A Cartesian Catalog of 30 Million Gaia Sources Based on Second-order and Monte Carlo Error Propagation

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Full Text

Preamble

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A Cartesian Catalog of 30 Million Gaia Sources Based on Second-order and Monte Carlo Error Propagation

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Abstract

Accurate measurements of stellar positions and velocities are crucial for studying galactic and stellar dynamics. We aim to create a Cartesian catalog from Gaia DR3 to serve as a high-precision database for further research using stellar coordinates and velocities. To avoid negative parallax values, we select 31,129,169 sources in Gaia DR3 with radial velocity measurements where the fractional parallax error is less than 20% ($0 < \sigma / < 0.2$). To identify the most accurate and efficient method for propagating means and covariances, we use Monte Carlo results with 10^7 samples (MC7) as the benchmark and compare the precision of linear, second-order, and Monte Carlo error propagation methods. By assessing the accuracy of propagated means and covariances, we observe that second-order error propagation exhibits mean deviations of at most 0.5% compared to MC7, with variance deviations of up to 10%. Overall, this outperforms linear transformation. Although the Monte Carlo method with 10^4 samples (MC4) is an order of magnitude slower than second-order error propagation, its covariance propagation accuracy reaches 1% when $\sigma /$ is below 15%. Consequently, we employ second-order error propagation to convert the mean astrometry and radial velocity into Cartesian coordinates and velocities in both equatorial and galactic systems for 30 million Gaia sources, and apply MC4 for covariance propagation. The Cartesian catalog and source code are provided for future applications in high-precision stellar and galactic dynamics.

Key words: catalogs — astrometry — reference systems — methods: data analysis — astronomical databases: miscellaneous

1. Introduction

The demand for high-precision reconstruction of stellar orbits has gained prominence, particularly with the emergence of advanced astrometric missions like Gaia (Gaia Collaboration et al. 2016a, 2018). Gaia Data Release 3 (DR3) offers the largest collection of all-sky spectrophotometry, radial velocities, and astrophysical parameters for stars. It achieves highly accurate astrometric measurements for $G < 15$ mag, with median position uncertainties ranging from 0.01 to 0.02 mas, median parallax uncertainties between 0.02 and 0.03 mas, median proper motion uncertainties from 0.02 to 0.03 mas yr^{-1}, and radial velocity uncertainties of approximately 1 km s^{-1} (Gaia Collaboration et al. 2021, 2023). Such extraordinary precision has become a cornerstone in Galactic and stellar dynamics research, unlocking new possibilities for understanding celestial motion and providing unprecedented insights into the structures and dynamics of our galaxy.

We can resolve individual stars in and around our host Galaxy, the Milky Way (MW). These include stars in tidal debris or stellar streams, which are stripped by tidal forces from dwarf galaxies and globular clusters orbiting the MW. In the field of galactic dynamics, the modeling of stellar streams often combines an approximate treatment of stream formation with high-resolution simulations of galaxy formation. For streams whose progenitor galaxy or globular cluster is not fully disrupted by tides, the currently observed position and velocity of the progenitor can be integrated back in time with a fiducial MW potential model, then integrated forward in time with tracer particles released from the two Lagrangian points to simulate the formation and evolution of the stream in the galactic tidal field. By comparing the simulated stream to observations, one can constrain the underlying dark matter distribution (e.g., Küpper et al. 2012, 2015; Bonaca et al. 2014; Gibbons et al. 2017; Palau & Miralda-Escudé 2023). The modeling of stream orbits strongly depends on the initial conditions set by the progenitors, often achieved in action and action-frequency space (e.g., Sanders & Binney 2013; Bovy 2014; Sanders 2014; Bovy et al. 2016), and is therefore sensitive to error propagation.

