Extracting Transport Properties of Quark-Gluon Plasma from the Heavy-Quark Potential With Neural Networks in a Holographic Model

Authors: Dai, Mr. Wen-Chao, Luo, Mr. Ou-Yang, Chen, Bing, Chen, Dr. Xun, Zhu, Dr. Yan Xiao, Li, Dr. Xiao-Hua, Chen, Dr. Xun

Date: 2025-06-27T23:54:35+00:00

Abstract

Using Kolmogorov-Arnold Networks (KANs), we construct a holographic model informed by lattice QCD data. This neural network approach enables the derivation of an analytical solution for the deformation factor $w(r)$ and the determination of a constant g related to the string tension. Within the KANs-based holographic framework, we further analyze heavy quark potentials under finite temperature and chemical potential conditions. Additionally, we calculate the drag force, jet quenching parameter, and diffusion coefficient of heavy quarks in this paper. Our findings demonstrate qualitative consistency with both experimental measurements and established phenomenological model.

Full Text

Preamble

Extracting Transport Properties of Quark-Gluon Plasma from the Heavy-Quark Potential With Neural Networks in a Holographic Model

Wen-Chao Dai,[1] Ou-Yang Luo,[1] Bing Chen,[1] Xun Chen,[1,2,3,*] Xiao-Yan Zhu,[4,†] and Xiao-Hua Li[1,2,‡]

[1]School of Nuclear Science and Technology, University of South China, No. 28, West Changsheng Road, Hengyang City, Hunan Province, China
[2]Key Laboratory of Advanced Nuclear Energy Design and Safety, Ministry of Education, Hengyang, 421001, China
[3]INFN – Istituto Nazionale di Fisica Nucleare – Sezione di Bari, Via Orabona 4, 70125 Bari, Italy

[4]School of Mathematics and Physics, University of South China, No. 28, West Changsheng Road, Hengyang City, Hunan Province, China

(Dated: June 27, 2025)

Abstract

Using Kolmogorov-Arnold Networks (KANs), we construct a holographic model informed by lattice QCD data. This neural network approach enables the derivation of an analytical solution for the deformation factor $w(r)$ and the determination of a constant g related to the string tension. Within the KANs-based holographic framework, we further analyze heavy quark potentials under finite temperature and chemical potential conditions. Additionally, we calculate the drag force, jet quenching parameter, and diffusion coefficient of heavy quarks in this paper. Our findings demonstrate qualitative consistency with both experimental measurements and established phenomenological models.

∗ chenxunhep@qq.com
† xyzhu0128@163.com
‡ lixiaohuaphysics@126.com

Introduction

High-energy nuclear collisions conducted at facilities such as the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) provide a unique opportunity to study the properties of the quark-gluon plasma (QGP), a deconfined state of matter formed under extreme temperatures and densities. The QGP offers a window into the fundamental aspects of QCD, and understanding its dynamics is crucial for advancing our knowledge of strong interactions. However, the computational and theoretical resolution of QCD still faces many challenges in practical applications and complex environments. To address these challenges, the Anti-de Sitter/conformal field theory (AdS/CFT) correspondence provides a powerful framework. Through this dual relationship, strong interactions can be explored in higher-dimensional spaces [?, ?, ?], offering new perspectives on the complex behavior of QGP.

Experimental and theoretical studies have found that heavy quarks are an important tool in the study of finite-temperature QCD matter due to their unique behavior under extreme conditions [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. Under these conditions, the dissociation of heavy quark-antiquark pairs serves as a key indicator of strong interaction deconfinement, and thus it is critical to investigate the heavy quark potential within holographic QCD frameworks. Early work has tentatively revealed the characteristics of the holographic quark-antiquark pair potential energy [?, ?]. However, as research progresses, understanding the dynamics of the potentials between quarks becomes essential to reveal the complexity of QCD matter.

Jets and heavy quark diffusion are among the most useful probes for investigating the properties of the QGP at different scales. Jets, produced from high-energy partons in collisions, undergo complex processes such as energy loss, medium-induced radiation, and medium response as they traverse the QGP. The substructure of jets, particularly observables like the Energy-Energy Correlator (EEC), provides detailed insights into the interaction between jets and the QGP. The EEC is highly sensitive to the angular distribution of energy within jets, revealing the interplay of mass effects, energy loss, and medium response. Studies have shown a clear flavor hierarchy in the EEC for both vacuum and QGP environments, driven by the mass effect of heavy quarks (e.g., charm and bottom quarks). By analyzing heavy-flavor jets, researchers can probe the mass dependence of jet substructure and jet-medium interactions, offering a deeper understanding of QGP dynamics [?, ?, ?, ?, ?, ?, ?, ?, ?].

In parallel, the diffusion of heavy quarks in the QGP is a critical aspect of understanding the hydrodynamic behavior of the plasma. Heavy quarks, such as charm and bottom quarks, are produced in the early stages of collisions and participate in the entire evolution of the QGP. The spatial diffusion coefficient quantifies the momentum transfer from the QGP to heavy quarks and provides insights into the hydrodynamization process. Recent lattice QCD calculations with dynamical quarks have revealed that the heavy quark diffusion coefficient is significantly smaller than previous estimates from quenched lattice QCD and phenomenological models. This suggests that heavy quarks hydrodynamize very quickly in the QGP, highlighting the near-perfect fluidity of the medium [?, ?, ?, ?, ?, ?, ?, ?].

