Test of conformal gravity as an alternative to dark matter from the observations of elliptical galaxies Postprint

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Date: 2025-06-04T17:16:18+00:00

Abstract

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Full Text

Test of Conformal Gravity as an Alternative to Dark Matter from the Observations of Elliptical Galaxies

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Received 2025 April 5; revised 2025 April 26; accepted 2025 May 6; published 2025 June 2

Abstract

As an alternative gravitational theory to general relativity (GR), conformal gravity (CG) has recently been successfully verified by observations of type Ia supernovae (SN Ia) and the rotation curves of spiral galaxies. The observations of galaxies only pertain to the non-relativistic form of gravity. In this context, within the framework of the Newtonian theory of gravity (the non-relativistic form of GR), dark matter (DM) is postulated to account for the observations. On the other hand, the non-relativistic form of CG predicts an additional potential: besides the Newtonian potential, there is a so-called linear potential term, characterized by the parameter γ, as an alternative to DM in Newtonian gravity. To test CG in its non-relativistic form, much work has been done by fitting the predictions to the observations of circular velocity (rotation curves) for spiral galaxies. In this paper, we test CG with the observations from elliptical galaxies. Instead of the circular velocities for spiral galaxies, we use the velocity dispersion for elliptical galaxies. By replacing the Newtonian potential with that predicted by the non-relativistic form of CG in the Hamiltonian, we directly extend the Jeans equation derived in Newtonian theory to that for CG. By comparing the results derived from the ellipticals with those from spirals, we find that the extra potential predicted by CG is not sufficient to account for the observations of ellipticals. Furthermore, we discover a strong correlation between γ and the stellar mass M* in dwarf spheroidal galaxies. This finding implies that the variation in γ violates a fundamental prediction of CG, which posits that γ should be a universal constant.

Key words: gravitation – galaxies: dwarf – galaxies: elliptical and lenticular, cD – Galaxy: disk

1. Introduction

Einstein's general relativity (GR) has been verified very successfully on the scale of the solar system, where the vacuum solutions of Einstein's equation, known as the Schwarzschild metric, are applied. On larger scales, in particular when it comes to the studies of galaxies and cosmology, dark matter (DM) and dark energy (DE) are assumed to account for observations. Since both DM and DE lack direct theoretical support and observational evidence, many efforts are devoted to modified gravity alternatives to GR and its non-relativistic form, Newtonian gravity.

For instance, one can enhance the standard Lagrangian in GR by incorporating higher-order curvature terms (Lovelock 1971, 1972; Boulware & Deser 1985; Kobayashi 2005; Oikonomou 2021; Brassel et al. 2022), or formulate nonlinear Lagrangians (Buchdahl 1970; Goswami et al. 2014). Other relevant theories include modified Newtonian dynamics (MOND) (Milgrom 1983; Famaey & Mcgaugh 2012) and its relativistic version (Bekenstein 2004), conformal gravity (CG; Mannheim 1997, 2006), as well as quantum effects on cosmic scales as an alternative to DM and DE (Chen 2022; Chen & Wang 2024). Clearly, any modifications or extensions to GR should be verified by observations, in particular by observations from the solar system. However, in the solar system, higher-order corrections to GR should be negligible since on this scale, GR turns out to be exact when predicting observations. On galactic scales, the non-relativistic theory of gravity suffices. For Newtonian theory, DM is introduced to produce extra gravitational potential so that when combined with the potential created by the luminous matter, the total gravitational potential can account for the observations of galaxies. On the other hand, in any modified theory of gravity, it is required that, besides the usual Newtonian potential, the luminous matter must produce extra gravitational potential to replace the potential produced by DM in Newtonian theory.

In recent years, CG has attracted much interest in testing it as an alternative to DM and DE with astronomical observations (for a review, see Mannheim 2006). As a relativistic theory alternative to GR, CG can solve the long-standing cosmological constant problem encountered in the standard ΛCDM cosmological model (Mannheim 1992, 2000, 2001), and the CG cosmology has been tested with SN Ia data (Mannheim 2006; Yang et al. 2013). In its non-relativistic limit, luminous matter generates additional gravitational potential beyond the conventional Newtonian potential (Mannheim & Kazanas 1989). This could potentially resolve the missing mass problem observed in galaxies and galaxy clusters without the need for DM. To assess CG in its non-relativistic form, a significant amount of research has been conducted. This involved fitting the theoretical predictions to the observed circular velocities (rotation curves) of spiral galaxies (Mannheim & O'Brien 2012, 2013; O'Brien & Moss 2015).

In this paper, we take a different approach. We test CG using the observations from elliptical galaxies. Instead of relying on the circular velocities characteristic of spiral galaxies, we utilize the velocity dispersion of elliptical galaxies. Specifically, in the Hamiltonian, we substitute the Newtonian potential with the one predicted by the non-relativistic form of CG. By doing so, we directly extend the Jeans equation, which was originally derived within the framework of Newtonian theory, to the context of CG.