In the pursuit of understanding wide binaries, achieving precise error propagation is paramount, given the subtle velocity differences, often as minimal as 1 km s^{-1}, exhibited by these binary systems. Harnessing the high-precision astrometric data provided by Gaia (Gaia Collaboration et al. 2016b), millions of wide binaries have been identified through meticulous examination of their common proper motion and parallax (e.g., El-Badry & Rix 2018; Tian et al. 2020; El-Badry et al. 2021). Particularly noteworthy is the utilization of the relative velocity and relative position angle of wide binaries to explore their eccentricity distribution (Hwang et al. 2022). Since these investigations heavily rely on accurate Cartesian coordinates and velocities, precise transformation of coordinates from Gaia astrometry with careful error propagation becomes imperative. This is crucial not only for avoiding false positives in wide binary selection but also for enhancing the reliability of statistical studies concerning their properties.

Unlike wide binaries, stellar encounters encompass the serendipitous alignment of stars occurring when they come into close proximity over relatively short periods. Notably, encounters within the solar system significantly contribute to shaping the structure of the Oort Cloud (e.g., García-Sánchez et al. 2001; Dones et al. 2004; Rickman et al. 2008, 2012; Feng & Bailer-Jones 2015; Dybczyński et al. 2022). The exploration of slow and close encounters involving interstellar objects like 'Oumuamua yields valuable insights into identifying their home systems and understanding their dynamical history (e.g., Feng & Jones 2018b; Bailer-Jones et al. 2018; Zwart et al. 2018; Loeb 2022). Accurate stellar orbital integration proves essential for pinpointing stellar encounters, necessitating precise propagation of stellar motion from an initial epoch to millions of years in the past or future. Given the nonlinear nature of stellar orbits over million-year timescales, the encounter time and distance are highly sensitive to the initial Cartesian state of the stars (Dybczyński & Berski 2015).

Recognizing the critical role of error propagation in numerous astrophysical applications, we undertake a comparative analysis of different methodologies for error propagation in the transformation of Gaia astrometry into Cartesian positions and velocities of stars. Specifically, we investigate linear, second-order, and Monte Carlo (MC) error propagation methods. While advanced techniques such as the Kalman filter and unscented transformation (e.g., Smith et al. 1962; Schmidt 1966; Julier & Uhlmann 2004; Chen et al. 2017; Michelotti et al. 2024) are commonly applied in nonlinear-system tracking and navigation, second-order error propagation consistently achieves precision comparable to these approaches, as exemplified by Feng & Jones (2018a), who conducted a comprehensive comparison of various methods in stellar orbital integration. Our study reaffirms that second-order error propagation surpasses linear propagation in accuracy. Consequently, we use second-order error propagation to derive Cartesian positions and velocities in both Equatorial and Galactic coordinate systems from Gaia astrometry, and apply MC with 10^4 samples for covariance conversion. This approach enhances both the efficiency and accuracy of coordinate transformation while establishing a robust foundation for research across diverse astrophysical fields.

The structure of this paper unfolds as follows: In Section 2, we provide a detailed overview of the data employed for error propagation. Section 3 delves into the techniques utilized for coordinate transformation and covariance propagation, encompassing MC, linear, and second-order error propagation methods. Section 4 presents and discusses the extensive outcomes derived from our methodology, focusing on approximately 30 million sources in Gaia DR3. Finally, Section 5 encapsulates a succinct discussion and conclusion. Definitions of relevant variables and abbreviations are provided in Table E1.

2. Data

Gaia DR3 (Gaia Collaboration et al. 2023) encompasses 33,812,183 sources, providing measurements of radial velocities (v) and five-parameter astrometric data, including right ascension (α), declination (δ), parallaxes ( ), proper motions in right ascension ( ), and proper motions in declination ($_8$). Due to more observational data and an improved data reduction pipeline, Gaia DR3 provides many more targets with radial velocity data than the 7,224,631 supplied by Gaia Data Release 2 (DR2; Gaia Collaboration et al. 2018), and the accuracy of radial velocity has also been significantly improved (Katz et al. 2023).

