quantum Gravity,

2014, 31: 5010
[23] Sato S, Kawamura S, Kokeyama K, et al. Physical Review D, 2007, 75(8): 082004
[24] Bassan M. Astrophysics and Space Science Library, 2014, 404: 275
[25] Barsotti L, Evans M. https://dcc.ligo.org/LIGO-T1000294/public, 2025
[26] Acernese F, Alshourbagy M, Amico P, et al. Astroparticle Physics, 2008, 30(1): 29
[27] Heinzel G, Mizuno J, Schilling R, et al. Physics Letters A, 1996, 217(6): 305
[28] Schnupp L. Presentation at European Collaboration Meeting on Interferometric Detection of Gravitational Waves. Sorrent, Italy, 1988
[29] Hild S, Grote H, Degallaix J, et al. Classical and Quantum Gravity, 2009, 26(5): 055012
[30] Aso Y, Michimura Y, Somiya K, et al. Physical Review D, 2013, 88(4): 043007
[31] Aso Y, Somiya K, Miyakawa O. Classical and Quantum Gravity, 2012, 29(12): 124008
[32] Acernese F, Agathos M, Aiello L, et al. Astroparticle Physics, 2020, 116: 102386
[33] Fritschel P, Coyne D. https://dcc.ligo.org/LIGO-T010075/public, 2025
[34] Evans M. https://dcc.ligo.org/LIGO-T070260-x0/public, 2025
[35] Abbott R, Adhikari R, Ballmer S, et al. https://dcc.ligo.org/LIGO-T070247/public, 2025
[36] Ward R L. https://inspirehep.net/files/ca235e19469066a8d6241e1231b29576, 2025
[37] Barr B, Miyakawa O, Kawamura S, et al. Classical and Quantum Gravity, 2006, 23(18): 5661
[38] Bowman D. https://dcc.ligo.org/LIGO-T2300218/public, 2025
[39] Tsang T, Arellano F E P, Ushiba T, et al. https://arxiv.org/abs/2407.15972, 2025
[40] Ushiba T, Kagra Collaboration, et al. Proceedings of the XVIII International Conference on Topics in Astroparticle and Underground Physics 2023, Italy: Proceedings of Science, 2024: 116
[41] Valentini M. Dissertation. Trento: University of Trento, 2023: 178
[42] Zhang T, Yang H, Martynov D, et al. Physical Review X, 2023, 13(2): 021019
[43] Guo X, Zhang T, Martynov D, et al. Classical and Quantum Gravity, 2023, 40(23): 235005

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