The remainder of this paper is structured as follows. In Section 2, we review the fundamentals of CG and present the necessary formulas. In Section 3, we first give a brief introduction to the test of CG using spiral galaxies. Subsequently, we elaborate in detail on the procedures we adopted when applying CG to elliptical galaxies. The conclusions and discussions are presented in Section 4.

2. Conformal Gravity

In comparison to GR, CG is formulated by maintaining the metric as the gravitational field. However, it endows gravity with an additional symmetry, namely conformal symmetry, which extends beyond ordinary coordinate invariance. By imposing the principle of local conformal invariance as the requisite principle to restrict the choice of action for the gravitational field in curved spacetime, one requires the uniquely selected fourth-order gravitational action

$$I_W = -\alpha_g \int d^{4x}\, (-g)^{1/2} C^{\lambda\mu\nu\kappa} C_{\lambda\mu\nu\kappa}$$

to remain invariant under any local metric transformation $g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)} g_{\mu\nu}(x)$ (called conformal transformation), and thus an action satisfying conformal symmetry. In Equation (1), α_g is a dimensionless coupling constant, and $C_{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor defined by

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \frac{1}{6}R(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu})$$

i.e., a tensor constructed by a particular combination of the Riemann and Ricci tensors and the Ricci scalar. The particular property of the Weyl tensor is that it has the kinematic relation $C^{\mu}{}_{\nu\mu\kappa} = 0$. In other words, the Weyl tensor is traceless.

CG requires the energy-momentum tensor $T_{\mu\nu}$ to be traceless, i.e., $T^{\mu}_{\ \mu} = 0$. On the other hand, elementary particle masses are not kinematic, but rather are acquired dynamically by spontaneous breakdown. Hence, consider a massless, spin-1/2 matter field fermion $\psi(x)$ which is to get its mass through a massless, real spin-0 Higgs scalar boson field $S(x)$. The required matter field action I_M can be defined by

$$I_M = -\int d^{4x}\, (-g)^{1/2} \left[\frac{1}{2} h S^{2R} + \frac{1}{12} h S^{2R} + \lambda S^4 + i\bar{\psi}\gamma^{\mu}(x)[\partial_{\mu} + \Gamma_{\mu}(x)]\psi - hS\bar{\psi}\psi\right]$$

where h and λ are dimensionless coupling constants, $\gamma^{\mu}(x)$ are the Dirac matrices and $\Gamma_{\mu}(x)$ are the fermion spin connection.

Variation of I_M with respect to the metric yields the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{(-g)^{1/2}} \frac{\delta I_M}{\delta g^{\mu\nu}}$$

The total action is $I = I_W + I_M$. Variation of the total action with respect to the metric then yields

$$4\alpha_g W_{\mu\nu} = T_{\mu\nu}$$

where $W_{\mu\nu} = \frac{1}{2} g_{\mu\nu} C^{\lambda\mu\nu\kappa} C_{\lambda\mu\nu\kappa} - 2C_{\mu\lambda\nu\kappa} R^{\lambda\kappa} + 2C_{\mu\lambda\kappa\sigma} C_{\nu}{}^{\lambda\kappa\sigma}$.

2.1. Applying to Cosmology

In applying CG to cosmology, the Weyl tensor vanishes in a Robertson–Walker metric. Thus $W_{\mu\nu} = 0$, and we see from Equation (5) that $T_{\mu\nu} = 0$. It turns out that the conformal symmetry forbids the presence of any fundamental cosmological term, and is thus a symmetry that is able to control the cosmological constant. Even after the conformal symmetry is spontaneously broken (as is needed to generate particle mass), the contribution of an induced cosmological constant to cosmology will still be under control (Mannheim 2006). Consequently, CG is potentially capable of solving the cosmological constant problem. The full content of the theory can be obtained by choosing a particular gauge in which the scalar field takes the constant value S_0. In this case, the energy-momentum tensor of Equation (4) becomes

$$T_{\mu\nu} = \frac{hS_0^2}{6} \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \text{fermionic contributions}$$

An averaging of over all the fermionic modes propagating in a Robertson–Walker background will bring the fermionic contribution to $T_{\mu\nu}$ to the form of a kinematic perfect fluid

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

Thus the conformal cosmology equation of motion can be written as

$$\frac{3}{8\pi G_{\text{eff}}} \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) = \rho$$

For future reference, we define the angular diameter distance as

$$D_A(0, z) = \frac{c}{H_0(1+z)} \int_0^z \frac{dz'}{E(z')}$$

where $E(z) = H(z)/H_0$.