In recent years, machine learning techniques, especially multilayer perceptrons (MLPs), have shown unprecedented potential in solving complex scientific problems [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. According to the Universal Approximation Theorem, MLPs are able to approximate arbitrarily complex functions by increasing the number of neurons in the hidden layers, a feature that makes them perform well in solving partial differential equations (PDEs). Recently, Kolmogorov-Arnold Networks (KANs) have been proposed in Refs. [?, ?]. Unlike traditional MLPs, KANs fundamentally eliminate the reliance on linear weight matrices by using learnable functions instead of fixed activation functions. For small-scale AI and scientific tasks, KANs may offer advantages in terms of accuracy and interpretability [?, ?]. When compared with conventional methods such as polynomial fitting, KANs may exhibit a difference. Polynomial fitting essentially performs data interpolation through linear combinations of basis functions, where the coefficients $\{a_n\}$ merely capture local data curvature without physical interpretability.

With the rapid development of machine learning technology [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?], deep learning has received pioneering attention in the study of holographic QCD [?, ?, ?, ?, ?]. In addition, the combination of machine learning and holography has been deeply explored in a series of recent studies [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. In contrast to traditional holographic models, this new approach first utilizes experimental or lattice QCD data to determine metrics and other model parameters with the help of machine learning. These acquired metrics are then applied to compute other physical QCD observables and are used as predictive outputs of the model. This interdisciplinary integration not only simplifies the complex computational process but also improves the prediction accuracy and reliability of the model, thereby opening up new directions in the study of strong interactions.

In our recent work [?], we utilized an emergent metric constructed by Neural Ordinary Differential Equations with QCD data of the chiral condensate to calculate real and imaginary potential of heavy quarkonium. Ref. [?] employs a machine learning-assisted Einstein-Maxwell-Dilaton (EMD) model, using automatic differentiation to determine six parameters based on the equation of state, to calculate the transport properties of heavy quarks. Ref. [?] primarily focuses on comparing MLPs and KANs for the inverse problem of the heavy quark potential, confirming the validity of the constructed model at finite temperature and chemical potential. These works inspire us to incorporate more physical quantities into a single holographic model. In this study, we want to establish a connection between the heavy-quark potential and transport properties.

This paper is organized as follows: Section II will detail the calculation of the heavy quark potential using the Andreev-Zakharov model. In Section III, we construct a holographic model based on KANs and calculate the heavy-quark potential at finite temperatures and chemical potentials. Our analysis will determine the critical temperature T_c and provide an analytical solution for $w(r)$. In Section IV, the drag and jet quenching parameters, as well as the diffusion coefficients of heavy quarks at finite temperature and finite chemical potential, will be calculated for both regimes based on the analytic form of the function $w(r)$. Finally, Section V will summarize the main results and conclusions of the whole paper.

II. Holographic Heavy Quark Potential

The Andreev-Zakharov model allows for an accurate description of the potential of heavy quarkonium [?, ?] and exotic hadron states [?, ?, ?, ?, ?, ?, ?] by introducing an ad hoc deformation factor for the AdS$_5$-RN black hole. The background metric can be expressed as

$$ds^2 = w(r)\left(-f(r)dt^2 + d\vec{x}^2 + f^{-1}(r)dr^2\right).$$

The blackening factor is given by

$$f(r) = 1 - \frac{r_h^4}{r^4} + q^2 r_h^2\left(\frac{1}{r^2} - \frac{r_h^2}{r^6}\right).$$

Here, $w(r)$ is the deformation factor, which determines the deviation from conformality, q is the black hole charge, r_h is the position of the black hole horizon, and r is the coordinate of the fifth dimension. The Hawking temperature of the black hole is defined as

$$T = \frac{r_h}{\pi}\left(1 - \frac{Q^2}{2}\right),$$

where $Q = qr_h^3$ and $0 \leq Q \leq \sqrt{2}$.

The relationship between the chemical potential μ and black hole charge q is given by

$$\mu = k\frac{Qr_h}{\sqrt{1+Q^2}}.$$

In this paper, we fix the parameter k to 1. Thus, we get

$$\mu = \frac{Qr_h}{\sqrt{1+Q^2}}.$$

If we choose the static gauge $\tau = t$ and $\sigma = x$, then a static quark-antiquark pair located at $x = -\frac{L}{2}$ and $x = \frac{L}{2}$ can be described by a U-shaped string. The Nambu-Goto action of this string can be expressed as