While distance is inherently a positive quantity, some sources may exhibit negative parallaxes due to various reasons. Bayesian inference has been adopted to estimate distances for stars with negative parallax measurements (e.g., Astraatmadja & Bailer-Jones 2016; Luri et al. 2018; Bailer-Jones et al. 2018, 2021). Following the recommendation of Bailer-Jones (2015), incorporating appropriate priors becomes crucial for Gaia sources with fractional parallax error (σ /) exceeding 20% for distance inference. However, precise parallax measurements ($0 < \sigma$ / < 0.2) with high signal-to-noise ratio (S/N > 5) are not sensitive to priors (e.g., Bromley et al. 2018). Consequently, after correcting for zero-point parallax offset (Lindegren et al. 2021; Ding et al. 2024), as well as magnitude and color-dependent proper motion bias (Cantat-Gaudin & Brandt 2021), we select a subset of 31,129,169 stars from Gaia DR3 that have valid radial velocity measurements and fractional parallax error satisfying $0 < \sigma$ / < 0.2. This allows distance to be directly represented by the reciprocal of parallax, ensuring reliability and accuracy while minimizing potential bias from priors.

3. Error Propagation

In this section, we delineate three error propagation methods utilized in this study. Our approach involves converting the five-parameter astrometry and radial velocities of Gaia sources to Cartesian coordinates and velocities in both the Equatorial and Galactic coordinate systems. Additional details regarding the transformation of Galactic coordinates can be found in Appendix B.

The transformation from spherical coordinates to Cartesian coordinates requires utilization of the Jacobian matrix.

3.1. Linear Error Propagation

Linear error propagation is the default method used by most astronomical data analyses. Considering that linear error propagation is broadly used in the community (e.g., Butkevich & Lindegren 2014), we only briefly introduce it as follows.

First, we define the vectors and matrices used in error propagation. The spherical position and velocity vector in the Equatorial coordinate system is defined as [spherical coordinates]. The conversion from a covariance matrix in spherical coordinates to one in Cartesian Equatorial coordinates can be expressed as **J C J** , where **J** is the Jacobian matrix (see Appendix C for details).

3.2. Second-order Error Propagation

The distance r is derived from parallax according to r = A/ , where A = 1 au. The position and velocity vectors in Cartesian equatorial coordinates are linked to the corresponding spherical coordinates through nonlinear transformations involving trigonometric functions of α and δ.

The Gaia DR3 catalog provides uncertainties and correlation coefficients for the five-parameter astrometric solutions, plus radial velocity errors. Since radial velocity and the five astrometric parameters are measured independently, we treat them as independent, meaning correlation coefficients between v and the five astrometric parameters are zero. From Gaia DR3, we use $\sigma\alpha^*$ to denote $\sigma\alpha$, and σ to denote errors in position and velocity vectors, with (=) representing correlation coefficients where i or j corresponds to α, δ, , α^*, δ, or v . The six-dimensional covariance matrix is constructed accordingly.

Although linear propagation is widely used, it may not always yield optimal results for accurate error propagation (e.g., Ilyin 2012). Higher-order error propagation becomes necessary to ensure optimal performance, particularly when dealing with measurements having relatively large errors where Taylor series truncation errors become significant. In such scenarios, linear transformation may result in decreased accuracy. To achieve enhanced precision, we employ a second-order Taylor series for propagating statistical errors.

Second-order error propagation is commonly used in science and engineering for precise calculations and data processing (Wang & Chirikjian 2008; Le Dimet et al. 2014). In astronomy, it is frequently applied to measure positions and motions of celestial bodies, such as in satellite orbit calculations (e.g., Sengupta et al. 2007; Li & Sang 2020).

Following Putko et al. (2001), the second-order mean and variance of the output vector **F**, transformed from the input vector **b**, can be expressed as:

_F = f(_b) + ½ tr(**H C**) **C**_F = **J C J** + ½ tr(**H C H C**)

where _b and **C** are the first-order mean and variance respectively. When the Hessian matrix **H** and covariance matrix **C** are available, calculations of second-order moments can be performed using matrix operations (see Zhang et al. 2011 for detailed definitions and derivation).

In matrix form, the second-order terms can be expressed as:

[Second-order terms in matrix form]

Performing a Taylor series expansion at a particular point provides results valid only in the immediate proximity of that point. Consequently, approximation precision tends to diminish as deviation from the mean grows, particularly with non-Gaussian output distributions. While it is feasible to analytically derive higher moments of a non-Gaussian distribution, reconstructing the distribution uniquely from these moments poses considerable challenges. Given that many astronomical applications primarily focus on mean and covariance, we opt not to undertake error propagation for higher moments in this study.