2.2. Non-relativistic Limit

To conduct a test of CG using galaxy observations, it is necessary to derive the non-relativistic limit of CG. Mannheim and Kazanas (1989, 1994) found an exact CG analog of the Schwarzschild exterior and interior solutions to standard gravity by solving the equation $4\alpha_g W_{\mu\nu} = T_{\mu\nu}$ for a static, spherically symmetric source. It turns out that the full kinematic content of CG is contained in the line element

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Evaluating the form that $W_{\mu\nu}$ takes in this line element leads to the equation of motion. Comparing with the standard Einstein equation, we only need to replace the gravitational constant G by an effective, dynamically induced constant G_{eff}. We define conformal analogs of the standard $\Omega_M(t)$, $\Omega_\Lambda(t)$ and $\Omega_K(t)$ via

$$\Omega_M(t) = \frac{8\pi G_{\text{eff}}\rho(t)}{3H^2(t)}, \quad \Omega_\Lambda(t) = \frac{\Lambda(t)}{3H^2(t)}, \quad \Omega_K(t) = -\frac{k}{a^2(t)H^2(t)}$$

where the Hubble parameter $H(t) = \dot{a}/a$ and $\Lambda(t)$ is an effective cosmological term.

As usual, in a Robertson–Walker geometry the expression of the Hubble parameter at redshift z is

$$H(z) = H_0\sqrt{\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda}$$

In subsequent calculations, we adopt the values $\bar{\Omega}_M = 0.33$, $\bar{\Omega}_\Lambda = 0.67$, and $H_0 = 69.3$ km s^{-1} Mpc^{-1}, as per reference (Yang et al. 2013).

It is convenient to define a source function $f(r)$ via

$$\nabla^4 B(r) = f(r)$$

so that the equations of motion can be written as

$$\nabla^4 B(r) = f(r)$$

We are interested in the exterior solution for a static, spherically symmetric source of radius r_0, which is readily given by

$$B(r) = w - \frac{2\beta}{r} + \gamma r - kr^2$$

where the $-kr^2$ term is the general solution to the homogeneous equation $\nabla^4 B(r) = 0$. On dropping the kr^2 term and setting $w = 1$, the metric can be written, without any approximation, as

$$ds^2 = -\left(1 - \frac{2\beta}{r} + \gamma r\right) dt^2 + \frac{dr^2}{1 - \frac{2\beta}{r} + \gamma r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

The Schwarzschild-like vacuum solutions of any modified theory of gravity offer us an opportunity to verify the theory in its non-relativistic form. Specifically, this verification can be carried out on the scales of solar systems, galaxies, and galaxy clusters. In such scenarios, the metric $g_{\mu\nu}$ is reduced to gravitational potential V. In terms of gravitational potential $V(r)$, we can rewrite the metric as

$$ds^2 = -(1 + 2V/c^2)dt^2 + (1 - 2V/c^2)(dx^2 + dy^2 + dz^2)$$

with

$$V(r) = -\frac{\beta c^2}{r} + \frac{\gamma c^2 r}{2}$$

In the region where $2\beta/r \gg \gamma r$, when $\beta = GM/c^2$, the Schwarzschild solution can be recovered. Departures from this solution, specifically the linear potential $V_\gamma = \gamma c^2 r/2$, only occur at large distances. As a result, the standard solar system Schwarzschild phenomenology is preserved.

3. Test of Conformal Gravity with Observations of Galaxies

As previously shown, when verifying a new relativistic theory of gravity through galaxy observations, one must transition from the geometric perspective (utilizing the metric $g_{\mu\nu}$) to that of Newtonian dynamics (employing the gravitational potential V). Consequently, in the realm of galactic dynamics, the kinematic aspects are determined by the gravitational potential. This holds true regardless of the form the potential assumes and its origin. The potential shown in Equation (24) represents the potential generated by a point mass M in CG. Besides the conventional Newtonian potential $V_\beta = -\beta c^2/r$, there is also a linear potential $V_\gamma = \gamma c^2 r/2$. This linear potential is proposed as an alternative to the potential generated by DM in Newtonian theory and thus requires verification through observations of galaxies. For a typical star of solar mass $M_\odot$, we write its potential as

$$V_*(r) = -\frac{\beta_* c^2}{r} + \frac{\gamma_* c^2 r}{2}$$

where $\beta_* = GM_\odot/c^2 = 1.48 \times 10^3$ m and γ_* can be determined by observations. If we denote $\gamma = (M/M_\odot)\gamma_*$, then for any point mass M, the expression for its potential can be rewritten as

$$V(r) = -\frac{\beta c^2}{r} + \frac{\gamma c^2 r}{2} = -\frac{GM}{r} + \frac{\gamma_* M c^2 r}{2M_\odot}$$

3.1. Test with Spiral Galaxies

Up to now, the value of γ in Equation (26) has been uniquely determined by the rotation curve observations of the circular velocity $v_c(R)$ of spiral galaxies. For example, the total potential contributed by luminous matter in the equatorial plane $z = 0$ of an axisymmetric disk galaxy of surface mass density $\Sigma(R)$ is

$$V_{\rm LOC}(R) = V_\beta(R) + V_\gamma(R)$$

with

$$V_\beta(R) = -\int_0^\infty \frac{G\Sigma(R')}{R'} J_0(kR')J_0(kR)e^{-k|z|}dk$$

$$V_\gamma(R) = \frac{\gamma_* c^2}{2M_\odot}\int_0^\infty \Sigma(R')R' J_0(kR')J_0(kR)e^{-k|z|}dk$$