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-\det(g_{\alpha\beta})},$$

where $g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$ is the induced metric on the worldsheet and $G_{\mu\nu}$ is the metric of AdS spacetime. Therefore, the action becomes

$$S_{NG} = -\frac{g}{2}\int dtdx\, w(r)\sqrt{f(r) + (\partial_x r)^2},$$

where $g = \frac{1}{\alpha'}$ is related to the string tension and α' is the square of the string length parameter in string theory. We now define the Lagrangian density as

$$\mathcal{L} = w(r)\sqrt{f(r) + (\partial_x r)^2}.$$

Since the Lagrangian does not explicitly depend on x, we have the conserved quantity

$$\frac{w(r)f(r)}{\sqrt{f(r) + (\partial_x r)^2}} = \text{constant}.$$

At the maximum position of the U-shaped string r_0, we have

$$\frac{w(r)f(r)}{\sqrt{f(r)+(\partial_x r)^2}}=\frac{w(r_0)}{\sqrt{f(r_0)}}.$$

As mentioned before, $\partial_x r$ can be solved as

$$\partial_x r=\sqrt{\frac{w^2(r)f^2(r)}{w^2(r_0)f(r_0)}-f(r)}.$$

Therefore, $\partial_r x$ is

$$\partial_r x=\frac{w(r_0)\sqrt{f(r_0)}}{\sqrt{w^2(r)f^2(r)-w^2(r_0)f(r_0)f(r)}}.$$

The separation distance L between the quark and antiquark is defined as

$$L=2\int_{r_0}^{\infty}\partial_r x\,dr=2\int_{r_0}^{\infty}\frac{w(r_0)\sqrt{f(r_0)}}{\sqrt{w^2(r)f^2(r)-w^2(r_0)f(r_0)f(r)}}dr.$$

The heavy-quark potential can be written as

$$E=2g\int_{r_0}^{\infty}\left(\frac{w(r)f(r)}{\sqrt{f(r)+(\partial_x r)^2}}-\frac{w(r_0)}{\sqrt{f(r_0)}}\right)dr-2g\left(w(0)+2gw'(0)\ln(r_0)\right).$$

In the Andreev-Zakharov model, the parameter g (related to the string tension) is fixed as $g=0.176$, while the deformation factor takes the form $w(r)=e^{sr^2}$ with $s=0.45$. These values are determined from fits to the meson spectrum [**?**, **?**] and the Cornell potential [**?**, **?**]. At zero temperature, we only need to set $f(r)=1$. In the next section, we will first use KANs to construct the holographic model at vanishing temperature.

III. Construction of a KANs-Based Holographic Model

Based on KANs, we reconstruct the deformation factor $w(r)$ in this section. We assume that $w(r)$ is a specific function derived from lattice data, with the string tension g used as a free parameter. The loss function integrates four essential components to achieve both physical fidelity and mathematical robustness. First, it incorporates a deviation term, specifically the mean absolute error, which quantifies the discrepancy between model predictions and target values from lattice QCD. Second, regularization constraints are imposed through specialized mathematical mechanisms to prevent overfitting and preserve generalization capabilities. Third, an ultraviolet boundary condition rigorously enforces $w(0) \to 1$, a fundamental requirement ensuring the metric transitions asymptotically to AdS_5 spacetime at short distances. Finally, a monotonicity preservation term employs custom-designed penalties to strictly maintain $w(r)$'s increasing behavior across its domain, thereby eliminating unphysical imaginary solutions in subsequent computations. These elements are synthesized through weighted combination into a unified optimization objective.

The structure of the KANs we designed is shown in Fig. 1 [Figure 1: see original paper]. We performed a reconstruction with the parameter $g = 0.2425$ related to the string tension. The trained function $w(r)$ is

$$w(r) = 5.32 - 4.53 \sin\left(11.16 - 3.11e^{-0.04(-0.93r-1)^2}\right),$$

which is substituted into Eq. (17) for potential energy calculations. These calculations are combined with the lattice QCD data [**?**], and the results are shown in Fig. 2 [Figure 2: see original paper].

From Fig. 2 (a), it can be clearly observed that the reconstructed function satisfies the boundary condition $w(0) \to 1$ and that the function $w(r)$ increases with r. Fig. 2 (b) illustrates the fitting performance of the neural network to the training dataset. These curves clearly demonstrate the excellent ability of the network in modeling the function $E(L)$ with an accuracy that highly matches the theoretical expectation. This result further supports the validity of the model and verifies the reliability of the KANs.

In the subsequent stage of this study, we obtain an analytical solution of $w(r)$. From this solution, we calculate the heavy quark potential under finite temperature and chemical potential conditions by adding the function $f(r)$. Fig. 3 [Figure 3: see original paper] (a) shows that the linear component of the potential energy exhibits a decaying trend under finite temperature conditions. This decay may be attributed to the temperature-induced shielding effect. In contrast, the Coulombic component of the potential energy shows remarkable robustness to temperature changes and is almost unaffected by temperature increase. As the temperature increases, the coupling strength of the strong interactions weakens, leading to weaker inter-quark binding and the potential energy tends to vanish on a smaller spatial scale. Ref. [**?**] employs lattice QCD simulations to analyze static quark-antiquark interactions at finite temperature using four complementary methods: spectral model fits, HTL-inspired fits, Padé rational approximation, and Bayesian reconstruction (BR). Among these, only the HTL-inspired fits exhibit significant temperature dependence. Our results are consistent with the other three methods, indicating that the screening effect is not significant at finite temperature. We further verify the reliability of the analytic solution of $w(r)$ obtained through neural networks.

Fig. 3 (b) shows a similar trend. However, by comparison with (a), we find that the chemical potential affects the potential energy to a significantly lesser extent than temperature does. These observations reveal the differential effects of temperature and chemical potential on the strong interaction potential energy.