3.3. Monte Carlo (MC) Error Propagation

To determine the precision of linear and second-order error propagation methods, we use the MC error propagation method as our reference standard. Instead of directly sampling absolute positions (α_0, δ_0), we sample deviations relative to observed positions according to observational errors. To prevent bias toward the equatorial poles, we generate samples of coordinate and velocity deviations ($\Delta\alpha$, $\Delta\delta$, , α, δ, v) from six-dimensional joint Gaussian distributions centered on observed values (0, 0, $_0$, α^*_0, δ_0, v $_0$). The covariance matrix is defined using standard deviations and correlation coefficients.

To obtain samples of spherical positions, we add sampled position deviations to observed positions: $\alpha = \alpha_0 + \Delta\alpha^*$ and $\delta = \delta_0 + \Delta\delta$. This avoids rounding errors because observational errors ($\sigma\alpha$, $\sigma\delta$) are very small compared to α_0 and δ_0 themselves.

For Gaia DR3 sources with radial velocity and fractional parallax error satisfying $0 < \sigma\ /\ < 0.2$, only about 0.001% of data show negative parallaxes in MC samples. We resample until achieving positive parallaxes, which has minimal impact on mean and covariance as the fractional bias is approximately 0.01%. We calculate Cartesian coordinates for each MC sample according to the transformation equations, obtaining the distribution of Cartesian coordinates.

To more accurately evaluate error propagation methods, we generate a reference set of 10 million MC samples (MC7) for comparison. Additionally, we employ MC simulations with 10^3 (MC3), 10^4 (MC4), 10^5 (MC5), and 10^6 (MC6) samples. Transforming each sample from spherical to Cartesian coordinates in the Equatorial system and using their means as MC propagation results, we define fractional deviations of coordinates and velocities (D) and fractional deviations of their variances (s) relative to MC7 for various methods.

4. Results

To investigate the dependence of error propagation on fractional parallax error, we evenly divide $\sigma\ /\ $ (range 0–0.2) into 10 bins and randomly select 1000 stars from Gaia DR3 for each bin. We employ linear, second-order, and MC error propagation methods with different sample sizes to transform stars from spherical coordinates and velocities to Cartesian coordinates and velocities in the Equatorial system. We compare fractional deviations of means and variances relative to MC7 in Figure 1 [Figure 1: see original paper].

Analysis reveals the following features:

- **Propagation of coordinates and velocities:** According to the top panels of Figure 1, linear error propagation leads to >1% fractional deviations in mean coordinate and velocity for targets with $\sigma\ /\ > 0.11$. Mean coordinate and velocity propagation using second-order error propagation and MC with >10,000 samples achieves <0.2% precision for sources with $0 < \sigma\ / \ < 0.15$. Second-order error propagation achieves <0.5% precision for sources with $0 < \sigma\ / \ < 0.2$.

- **Propagation of variance:** As seen in the bottom panels of Figure 1, linear and second-order variance propagation can achieve 10% precision for sources with <15% fractional parallax error. In contrast, MC with >10,000 samples propagates variance with <1% precision for sources with $0 < \sigma\ / \ < 0.15$. While second-order error propagation achieves higher precision for mean transformation than MC4 for sources with small fractional parallax error, it fails to propagate variance as precisely as MC4. This occurs because Gaussian distributions in spherical coordinates transform into non-Gaussian distributions in Cartesian coordinates. Since higher moments are not accounted for when calculating variance of non-Gaussian distributions, second-order error propagation is less accurate in propagating variance (and covariance) than in propagating the mean.

To evaluate the efficiency of various propagation methods, we calculate computational time for each method in Figure 2 [Figure 2: see original paper]. MC method time consumption is linearly proportional to the number of samples, while second-order error propagation execution time is one order of magnitude lower than MC4. Although MC3 requires comparable time to second-order error propagation, the latter exhibits significantly higher precision, as illustrated in Figure 1.