However, when fitting to the rotation curves of spiral galaxies, it has been found that there exists a universal, galaxy-independent linear potential, $\gamma_0 c^2 r/2$. This potential can be ascribed to the effect of the potentials due to the rest of the matter in the universe on any local galaxies (Mannheim 1997). Consequently, around a point mass M, the total potential on a test particle is

$$V_{\rm total}(r) = V_\beta(r) + V_\gamma(r) + V_{\gamma_0}(r)$$

with

$$V_{\gamma_0}(r) = \frac{\gamma_0 c^2 r}{2}$$

By fitting rotation curves of spiral galaxies (Mannheim 2006), it is found that

$$\gamma_* = 5.42 \times 10^{-39}\ \mathrm{m}^{-1}, \quad \gamma_0 = 3.06 \times 10^{-28}\ \mathrm{m}^{-1}$$

In what follows, we will determine γ_* and γ_0 via a different approach, namely by using the observations of elliptical galaxies.

3.2. Test with Elliptical Galaxies: Theory

The observable quantities of elliptical galaxies that we can utilize are the surface brightness and velocity dispersion. To validate CG using these observations, we begin with the Jeans equation for static gravitational systems. Generally speaking, for static systems, the modified Hamiltonian of any new gravitational theory can be straightforwardly constructed by the replacement of $V_N \to V_{\mathrm{CG}}$. In this paper, we make use of the potential presented in Equation (30).

The collisionless Boltzmann equation (CBE) is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla V \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

where f is the distribution function (DF) in phase space and the square bracket is a Poisson bracket. In terms of inertial Cartesian coordinates, in which $\nabla = \partial/\partial \mathbf{x}$ and $\partial/\partial \mathbf{v}$ is the gradient in velocity space, the CBE for a static system is

$$\mathbf{v} \cdot \nabla f - \nabla V \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

The Jeans equation is derived from the CBE, and for static, spherical systems, it reads (Binney & Tremaine 2011)

$$\frac{1}{\rho}\frac{d(\rho\sigma_r^2)}{dr} + \frac{2\beta(r)\sigma_r^2}{r} = -\frac{dV}{dr}$$

where ρ is the matter density, σ_r is the radial velocity dispersion, and $\beta(r)$ is the anisotropy parameter (not to be confused with the β potential). Note that the gravitational potential V in Equation (34) is the one for CG, as shown in Equation (30). For simplicity, we assume that the systems are isotropic ($\beta = 0$) and that the velocity dispersion σ_* (the line-of-sight dispersion) is a constant for each system. Thus, the Jeans equation is simplified as

$$\frac{\sigma_*^2}{\rho}\frac{d\rho}{dr} = -\frac{dV}{dr}$$

What we actually observe is the surface brightness $I(R)$, so we must extract $\rho(r)$ from it. For dwarf spheroidal galaxies (dSphs), we employ the Plummer profile (Walker et al. 2009; Moskowitz & Walker 2020)

$$I(R) = \frac{L}{\pi R_e^2}\left(1 + \frac{R^2}{R_e^2}\right)^{-2}$$

where L is the total luminosity and R_e is the effective radius, i.e., the projected radius encircling half of the total luminosity. The luminosity density $j(r)$ can be extracted from $I(R)$ via an Abel transform (Binney & Tremaine 2011)

$$j(r) = -\frac{1}{\pi} \int_r^\infty \frac{dI}{dR} \frac{dR}{\sqrt{R^2 - r^2}}$$

By considering the mass-to-light ratio $\Upsilon = M/L$, we obtain the mass density for the Plummer profile

$$\rho(r) = \frac{3M}{4\pi a^3} \left(1 + \frac{r^2}{a^2}\right)^{-5/2}$$

where a is related to the effective radius by $a = R_e/1.305$.

For other general elliptical galaxies, we employ the Sérsic profile (Sérsic 1963; Sersic 1968)

$$I(R) = I_0 \exp\left[-b_n \left(\frac{R}{R_e}\right)^{1/n}\right]$$

where I_0 is the central intensity, R_e is the effective radius, n is the Sérsic index, and b_n is the scale factor, the fitted approximate value of which is $b_n = 2n - 1/3 + 4/405n + 46/25515n^2$ (Ciotti & Bertin 1999). By making use of the formula $L = 2\pi \int_0^\infty I(R) R dR$, one can derive the central intensity

$$I_0 = \frac{L}{2\pi R_e^2} \frac{b_n^{2n}}{\Gamma(2n)}$$

Consequently, the Sérsic density profile can be computed once more through an Abel transform of Equation (37) (Prugniel & Simien 1997)

$$\rho(r) = \rho_0 \left(\frac{r}{R_e}\right)^{-p} \exp\left[-b_n \left(\frac{r}{R_e}\right)^{1/n}\right]$$

where the parameter p satisfies the relationship $p = 1 - 1.188/2n + 0.22/4n^2$.