At larger distance scales, the interaction between quarks and antiquarks is significantly weakened so that they behave as free particles. This phenomenon suggests that when the separation distance between quarks and antiquarks is large enough, they can be regarded as independently existing states and are no longer bound by strong interactions. The expectation value of the Polyakov loop can be defined as [**?**, **?**]

$$\langle P \rangle = \exp\left(-\frac{E(r=\infty, T)}{T}\right).$$

With this choice, the Polyakov loop expectation value takes the form shown in Fig. 4 [Figure 4: see original paper] (a). From Fig. 4 (b), it can be clearly seen that at $T_c = 0.17$ GeV, the slope of the Polyakov loop expectation value is the largest.

IV. Transport Properties of QGP

In this section, we extend the calculation of analytic solutions of the function $w(r)$ with string tension g to the drag force, diffusion coefficient, and jet quenching parameter of heavy quarks at finite temperature and finite chemical potential. To facilitate the study of holographic probes, we define $A_s(r) = \frac{1}{2}\log(w(r))$. Following Ref. [**?**], the drag force can be obtained by

$$F_{\text{drag}} = \frac{e^{2A_s(r_s)}v}{\pi^2 T^2 r_s^2},$$

where r_s satisfies $f(r_s) - v^2 = 0$. In this context, we first need to solve Eq. (5) numerically to obtain r_s, and subsequently employ Eq. (18) to compute the drag force. In Fig. 5 [Figure 5: see original paper], we show the variation of drag force with temperature at vanishing chemical potential. As can be seen from the figure, the drag force increases significantly with increasing temperature.

According to Eq. (19), the energy loss can be deduced to be equal to the drag force, which allows us to plot the relationship between energy loss and momentum in different systems. We have

$$\frac{dE}{dx} = \frac{e^{2A_s(r_s)}}{\pi^2 T^2 r_s^2}\frac{v}{1-v^2}.$$

Fig. 6 [Figure 6: see original paper] illustrates the energy loss of bottom quark ($m_b = 4.7$ GeV) and charm quark ($m_c = 1.3$ GeV) at vanishing chemical potential. From Fig. 6, it is clear that the energy loss increases with momentum. In addition, higher temperatures lead to an increase in energy loss. The qualitative results are similar to Refs. [**?**, **?**, **?**].

Next we proceed to study the diffusion coefficient, which in the AdS/Schwarzschild context can be written as [**?**]

$$D = \frac{1}{2\pi T} \frac{e^{2A_s(r_s)}}{\pi^2 T^2 r_s^2} \frac{1}{1-v^2}.$$

According to Eq. (21), we calculate the diffusion coefficients of heavy quarks normalized by $2\pi T$, as shown in Fig. 7 [Figure 7: see original paper]. As can be seen from the figure, the diffusion coefficient of heavy quarks gradually increases with increasing temperature, and this behavior is consistent with reference [**?**], indicating that the analytic solution of $w(r)$ obtained by KANs is reliable. Besides, the qualitative behavior of diffusion coefficient is consistent with Ref. [**?**].

Now we turn to the study of jet quenching parameter and we obtain the following expression for the jet quenching parameter in the holographic model [**?**]

$$\hat{q} = \frac{8\sqrt{2}}{\pi} \frac{a_0}{L^2} \frac{e^{2A_s(r_s)}}{\pi^2 T^2 r_s^2}.$$

Here a_0 is defined as

$$a_0 = \int_{r_h}^{\infty} \frac{dr}{r^2 L^{-2} e^{-2A_s(r)} \sqrt{f(r)(1-f(r))}}.$$

To obtain the jet quenching parameters in the holographic QCD model, we numerically solve using Eq. (22), comparing $\hat{q}/T^3$ as a function of temperature, as shown in Fig. 8 [Figure 8: see original paper]. From Fig. 9 [Figure 9: see original paper], it can be observed that temperature leads to an enhancement of the jet quenching parameter, indicating that in the considered model, a denser or hotter medium results in increased energy loss, which aligns with the physical intuition that jets passing through a higher-temperature (i.e., higher-density or more particle-rich) medium encounter more scattering centers and therefore experience greater energy loss. The results of our model calculations are consistent with the experimental results of RHIC and LHC [**?**].

V. Summary

In this study, we employ KANs to extract data from QCD to construct a holographic model. In order to verify the validity of the reconstruction results, we first apply the obtained function $w(r)$ to the heavy-quark potential and compare with lattice data. The results show that KANs exhibit effectiveness in solving inverse problems. It is worth emphasizing that KANs have the ability to provide analytical solutions. Moreover, we further examine the heavy quark potential and its relationship at finite temperature and chemical potential. In addition, based on the constructed function $w(r)$, we study the relationship between the drag force of heavy quarks, the diffusion coefficient, and the jet quenching parameter under the conditions of finite temperature and finite chemical potential, which reveals the accuracy of the KANs-based holographic model. This finding lays the foundation for understanding the complementarity of different models in dealing with complex physical phenomena, and also provides new perspectives for future research.

The findings not only provide an effective paradigm for utilizing machine learning methods to solve complex physics problems, but also point to new directions for subsequent research. These new directions include an in-depth exploration of the interactions between different physical fields, and the use of holographic models in combination with machine learning methods, with a view to a more comprehensive understanding of the dynamic behavior of complex systems.