Therefore, considering both efficiency and precision, we recommend second-order error propagation for mean transformation with ~0.5% precision and for covariance propagation with 10% precision. If higher variance propagation precision is needed, we recommend the MC4 method. Combining advantages of both methods, we apply second-order error propagation to convert means and employ MC4 to propagate covariances of astrometric parameters and radial velocities for 30 million Gaia sources with $0 < \sigma\ / \ < 0.2$ into Cartesian coordinates and velocities in both Equatorial and Galactic coordinate systems. This approach requires only 0.0065 s average CPU time per calculation for a complete set of Cartesian catalog data (including both coordinate systems).

Figure 3 [Figure 3: see original paper] shows the galactocentric Cartesian coordinate distribution of 31,066,855 stars from Gaia DR3 with radial velocity, where parallax satisfies $0 < \sigma\ / \ < 0.2$ and duplicated sources are removed. Although the face-on view (XY plane) is similar to Figure 2 in Katz et al. (2023), Figure 3 lacks elongated features in face-on and edge-on views because the catalog excludes Large and Small Magellanic Cloud stars due to their fractional parallax errors exceeding 20% (as discussed in Bailer-Jones et al. 2021).

The Hertzsprung–Russell (HR) diagram of the Cartesian catalog is displayed in Figure 4 [Figure 4: see original paper], including only data with M_G and G_{BP} − G_{RP}. The figure shows the catalog contains stars with absolute magnitudes ranging from 15 to –5, including numerous red giants and main-sequence stars, but no white dwarfs. The Gaia spectroscopic pipeline lacks appropriate templates for white dwarfs, and mismatch between observed spectra and templates can lead to significant systematic errors in radial velocity measurements (Katz et al. 2023). Consequently, white dwarfs lack radial velocity data and are not included in this catalog.

The Cartesian catalog in Appendix A (Table A1) contains mean Cartesian coordinates determined by second-order error propagation and covariance of Cartesian coordinates determined by the MC4 method. We also include transversal velocity and distance values in the catalog.

5. Discussion and Conclusion

In our Gaia DR3-based study, we compare various error propagation methods and identify second-order error propagation as the most efficient for achieving nearly 0.5% precision in propagating mean coordinates and velocities. Linear propagation, commonly used, fails to achieve 1% precision in coordinate and velocity means when fractional parallax error exceeds 10%.

However, second-order error propagation precision for variance is inferior to that for mean transformation. This discrepancy arises because nonlinear coordinate transformation introduces non-Gaussian errors in new coordinate systems. Although MC4 is about 10 times more computationally expensive than the second-order method, it achieves more accurate covariance propagation. Therefore, for propagating covariance with nearly 1% precision, we suggest employing MC propagation with a minimum of 10,000 samples.

Balancing efficiency and precision, we employ second-order error propagation for mean values of 31,129,169 Gaia sources with radial velocity and fractional parallax error below 20%, and utilize MC4 for covariance conversion. We present Cartesian coordinates, velocities, and their covariances in both Equatorial and Galactic coordinate systems. This catalog offers highly precise mean coordinates and velocities, facilitating applications such as accurate integration of stellar orbits and studies of wide binaries.

Subsequent investigations into precise error propagation should address the non-Gaussian nature of errors and incorporate higher moments, including skewness and kurtosis, in the propagation process. Advanced techniques such as Kalman and unscented filters (mentioned in Section 1) may offer more efficient and accurate results for error propagation in stellar motions.

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Data availability: The complete data in Table A1 will be available at CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via https://cdsarc.cds.unistra.fr/viz-bin/cat/I/363 and the source code can be found on GitHub (https://github.com/LuyaoZhang-sjtu/GaiaDR3-error-propagation).

Appendix A Data Sample

This appendix presents a sample of the data catalog in Table A1, including mean Cartesian coordinates derived from second-order error propagation, the covariance matrix obtained using the MC4 method, and transversal velocity and distance for each source.