Substituting $\rho(r)$ from Equations (38) or (42) into Equation (35), we obtain the Jeans equation for the Plummer profile

$$\frac{\sigma_*^2}{\rho(r)} \frac{d\rho(r)}{dr} = -\frac{dV}{dr}$$

and for the Sérsic profile

$$\frac{\sigma_*^2}{\rho(r)}\frac{d\rho(r)}{dr} = -\frac{dV}{dr}$$

We now shift our focus to the right-hand side of Equations (35), (39) or (43) and compute the derivatives of V_β and V_γ. The derivative of the Newtonian potential V_β is readily given by (Mannheim 2006)

$$\frac{dV_\beta}{dr} = \frac{GM(r)}{r^2}$$

where $M(r) = 4\pi \int_0^r \rho(r')r'^2 dr'$ is the total mass enclosed within radius r. For the linear potential of the system, we have

$$V_\gamma(r) = \frac{\gamma_* c^2}{2M_\odot}\int_0^r \rho(r')r'dr'$$

We thus obtain the derivative of the linear potential V_γ (Mannheim 2006)

$$\frac{dV_\gamma}{dr} = \frac{\gamma_* c^2 r}{2M_\odot}\rho(r)$$

By substituting Equations (44) and (46) into the right-hand side of Equation (39), we can determine γ_* and γ_0 using the data of dSphs. Similarly, when aiming to determine γ_* and γ_0 from the data of bright spheroidal galaxies, we can perform the same procedure for the Sérsic profile of Equation (43).

On the other hand, it is intriguing to compare the results of our CG analysis with those predicted by the conventional DM model. To carry out this comparison, similar to the approach in Equation (35), we assume that the system is isotropic and the velocity dispersion remains constant. The key distinction here is that the gravitational potential V follows the Newtonian form, which is generated by the combined mass of DM and luminous matter, denoted as dynamic mass M_{dyn}. So for Newtonian theory of gravity, Equation (35) becomes

$$\frac{\sigma_*^2}{\rho}\frac{d\rho}{dr} = -\frac{GM_{\text{dyn}}(r)}{r^2}$$

Of course, this equation is valid only when we assume that mass distribution follows the light distribution. However, this assumption is generally not true because, in most cases, a significant portion of DM is distributed outside the region of luminous matter, forming a dark halo (Walker et al. 2009; Moskowitz & Walker 2020). Nevertheless, from the perspective of gravitational force, as a toy model, such a simplification can help us verify whether CG has the ability to account for the observations without invoking DM. Consequently, to compare the results of CG with those of Newtonian theory, we have to replace the potential in Equations (39) and (43) with the Newtonian potential. Specifically, Equation (39) is replaced by

$$\frac{\sigma_*^2}{\rho(r)}\frac{d\rho(r)}{dr} = -\frac{GM_{\rm dyn}(r)}{r^2}$$

for the Plummer profile assumed for dSphs, and Equation (43) is replaced by

$$\frac{\sigma_*^2}{\rho(r)}\frac{d\rho(r)}{dr} = -\frac{GM_{\rm dyn}(r)}{r^2}$$

for the Sérsic profile.

3.3. Test With Elliptical Galaxies: Fitting Data

To assemble a sample for dSphs, we choose 43 dSphs from the sample of all dwarf galaxies in and around the Local Group, as presented in McConnachie (2012). The sample we have selected includes information such as the effective radius R_e, velocity dispersion σ_*, and stellar mass M_*. This information is required when attempting to determine γ_* and γ_0 in accordance with Equation (39). We denote this sample as sample dSphs.

Before proceeding further, it is essential to modify the effective radius R_e for future use in CG. In actual observations, the angular radius $\theta_e = R_e/D_A(0,z)$ is measured, where $D_A(0,z)$ is the angular diameter distance to an object at redshift z. This angular diameter distance is derived in CG, and its general formula is given in Equation (13). Given that $D_A(0,z)$ varies across different theories of gravity, if the data is presented in the framework of GR, we should modify the value of R_e according to

$$R_e^{\rm CG} = R_e^{\rm GR}\frac{D_A^{\rm CG}(0,z)}{D_A^{\rm GR}(0,z)}$$

where $R_e^{\rm GR}$ and $D_A^{\rm GR}(0,z)$ are the values evaluated in GR.