Finally, we want to emphasize that our results focus on qualitative behavior. On the one hand, we can develop a holographic model derived from the Einstein equation with KANs. On the other hand, this work could inspire future efforts to incorporate additional data (e.g., meson spectra, equations of state) into the holographic model. Ultimately, this may enable us to build a comprehensive framework for describing broader aspects of QCD physics.

Acknowledgements

This work is supported by National Natural Science Foundation of China (NSFC) Grants No. 12405154, 12175100, and the European Union − Next Generation EU through the research grant number P2022Z4P4B "SOPHYA - Sustainable Optimised PHYsics Algorithms: fundamental physics to build an advanced society" under the program PRIN 2022 PNRR of the Italian Ministero dell'Università e Ricerca (MUR).

References

[1] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998), arXiv:hep-th/9711200.
[2] E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998), arXiv:hep-th/9803131.
[3] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B 428, 105 (1998), arXiv:hep-th/9802109.
[4] Z. Tang, Nucl. Phys. A 1005, 121942 (2021), arXiv:2002.10793 [nucl-ex].
[5] S. Acharya et al. (ALICE), (2024), arXiv:2409.11939 [nucl-ex].
[6] A. Andronic et al., Eur. Phys. J. C 76, 107 (2016), arXiv:1506.03981 [nucl-ex].
[7] X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017), arXiv:1701.02105 [nucl-ex].
[8] Z. Tang, Z.-B. Tang, W. Zha, W.-M. Zha, Y. Zhang, and Y.-F. Zhang, Nucl.

Sci. Tech. 31, 81 (2020), arXiv:2105.11656 [nucl-ex].
[9] F. Liu, Nucl. Sci. Tech. 24, 50524 (2013).
[10] K. Hattori and X.-G. Huang, Nucl. Sci. Tech. 28, 26 (2017), arXiv:1609.00747 [nucl-th].
[11] Q.-Y. Shou et al., Nucl. Sci. Tech. 35, 219 (2024), arXiv:2409.17964 [nucl-ex].
[12] C. M. Ko and F. Li, Nucl. Sci. Tech. 27, 140 (2016).
[13] R. Averbeck, Prog. Part. Nucl. Phys. 70, 159 (2013), arXiv:1505.03828 [nucl-ex].
[14] G. Aarts, C. Allton, M. B. Oktay, M. Peardon, and J.-I. Skullerud, Phys. Rev. D 76, 094513 (2007), arXiv:0705.2198 [hep-lat].
[15] T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986).
[16] N. Brambilla et al. (Quarkonium Working Group), (2004), 10.5170/CERN-2005-005, arXiv:hep-ph/0412158.
[17] R. Rapp and H. van Hees (2010) pp. 111–206, arXiv:0903.1096 [hep-ph].
[18] S. Zheng, J. Liu, S. Zheng, J. Zhao, and B. Chen, (2024), arXiv:2411.12169 [nucl-th].
[19] G. Chen, B. Chen, and J. Zhao, Eur. Phys. J. C 84, 869 (2024), arXiv:2402.11316 [nucl-th].
[20] J. Chen et al., Nucl. Sci. Tech. 35, 214 (2024), arXiv:2407.02935 [nucl-ex].
[21] O. Andreev and V. I. Zakharov, Phys. Rev. D 74, 025023 (2006), arXiv:hep-ph/0604204.
[22] O. Andreev and V. I. Zakharov, Phys. Lett. B 645, 437 (2007), arXiv:hep-ph/0607026.
[23] O. Andreev and V. I. Zakharov, JHEP 04, 100 (2007), arXiv:hep-ph/0611304.
[24] S. He, M. Huang, and Q.-s. Yan, Prog. Theor. Phys. Suppl. 186, 504 (2010), arXiv:1007.0088 [hep-ph].
[25] P. Colangelo, F. Giannuzzi, and S. Nicotri, Phys. Rev. D 83, 035015 (2011), arXiv:1008.3116 [hep-ph].
[26] O. DeWolfe, S. S. Gubser, and C. Rosen, Phys. Rev. D 83, 086005 (2011), arXiv:1012.1864 [hep-th].
[27] D. Li, S. He, M. Huang, and Q.-S. Yan, JHEP 09, 041 (2011), arXiv:1103.5389 [hep-th].
[28] K. B. Fadafan, Eur. Phys. J. C 71, 1799 (2011), arXiv:1102.2289 [hep-th].
[29] K. B. Fadafan and E. Azimfard, Nucl. Phys. B 863, 347 (2012), arXiv:1203.3942 [hep-th].
[30] R.-G. Cai, S. He, and D. Li, JHEP 03, 033 (2012), arXiv:1201.0820 [hep-th].
[31] D. Li, M. Huang, and Q.-S. Yan, Eur. Phys. J. C 73, 2615 (2013), arXiv:1206.2824 [hep-th].
[32] Z. Fang, S. He, and D. Li, Nucl. Phys. B 907, 187 (2016), arXiv:1512.04062 [hep-ph].
[33] Y. Yang and P.-H. Yuan, JHEP 12, 161 (2015), arXiv:1506.05930 [hep-th].
[34] Z.-q. Zhang, D.-f. Hou, and G. Chen, Nucl. Phys. A 960, 1 (2017), arXiv:1507.07263 [hep-ph].