Table A1 Examples of the Catalog of Coordinates and Velocities in Equatorial and Galactic Coordinate Systems Based on Second-order Error Propagation and their Covariances Based on MC4

[Table A1 content preserved exactly as in original]

Note: The correlation coefficients between Cartesian coordinates are explained in Appendix D.

Appendix B Transformation from Equatorial to Galactic Coordinate System

The following relationship exists between coordinates in Equatorial and Galactic coordinate systems:

$$\cos b \cos(l - l_{NGP}) = \cos\delta \cos(\alpha - \alpha_{NGP})$$
$$\cos b \sin(l - l_{NGP}) = \cos\delta \sin(\alpha - \alpha_{NGP}) \cos i + \sin\delta \sin i$$
$$\sin b = -\cos\delta \sin(\alpha - \alpha_{NGP}) \sin i + \sin\delta \cos i$$

During calculation, the galactic pole coordinates are consistent with those in the Python package astropy, where $\alpha_{NGP} = 192.85947789°$, $\delta_{NGP} = 27.12825118°$, and $i = 122.93192526°$.

The position coordinate matrices in Equatorial (eqt) and Galactic (gal) coordinate systems are defined as:

$$\mathbf{P}_{eqt} = [x_{eqt}, y_{eqt}, z_{eqt}]$$
$$\mathbf{P}_{gal} = [x_{gal}, y_{gal}, z_{gal}]$$

By separating position coordinate matrices from the transformation equations, we extract the rotation matrix **R**:

R = [rotation matrix elements]

The velocity coordinate matrix **V**_{eqt} is the derivative of the equatorial position coordinate matrix with respect to time. Therefore, for both position and velocity propagation, we simply multiply by the rotation matrix:

P_{gal} = **R** **P**_{eqt}
V_{gal} = **R** **V**_{eqt}

Hence, linear and nonlinear calculation of Galactic coordinates depends only on Cartesian coordinate matrices **P** and velocities **V** in the Equatorial system. The Galactic variance-covariance matrix is calculated as:

C_{gal} = **R** **C**_{eqt} **R**

Appendix C Jacobian Matrix

The elements of the Jacobian matrix are given below, where A = 1 au:

x/ α = -A sin α / cos δ
x/ δ = -A cos α sin δ /
x/ = -A cos α cos δ / ^{2}
[Additional Jacobian elements preserved as in original]

Appendix D Correlation Coefficient

In the covariance transformation, the correlation coefficient of x and y is _{xy} = C_{xy} / (σ_x σ_y). This coefficient depends only on α, δ, and r (where r = A/ , A = 1 au), so the variables b_i, b_j = α, δ, .

Because $\sigma\alpha$ and $\sigma\delta$ are extremely small compared to α and δ values, as $\sigma\alpha$, $\sigma\delta$ $\rightarrow$ 0, the correlation coefficient of x and y simplifies to _{xy} -sin α cos α / (|sin α| |cos α|) = $±$1. The same applies to correlation coefficients _{xz} and _{yz}. Consequently, many x, y, and z correlation coefficients approach $±$1 in the resulting Cartesian catalog.

Figure D1 shows sample distributions from the MC4 method. For Gaia DR3 2149392370122774528 and Gaia DR3 2070799897454308224, correlation coefficients between x, y, and z are approximately $±$1 regardless of fractional parallax errors. Higher fractional parallax errors lead to more skewed distributions of six-dimensional parameters, indicating significant impact on higher-order moments.

Figure D1. Cornerplots showing distributions from the MC4 method for Equatorial Cartesian coordinates for Gaia DR3 2149392370122774528 (top) and Gaia DR3 2070799897454308224 (bottom), which have fractional parallax errors of 0.001 and 0.199, respectively. Blue dots represent MC4 samples while contours represent 0.5, 1, 1.5, and 2σ confidence levels. The mean is shown by horizontal and vertical gray lines.

Appendix E Acronyms and Variables

Definitions of all key variables and abbreviations referenced in this work are provided in Table E1.

Table E1 Glossary of Main Acronyms and Variables

[Table E1 content preserved exactly as in original]

References

[References section preserved exactly as in original]

Note: Figure translations are in progress. See original paper for figures.

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