We employ the least-squares method to evaluate γ_* and γ_0. From Equation (39), the χ^2 is defined by

$$\chi^2 = \sum_{i=1}^{N}\frac{[\sigma_{\rm obs}^2 - \sigma_{\rm model}^2(r_i)]^2}{\sigma_{\rm obs}^4}$$

In actual calculations, we choose $\sigma_{\rm obs} = 1$. Since both sides of Equation (39) are functions of radius r, we evaluate γ_* and γ_0 at $r = R_e$ for each galaxy. The optimized fitted values are as follows: $\gamma_*^{\rm dSph} = 1.22\times 10^{-35}$ m^{-1}, $\gamma_0^{\rm dSph} =$

5.27×10^{-28} m^{-1}. By comparing the results obtained from fitting dSphs with those from fitting spiral galaxies, as presented in Equation (31), we find that γ_* is four orders of magnitude larger, while γ_0 is of the same order. The universal value of γ_0 obtained from each dSph is anticipated. This is because it stems from the cosmological effect on the local system and, consequently, is independent of any specific local gravitational system. However, the fact that the fitted value of $\gamma_*^{\rm dSph}$ is much larger than that obtained from spiral galaxies implies that if the latter value is correct, it is insufficient to explain the dynamics of dSphs. In other words, when it comes to dSphs, a certain amount of DM must be introduced.

It should be noted that, as can be seen from the fitting results in reference (Mannheim & O'Brien 2012), when the stellar mass $M_* < 10^{11} M_\odot$, the γ_0 term dominates the linear potential. As M_* increases, the γ_* term gradually becomes dominant in the linear potential. In our sample of dSphs, the stellar mass of all galaxies satisfies the condition $M_* < 10^{11} M_\odot$. Therefore, the linear potential should be dominated by γ_0. On the other hand, in the conventional DM model, it is widely acknowledged that DM dominates the potential of dwarf galaxies, and this dominant tendency weakens as the stellar mass increases. In CG, the Newtonian potential generated by DM is replaced by two linear potentials (i.e., γ_0 and γ_* potentials). As shown in reference (Mannheim & O'Brien 2012), for dwarf galaxies, there is a trend in CG that is similar to that in the DM model.

According to CG, both γ_0 and γ_* should be universal constants. However, as we will demonstrate, for dSphs, γ_* is not a constant. Instead, it decreases with an increase in the stellar mass M_*. This tendency resembles the one in the DM model but violates the basic assumptions of CG. To achieve this, we fix γ_0 to be a smaller value of 3.97×10^{-29} m^{-1} (as opposed to the optimized value of $\gamma_0^{\rm dSph} = 5.27 \times 10^{-28}$ m^{-1}), and keep γ_* as a free parameter to be determined. This fixed value of γ_0 is obtained by setting $\gamma_* = 0$ for all galaxies and fitting the value of γ_0 according to the Jeans Equation (39), then finding the smallest one. Such a fixed value of γ_0 would ensure that the fitted value of γ_* cannot be negative. The rationale behind choosing to fix γ_0 instead of γ_* is as follows. γ_* represents the linear potential stemming from the local luminous mass. It can imitate the distribution of DM in Newtonian gravity within any local gravitational system. This enables us to draw a comparison between the DM distribution in Newtonian gravity and the linear potential generated by the luminous matter in CG.

Conversely, γ_0 measures the cosmological impact on local systems, and this impact remains independent of any particular local system. We would like to emphasize that, as is evident from Equation (30), for a given galaxy, the combination of γ_0 and $N\gamma_*$ must remain a constant. Thus, a decrease in the value of γ_0 necessarily implies an increase in the value of γ_*. Nonetheless, fixing γ_0 at its smallest value will not impede our ability to draw a correct conclusion regarding the correlation between M_* and γ_*. Meanwhile, we are aware that the actual optimal value of γ_0 for dSphs is 5.27×10^{-28} m^{-1}.

In CG, γ_* characterizes the linear potential of a unit mass and should therefore be a universal constant. However, the observed correlation between γ_* and the stellar mass M_* in galaxies closely resembles the correlation between $M_{\rm DM}$ and stellar mass M_*. This suggests that elliptical galaxies are better described by Newtonian theory (requiring DM) than by CG.

Subsequently, we will set γ_0 to 3.97×10^{-29} m^{-1} and calculate γ_* and M_* at $r = R_e$ using the Jeans Equations. We define χ^2 between the stellar mass $M_*(R_e)$ and $\gamma_*(R_e)$. For our selected sample of 43 dSphs, by applying Equations (39) to (52) we find an empirical formula

$$\log_{10} \gamma_* = a \log_{10}(M_*/M_\odot) + b$$

We obtain the optimal parameters a and b using the least-squares method. By doing so, we can establish the expected correlation

$$\log_{10} \gamma_* = -0.96 \log_{10}(M_*/M_\odot) - 27.56$$

The results are shown in Figure 1. Evidently, $\gamma_*^{\rm dSph}$ is not a constant as predicted by CG. Instead, it decreases as the stellar mass M_* increases.