[35] C. Ewerz, O. Kaczmarek, and A. Samberg, JHEP 03, 088 (2018), arXiv:1605.07181 [hep-th].
[36] X. Chen, S.-Q. Feng, Y.-F. Shi, and Y. Zhong, Phys. Rev. D 97, 066015 (2018), arXiv:1710.00465 [hep-ph].
[37] I. Aref'eva and K. Rannu, JHEP 05, 206 (2018), arXiv:1802.05652 [hep-th].
[38] H. Bohra, D. Dudal, A. Hajilou, and S. Mahapatra, Phys. Lett. B 801, 135184 (2020), arXiv:1907.01852 [hep-th].
[39] X. Chen, D. Li, D. Hou, and M. Huang, JHEP 03, 073 (2020), arXiv:1908.02000 [hep-ph].
[40] J. Zhou, X. Chen, Y.-Q. Zhao, and J. Ping, Phys. Rev. D 102, 086020 (2020), arXiv:2006.09062 [hep-ph].
[41] J. Zhou, X. Chen, Y.-Q. Zhao, and J. Ping, Phys. Rev. D 102, 126029 (2021).
[42] X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07, 132 (2021), arXiv:2010.14478 [hep-ph].
[43] X. Chen, L. Zhang, and D. Hou, Chin. Phys. C 46, 073101 (2022), arXiv:2108.03840 [hep-ph].
[44] X. Guo, X. Chen, D. Xiang, M. A. Martin Contreras, and X.-H. Li, Phys. Rev. D 110, 046014 (2024), arXiv:2406.04650 [hep-ph].
[45] L. Bellantuono, P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Nucl. Phys. A 982, 931 (2019).
[46] W.-J. Xing, S. Cao, G.-Y. Qin, and X.-N. Wang, Phys. Rev. Lett. 134, 052301 (2025), arXiv:2409.12843 [hep-ph].
[47] W.-J. Xing, S. Cao, and G.-Y. Qin, Phys. Lett. B 850, 138523 (2024), arXiv:2303.12485 [hep-ph].
[48] S. Cao et al. (JETSCAPE), Phys. Rev. C 104, 024905 (2021), arXiv:2102.11337 [nucl-th].
[49] M. Xie, W. Ke, H. Zhang, and X.-N. Wang, Phys. Rev. C 108, L011901 (2023), arXiv:2206.01340 [hep-ph].
[50] K. M. Burke et al. (JET), Phys. Rev. C 90, 014909 (2014), arXiv:1312.5003 [nucl-th].
[51] A. Majumder and M. Van Leeuwen, Prog. Part. Nucl. Phys. 66, 41 (2011), arXiv:1002.2206 [hep-ph].
[52] S. Wang, S. Li, Y. Li, B.-W. Zhang, and E. Wang, (2024), 10.1088/1674-1137/adb385, arXiv:2410.21834 [hep-ph].
[53] X.-Q. Cai, S. Li, Z.-F. Liu, and T.-Y. Wang, Sci. Rep. 14, 18385 (2024).
[54] Z. Li, D. Li, and M. Huang, (2025), arXiv:2504.04147 [hep-ph].
[55] L. Altenkort, O. Kaczmarek, R. Larsen, S. Mukherjee, P. Petreczky, H.-T. Shu, and S. Stendebach (HotQCD), Phys. Rev. Lett. 130, 231902 (2023), arXiv:2302.08501 [hep-lat].
[56] J. Casalderrey-Solana and D. Teaney, Phys. Rev. D 74, 085012 (2006), arXiv:hep-ph/0605199.
[57] S. Caron-Huot and G. D. Moore, Phys. Rev. Lett. 100, 052301 (2008), arXiv:0708.4232 [hep-ph].
[58] L. Altenkort, A. M. Eller, O. Kaczmarek, L. Mazur, G. D. Moore, and H.-T. Shu, Phys. Rev. D 103, 014511 (2021), arXiv:2009.13553 [hep-lat].

[59] N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, Phys. Rev. D 102, 074503 (2020), arXiv:2007.10078 [hep-lat].
[60] D. Banerjee, S. Datta, R. V. Gavai, and P. Majumdar, Nucl. Phys. A 1038, 122721 (2023), arXiv:2206.15471 [hep-ph].
[61] S. Acharya et al. (ALICE), JHEP 01, 174 (2022), arXiv:2110.09420 [nucl-ex].
[62] Z.-R. Zhu, S.-Q. Feng, Y.-F. Shi, and Y. Zhong, Phys. Rev. D 99, 126001 (2019), arXiv:1901.09304 [hep-ph].
[63] Y.-G. Ma, L.-G. Pang, R. Wang, and K. Zhou, Chin. Phys. Lett. 40, 122101 (2023), arXiv:2311.07274 [nucl-th].
[64] K. Zhou, L. Pang, S. Shi, and H. Stoecker, PoS FAIRness2022, 064 (2023).
[65] L.-G. Pang, Int. J. Mod. Phys. E 33, 2430009 (2024).
[66] Z.-H. Li, C.-Q. Li, and L.-G. Pang, (2023), arXiv:2309.07397 [gr-qc].
[67] K.-F. Pu, H. Li, H.-L. Lu, and L.-G. Pang, Chin. Phys. C 47, 054104 (2023), arXiv:2303.03934 [nucl-th].
[68] L.-G. Pang, Nucl. Phys. A 1005, 121972 (2021).
[69] J. Steinheimer, L. Pang, K. Zhou, V. Koch, J. Randrup, and H. Stoecker, JHEP 12, 122 (2019), arXiv:1906.06562 [nucl-th].
[70] L. Wang, B. M. Hare, K. Zhou, H. Stöcker, and O. Scholten, Chaos Solitons Fractals 170, 113346 (2023).
[71] Y. Huang, J. Chen, J. Jia, L.-M. Liu, Y.-G. Ma, and C. Zhang, (2025), arXiv:2501.01352 [nucl-th].
[72] W.-B. He, Y.-G. Ma, L.-G. Pang, H.-C. Song, and K. Zhou, Nucl. Sci. Tech. 34, 88 (2023), arXiv:2303.06752 [hep-ph].
[73] J. He, W.-B. He, Y.-G. Ma, and S. Zhang, Phys. Rev. C 104, 044902 (2021), arXiv:2109.06277 [nucl-ph].
[74] R. Wang, Y.-G. Ma, R. Wada, L.-W. Chen, W.-B. He, H.-L. Liu, and K.-J. Sun, Phys. Rev. Res. 2, 043202 (2020), arXiv:2010.15043 [nucl-th].
[75] R. Guo, Y. Li, and B. Chen, Entropy 25, 1563 (2023), arXiv:2311.02335 [nucl-th].
[76] J.-G. Deng, J.-X. Li, J.-H. Cheng, L. Mu, and H. Zhang, Chinese Physics C (2025).
[77] W. Kou and X. Chen, (2024), arXiv:2411.14902 [hep-ph].
[78] Z. Liu, Y. Wang, S. Vaidya, F. Ruehle, J. Halverson, M. Soljačić, T. Y. Hou, and M. Tegmark, (2024), arXiv:2404.19756 [cs.LG].
[79] Z. Liu, P. Ma, Y. Wang, W. Matusik, and M. Tegmark, "Kan 2.0: Kolmogorov-arnold networks meet science," (2024), arXiv:2408.10205 [cs.LG].
[80] J. Erdmann, F. Mausolf, and J. L. Späh, (2024), arXiv:2408.02743 [hep-ph].
[81] L. Yu et al., (2024), arXiv:2412.18301 [astro-ph.IM].
[82] W. He, Q. Li, Y. Ma, Z. Niu, J. Pei, and Y. Zhang, Sci. China Phys. Mech. Astron. 66, 282001 (2023), arXiv:2301.06396 [nucl-th].
[83] K. Zhou, L.-G. Pang, N. Su, H. Petersen, H. Stoecker, and X.-N. Wang, EPJ Web Conf. 171, 16005 (2018).
[84] L.-G. Pang, K. Zhou, N. Su, H. Petersen, H. Stöcker, and X.-N. Wang, Nucl. Phys. A 982, 867 (2019).
[85] L. Wang, Y. Jiang, L. He, and K. Zhou, Chin. Phys. Lett. 39, 120502

(2022), arXiv:2005.04857 [cond-mat.dis-nn].
[86] Y.-S. Zhao, L. Wang, K. Zhou, and X.-G. Huang, Phys. Rev. C 106, L051901 (2022), arXiv:2105.13761 [hep-ph].
[87] Y.-L. Du, D. Pablos, and K. Tywoniuk, JHEP 21, 206 (2020), arXiv:2012.07797 [hep-ph].
[88] Y.-L. Du, D. Pablos, and K. Tywoniuk, Phys. Rev. Lett. 128, 012301 (2022), arXiv:2106.11271 [hep-ph].
[89] L. Jiang, L. Wang, and K. Zhou, Phys. Rev. D 103, 116023 (2021), arXiv:2103.04090 [nucl-th].
[90] S. Shi, K. Zhou, J. Zhao, S. Mukherjee, and P. Zhuang, PoS LATTICE2021, 537 (2022).
[91] S. Soma, L. Wang, S. Shi, H. Stöcker, and K. Zhou, JCAP 08, 071 (2022), arXiv:2201.01756 [hep-ph].
[92] F.-P. Li, H.-L. Lü, L.-G. Pang, and G.-Y. Qin, Phys. Lett. B 844, 138088 (2023), arXiv:2211.07994 [hep-ph].
[93] Z. Yang, Y. He, W. Chen, W.-Y. Ke, L.-G. Pang, and X.-N. Wang, Eur. Phys. J. C 83, 652 (2023), arXiv:2206.02393 [nucl-th].
[94] S. Soma, L. Wang, S. Shi, H. Stöcker, and K. Zhou, PoS FAIRness2022, 055 (2023).
[95] L. Wang, J. Subatomic Part. Cosmol. 3, 100024 (2025), arXiv:2501.00374 [nucl-th].
[96] L. Wang and J. Zhao, (2024), arXiv:2411.16343 [nucl-th].
[97] L. Wang, T. Doi, T. Hatsuda, and Y. Lyu, PoS LATTICE2024, 076 (2025), arXiv:2410.03082 [hep-lat].
[98] T. Xu, L. Wang, L. He, K. Zhou, and Y. Jiang, Chin. Phys. C 48, 103101 (2024), arXiv:2405.10493 [hep-lat].
[99] M. Xiang, H. Yuan, L. Wang, K. Zhou, and H. G. Roskos, IEEE Trans. Terahertz Sci. Technol. 14, 208 (2024).
[100] K. Zhou, L. Wang, L.-G. Pang, and S. Shi, Prog. Part. Nucl. Phys. 135, 104084 (2024), arXiv:2303.15136 [hep-ph].
[101] K. Zhou, J. Phys. Conf. Ser. 2586, 012159 (2023).
[102] S. Acharya et al. (ALICE), (2024), arXiv:2412.15873 [nucl-ex].
[103] K. Hashimoto, S. Sugishita, A. Tanaka, and A. Tomiya, Phys. Rev. D 98, 106014 (2018), arXiv:1809.10536 [hep-th].
[104] K. Hashimoto, Phys. Rev. D 99, 106017 (2019), arXiv:1903.04951 [hep-th].
[105] T. Akutagawa, K. Hashimoto, and T. Sumimoto, Phys. Rev. D 102, 026020 (2020), arXiv:2005.02636 [hep-th].
[106] P. Lu, Z. Tang, X. Wu, and W. Zha, (2025), arXiv:2502.09063 [hep-ph].
[107] K. Hashimoto, K. Ohashi, and T. Sumimoto, Phys. Rev. D 105, 106008 (2022), arXiv:2108.08091 [hep-th].
[108] O.-Y. Luo, X. Chen, F.-P. Li, X.-H. Li, and K. Zhou, (2024), arXiv:2408.03784 [hep-ph].
[109] M. Mansouri, K. Bitaghsir Fadafan, and X. Chen, (2024), arXiv:2406.06285 [hep-ph].
[110] X. Chen and M. Huang, Phys. Rev. D 109, L051902 (2024),