[Figure 1: see original paper]

It would be intriguing to explore the correlation between the DM mass $M_{\rm dyn}$ and the stellar mass M_* in Newtonian gravity and to check whether this correlation resembles that between γ_* and M_* in CG. To accomplish this, we apply the Jeans Equation (48) for dSphs. For simplicity's sake, we calculate the total mass $M_{\rm dyn}(R_e)$ and stellar mass $M_*(R_e)$ within $r = R_e$. Thus, the DM mass within R_e is $M_{\rm DM}(R_e) = M_{\rm dyn}(R_e) - M_*(R_e)$. The correlation between $M_{\rm DM}(R_e)$ and $M_*(R_e)$ is depicted in Figure 2. Indeed, this figure reveals that the DM mass $M_{\rm DM}(R_e)$ decreases as the stellar mass $M_*(R_e)$ increases, following exactly the same pattern as that of $\gamma_*^{\rm dSph}$ and M_* as shown in Figure 1.

For ease of reference, we list in Table 1 the following parameters for each galaxy in our selected sample, which is based on the sample in reference McConnachie (2012): the total stellar mass M_*, effective radius R_e, velocity dispersion σ_e, fitted value of $\gamma_*^{\rm dSph}$, dynamical mass $M_{\rm dyn}(R_e)$ within R_e, and the DM-to-stellar mass ratio $M_{\rm dyn}(R_e)/M_*(R_e)$ within R_e.

We now shift our focus to the investigation of γ_* for bright elliptical galaxies. We will apply our method used for dSphs to two samples of bright elliptical galaxies. The first sample is composed of 76 compact, high-velocity-dispersion, early-type galaxies from the Sloan Digital Sky Survey (SDSS) with $0.05 < z < 0.2$. We denote this sample as SDSS DR 10. This sample was established in reference (Saulder et al. 2015) by employing the de Vaucouleurs model (Sérsic profile with $n = 4$). Therefore, for bright galaxies, we utilize the Jeans Equation (43) to calculate $\gamma_*^{\rm SDSS}$ at $r = R_e$. As proposed in reference (Saulder et al. 2015), in this scenario, $n = 4$, $b_n = 7.66925$, and $p = 0.854938$. Meanwhile, the effective radius R_e is adjusted in accordance with Equation (50). Because of the large velocity dispersion, a correction is required, and we take advantage of the work of Shu et al. (2015) and Cappellari et al. (2005) to use the corrected velocity dispersion at R_e,

$$\sigma_e^2 = \sigma_{\rm obs}^2 \left(1 + \frac{\theta_{\rm fiber}^2}{\theta_e^2}\right)$$

where $\theta_{\rm fiber} = 1.5$ arcsec is the angular radius of the SDSS fiber, and θ_e is the effective radius.

The results indicate that the correlation between $\gamma_*^{\rm SDSS}$ and M_* can be described by a fitted formula

$$\log_{10} \gamma_* = -1.64 \log_{10}(M_*/M_\odot) - 17.79$$

γ_* is not a constant; instead, it decreases as M_* increases, as presented in Figure 3.

[Figure 3: see original paper]

Similar to the case of dSphs, $\gamma_*^{\rm SDSS}/\gamma_0$ decreases as M_* increases. Meanwhile, the mass of DM $M_{\rm DM}(R_e)$ is calculated according to Equation (49). The correlation between $M_{\rm DM}(R_e)$ and M_* is presented in Figure 4. As depicted, this correlation is weak. However, in a certain sense, it is still similar to that between $\gamma_*^{\rm SDSS}$ and M_*.

The relevant original and derived parameters from sample SDSS10 are listed in Table 2.

To extract more information about γ_* from bright galaxies, we use a new sample based on the data set of the Sloan Lens ACS (SLACS) Survey (Auger et al. 2009) to carry out the same procedure as we did for the sample SDSS DR 10. This dataset was originally used for gravitational lensing analysis, but it provides us with more information than we need to study the properties of γ_*. We denote this dataset as sample SLACS.

Compared with the sample SDSS DR 10, the galaxies in the sample SLACS are brighter and have a larger effective radius. The correlation between $\gamma_*^{\rm SLACS}$ and M_* based on sample SLACS is presented in Figure 5. As shown, the correlation between $\gamma_*^{\rm SLACS}$ and M_* is weaker than that for $\gamma_*^{\rm SDSS}$ and that for $\gamma_*^{\rm dSph}$. The correlation between $M_{\rm DM}(R_e)$ and M_* for sample SLACS is also shown in Figure 6. As is evident, the correlation is much weaker than that for sample dSphs and that for sample SDSS DR 10. The parameters for sample SLACS are also presented in Table 3.

[Figure 5: see original paper]

[Figure 6: see original paper]

4. Conclusions and Discussions

An exact CG analog of the Schwarzschild exterior solution was found to predict a linear potential $V = V_\beta + V_\gamma$ besides the conventional Newtonian potential (Mannheim & Kazanas 1989). It was also found that there exists a universal, galaxy-independent linear potential, $V_{\gamma_0} = \gamma_0 c^2 r/2$, due to the rest of matter in the universe on any local galaxies (Mannheim 1997). The parameter $\gamma = (M/M_\odot)\gamma_*$, where M is the mass of luminous matter that generates the corresponding linear potential V_γ, and γ_* is the value of γ if $M = M_\odot$. Hence, the values of γ_0 and γ_* should be universal constants independent of galaxies. These predictions of CG can be verified through galaxy observations. To date, in the literature, the tests have been successfully conducted only via the observations of spiral galaxies, specifically using the rotation curve data. The rich data of this kind uniformly gives $\gamma_* = 5.42 \times 10^{-39}$ m^{-1} and $\gamma_0 = 3.06 \times 10^{-28}$ m^{-1}.

In contrast, in this paper, we aim to test CG by utilizing the velocity dispersion data from the observations of elliptical galaxies. It is well known that within elliptical galaxies, an extra gravitational force is required to balance the observed velocity dispersion. The Jeans equation is a useful tool for describing the relationship between the velocity dispersion and the gravitational potential. The Jeans equation was originally developed in Newtonian theory. In this context, DM is introduced to account for the extra potential. To test CG, we extend the Jeans equation by simply replacing the Newtonian potential with the potential predicted by CG. In fact, when people apply CG to spiral galaxies, they follow the same approach. That is, they replace the Newtonian potential with the potential predicted by CG to explain the observed rotation curves.

We first select a sample, sample dSphs, consisting of 43 dSphs based on the reference McConnachie (2012). We found that the value of $\gamma_0 = 5.27\times10^{-28}$ m^{-1} derived from the observations of the elliptical galaxies has the same order as that derived from the observations of spiral galaxies. This result is not surprising, since the γ_0 term in linear potentials originates from the cosmological effect on any local gravitational systems, and thus should be independent of local systems. However, our sample dSphs gives the optimum value of $\gamma_*^{\rm dSph} = 1.22 \times 10^{-35}$ m^{-1}, which is about four orders of magnitude larger than that fitted by spiral galaxies ($\sim 10^{-29}$ m^{-1}). It suggests that the linear potential of luminous matter estimated from spiral galaxies is negligible when applied to elliptical galaxies. This inconsistent result between elliptical and spiral galaxies may indicate that CG fails as an alternative to the DM model, at least for elliptical galaxies.

Furthermore, as depicted in Figure 1, we discover a strong correlation between $\gamma_*^{\rm dSph}(R_e)$ and the stellar mass $M_*(R_e)$ for dSphs. This is accomplished by fixing γ_0 and treating γ_* as a free parameter. As evident from Figure 1, γ_* decreases as M_* increases. Interestingly enough, this situation is analogous to that in Newtonian gravity, where DM is introduced to provide the necessary extra potential. In Newtonian gravity, it is a widely accepted notion that the brighter the galaxy, the less DM is required. In fact, we applied Newtonian gravity to the same sample and calculated the DM mass and the luminous stellar mass within the effective radius for each galaxy. As shown in Figure 2, we found a correlation between DM mass $M_{\rm DM}$ and stellar mass M_* that is similar to the correlation between $\gamma_*^{\rm dSph}$ and stellar mass M_*. These results imply that, to explain the observations of dSphs, $\gamma_*^{\rm dSph}$ cannot be a constant. Instead, it behaves more like the amount of DM, which can vary with the amount of stellar matter. Regrettably, the varying value of γ_* violates the fundamental prediction of CG, which requires γ_* to be a universal constant.

The dSphs are dominated by an extra gravitational potential. It would be interesting to explore the correlations we discovered in dSphs using the data sets of bright elliptical galaxies. To this end, we selected two samples: sample SDSS DR 10 and sample SLACS. The galaxies in the sample SLACS are brighter than those in the sample SDSS DR 10. We carried out the same procedure as we did for dSphs. For sample SDSS DR 10, we found that the correlation between $\gamma_*^{\rm SDSS}$ and M_* (as shown in Figure 3) and the correlation between $M_{\rm DM}(R_e)$ and M_* (as shown in Figure 4) are weaker than the corresponding correlations for sample dSphs. We further found that the correlations for sample SLACS (as shown in Figures 5 and 6) are even weaker than those for sample SDSS DR 10. This indicates that when less extra potential is needed, the correlations are statistically more scattered, as expected.

For ease of reference, we list all the parameters of each sample in the corresponding table.

As shown in Equations (25) and (26), γ_* characterizes the linear potential of a unit mass and should therefore be a universal constant. However, the observed correlation between γ_* and the stellar mass M_* in galaxies closely resembles the correlation between $M_{\rm DM}$ and stellar mass M_*. This suggests that elliptical galaxies are better described by Newtonian theory (requiring DM) than by CG. Naturally, this does not necessarily mean that Newtonian gravity is correct unless DM particles are directly detected in experiments. Alternatively, the γ_*–M_* correlation could imply an additional scale-dependent quantum potential in large scale structures, as proposed by Chen (2022), Chen & Wang (2024).

Acknowledgments

We thank the anonymous referee for the valuable suggestions, which helped improve the manuscript. This work is supported by the National Natural Science Foundation of China (NSFC, grant No. 11988101) and the K. C. Wong Education Foundation.

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Note: Figure translations are in progress. See original paper for figures.