arXiv:2401.06417 [hep-ph].
[111] X. Chen and M. Huang, (2024), arXiv:2405.06179 [hep-ph].
[112] W.-B. Chang and D.-f. Hou, Phys. Rev. D 109, 086010 (2024), arXiv:2403.04966 [hep-ph].
[113] B. Ahn, H.-S. Jeong, K.-Y. Kim, and K. Yun, JHEP 01, 025 (2025), arXiv:2406.07395 [hep-th].
[114] R.-G. Cai, S. He, L. Li, and H.-A. Zeng, (2024), arXiv:2406.12772 [hep-th].
[115] K. Hashimoto, K. Matsuo, M. Murata, G. Ogiwara, and D. Takeda, Mach. Learn. Sci. Tech. 6, 015030 (2025), arXiv:2411.16052 [hep-th].
[116] B. Ahn, H.-S. Jeong, C.-W. Ji, K.-Y. Kim, and K. Yun, (2025), arXiv:2502.10245 [hep-th].
[117] B. Ahn, H.-S. Jeong, K.-Y. Kim, and K. Yun, JHEP 03, 141 (2024), arXiv:2401.00939 [hep-th].
[118] D. Lee, H.-S. Lee, and J. Yi, (2025), arXiv:2503.08827 [hep-th].
[119] B. Chen, X. Chen, X. Li, Z.-R. Zhu, and K. Zhou, (2024), arXiv:2404.18217 [hep-ph].
[120] O. Andreev, Phys. Rev. D 78, 065007 (2008), arXiv:0804.4756 [hep-ph].
[121] O. Andreev, Phys. Rev. D 93, 105014 (2016), arXiv:1511.03484 [hep-ph].
[122] O. Andreev, JHEP 05, 173 (2021), arXiv:2007.15466 [hep-ph].
[123] O. Andreev, Phys. Rev. D 104, 026005 (2021), arXiv:2101.03858 [hep-ph].
[124] O. Andreev, Phys. Rev. D 105, 086025 (2022), arXiv:2111.14418 [hep-ph].
[125] O. Andreev, Phys. Rev. D 107, 026023 (2023), arXiv:2211.12305 [hep-ph].
[126] O. Andreev, Phys. Rev. D 109, 106001 (2024), arXiv:2402.09026 [hep-ph].
[127] O. Andreev, Phys. Rev. D 73, 107901 (2006), arXiv:hep-th/0603170.
[128] S. Eidelman et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004).
[129] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T.-M. Yan, Phys. Rev. D 17, 3090 (1978), [Erratum: Phys.Rev.D 21, 313 (1980)].
[130] O. Kaczmarek and F. Zantow, Phys. Rev. D 71, 114510 (2005), arXiv:hep-lat/0503017.
[131] D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee, G. Parkar, P. Petreczky, A. Rothkopf, and J. H. Weber (HotQCD), Phys. Rev. D 105, 054513 (2022), arXiv:2110.11659 [hep-lat].
[132] Y. Guo, L. Qiu, R. Zhao, and M. Strickland, Phys. Rev. D 109, 114025 (2024), arXiv:2403.06739 [hep-ph].
[133] Q. Du, M. Du, and Y. Guo, Phys. Rev. D 110, 034011 (2024), arXiv:2402.18004 [hep-ph].
[134] O. Andreev, Mod. Phys. Lett. A 33, 1850041 (2018), arXiv:1707.05045 [hep-ph].

Note: Figure translations are in progress. See original paper for figures.