Investigating the Evolution of Planetary Mass Ranking Entropy in Exoplanet Systems: Postprint

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Abstract

This study defines entropy related to mass ordering in multi-planetary systems and investigates the impact of planetary system dynamical evolution on planetary mass-ordering entropy using Monte Carlo simulations and N-body dynamical numerical simulations. The results demonstrate that behaviors such as collisions and position exchanges during the dynamical evolution of planetary systems lead to changes in the mass-ordering entropy of planetary systems. Position exchanges cause the system's mass-ordering entropy to gradually increase, while collisions may lead to a decrease in the system's mass-ordering entropy. Although the system's entropy value may decrease, the ratio of the current system's mass-ordering entropy to the maximum mass-ordering entropy achievable by the current system consistently exhibits an increasing trend, indicating that the system is evolving toward equilibrium. Comparisons with Kepler multi-planetary systems reveal that approximately 16.9%±4.7% of Kepler multi-planetary systems still maintain an ordered mass arrangement, suggesting that these systems may not have undergone violent dynamical evolution.

Full Text

Abstract

This study defines an entropy related to mass ordering in multi-planetary systems and investigates, through Monte Carlo simulations and N-body dynamical numerical simulations, the influence of planetary system dynamical evolution on the entropy of planetary mass ordering. The results indicate that collisions, positional swaps, and other dynamical processes can modify the mass ordering entropy of planetary systems. Positional swaps tend to gradually increase the mass ordering entropy, whereas collisions may lead to its decrease. Although a system's entropy may decrease, the ratio of its current mass ordering entropy to the maximum attainable mass ordering entropy consistently increases, demonstrating that the system evolves toward equilibrium. Comparison with Kepler multi-planetary systems reveals that approximately of them still maintain an ordered mass arrangement, suggesting that these systems may not have experienced violent dynamical evolution.

Key words: methods: numerical, methods: statistical, planets and satellites: dynamical evolution and stability, entropy

1 Introduction

The second law of thermodynamics elucidates the directional nature of spontaneous physical processes in nature, namely that in isolated systems, these processes proceed in the direction of entropy increase. Statistical physics proves that the entropy of a thermodynamic system is related to thermodynamic probability through the Boltzmann relation:

$$S = k \ln W$$

where W represents the number of microscopic states corresponding to a given macroscopic state (the thermodynamic probability) and k is the Boltzmann constant. From a microscopic perspective, spontaneous physical processes essentially evolve from states of lower probability to states of higher probability, from macroscopic states containing fewer microscopic states to those containing more. Common thermodynamic processes such as heat conduction, work-to-heat conversion, and diffusion phenomena are all entropy-increasing processes—results of increased thermodynamic probability. This increase in thermodynamic probability fundamentally represents the evolution of a system from non-equilibrium to equilibrium states. Beyond these heat-related phenomena, many other natural processes, such as the dynamical evolution of planetary systems, also proceed toward equilibrium. This paper draws an analogy with the definition of Boltzmann entropy in thermodynamics to define an entropy related to planetary mass ordering in planetary systems, and combines Monte Carlo simulations with N-body dynamical numerical simulations to study the evolution of this mass ordering entropy.

2 Mass Ordering Entropy in Kepler Planetary Systems

Some studies [5] suggest that the arrangement of planets in Kepler systems is ordered, meaning that planets farther from the central star tend to have larger radii and masses. In thermodynamics, order typically implies low entropy. Many theories of planetary evolution [6, 7] propose that the planetary systems we observe today have undergone tens of millions of years of dynamical evolution. During this evolution, planets may experience mutual collisions, mergers, and positional exchanges, leading us to expect that the observed planetary arrangements should be highly disordered. If planetary systems exhibit very ordered arrangements, this suggests that they may not have experienced particularly violent dynamical processes, at least not frequent changes in their relative positions. Inferring the evolutionary history of planetary systems based on their current entropy is therefore crucial for understanding their dynamical evolution. But how do we define an entropy related to planetary ordering? An appropriate method is to use the microscopic interpretation of entropy to define a sorting-related entropy in multi-planetary systems.

Building upon the approach in reference [8], which defined an entropy related to planetary radius ordering based on the Boltzmann entropy form of equation (1), we similarly define an entropy related to planetary mass ordering. We choose mass over radius because mass is directly related to the dynamical evolution of planetary systems. In our definition, the macrostate is the planetary sorting score, and the corresponding microstate is the number of possible planetary arrangements for that score. In a system with N planets, the sorting score for adjacent planetary masses is defined as follows: if $m_{i+1} > m_i$, then $t_i = 1$; otherwise, $t_i = -1$, where m_i and m_{i+1} are the masses of the i-th and $(i+1)$-th planets counted from the inside outward (i.e., from near the star to farther away). The total mass ordering score for the system is $T = \sum_{i=1}^{N-1} t_i$. The thermodynamic probability corresponding to this score—that is, the number of microstates W_T—can be calculated using Eulerian numbers $A_{N,q}$ [8]. The Eulerian numbers are expressed as:

$$A_{n,q} = \sum_{j=0}^{q+1} (-1)^j \binom{n+1}{j} (q+1-j)^n$$

The mass ordering entropy for a planetary system with score T is then:

$$S_T = \ln W_T = \ln A_{N,(N+T-1)/2}$$

Note that this definition of ordering entropy omits the Boltzmann constant from the traditional Boltzmann relation. If the planets in a system are arranged from inside to outside in order of increasing mass, then $T = N - 1$ and there is only one microstate ($W_T = 1$), giving a mass ordering entropy of 0. Similarly, if planets are arranged from inside to outside in order of decreasing mass, then $T = -(N-1)$ and again there is only one microstate, yielding zero entropy. Thus, when planetary systems are in ordered arrangements, their ordering entropy is very small. While reference [8] considered additional factors such as the influence of planetary sorting integration paths and the effect of adjacent planetary pairs with opposite score signs on the number of states, these require numerical statistical methods and involve substantial computational cost. Since the evolutionary trends of these three types of entropy are similar, we have chosen the simplest first type for our work. Although the number of planets in a planetary system is generally less than 10, not constituting a "large-number" system, its entropy evolution can be analogized to the evolution of Boltzmann entropy in thermodynamics, reflecting the system's progression toward equilibrium.

3 Impact of Planetary System Dynamical Evolution on Mass Ordering Entropy

3.1 Monte Carlo Simulations

The core accretion model suggests that low-mass planets tend to form in the inner regions of planetary systems, while high-mass planets form in the outer regions [9]. However, if positional exchanges occur during planetary system evolution, the mass ordering entropy will change accordingly. Using Monte Carlo methods, reference [8] simulated the evolution of radius ordering entropy caused by positional exchanges during planetary system evolution and found that the radius ordering entropy gradually increased with system evolution. In reality, besides positional exchanges, planets may also undergo collisions and scattering processes, all of which can alter the mass ordering entropy. Here, we similarly employ Monte Carlo methods to simulate the dynamical evolution of 100 planetary systems, each containing 20 planets. The masses of planets in these systems are drawn from a Gaussian distribution with a mean of 6 Earth masses and a standard deviation of 1 Earth mass, with a minimum mass of 0.1 Earth masses. We initially arrange these planets from inside to outside in order of increasing mass. During each iteration, we assume that adjacent planets have a 10% probability of undergoing a positional exchange and a 0.5% probability of experiencing a collision, with at most one pair of adjacent planets interacting per iteration. Additionally, we stipulate that collisions cease when only 8 planets remain in the system. To compare the effects of collisions and positional exchanges on planetary system mass ordering entropy, we conduct two control simulations: one considering only interplanetary collisions and another considering only positional swaps. In our simulations, all planetary collisions are treated as mass mergers [10]. Each planetary system undergoes 1,000 iterations, and we track the evolution of mass ordering entropy in these systems.

Furthermore, we define a new quantity F_T to measure whether the system has reached a state of maximum thermodynamic probability—that is, equilibrium. F_T is defined as:

$$F_T = \frac{S_T}{S_T^{\max}}$$

where S_T is the current system's mass ordering entropy and $S_T^{\max}$ is the maximum entropy achievable for the mass ordering of planets in the current system. If $F_T = 1$, the system has reached equilibrium (the state of maximum entropy). A smaller F_T indicates greater distance from equilibrium and a more ordered system. The evolution of F_T and S_T is shown in Figure 1 [Figure 1: see original paper].

Figure 1 illustrates the evolution of entropy and the ratio of entropy to maximum entropy in 100 sets of planetary systems, each containing 20 planets with an initial entropy of 0. From top to bottom, the plots show scenarios considering only positional swaps, only collisions, and both positional swaps and collisions simultaneously. The red shaded areas represent the 68% confidence intervals for the 100 sets of systems.

As shown, when only positional exchanges are considered, the planetary system's mass ordering entropy S_T continuously increases with iteration number. After 1,000 iterations, most planetary systems reach maximum entropy, with planetary mass distributions becoming completely random. When only collisions are considered, the system's entropy first increases and then decreases with iteration number. The initial increase occurs because collisional mergers alter the initial mass ordering, increasing the number of microstates. The subsequent decrease results from the reduction in planetary number, which reduces the number of microstates accordingly. When both positional swaps and collisions are considered, the planetary system's entropy also first increases and then decreases. In our simulations, the reduction in planetary number has a significant impact on the system's entropy value. However, the evolution of F_T shows that regardless of whether collisions are included, F_T for planetary systems tends to increase with iteration number. For some systems, despite a decrease in planetary number, they remain in the state of maximum thermodynamic probability. Therefore, both collisions and positional exchanges drive planetary systems toward equilibrium.

3.2 N-body Numerical Simulations

The above simulations are idealized, where planetary positional exchanges and collisions occur with given probabilities and only between adjacent planets. Next, we examine the impact of planetary system dynamical evolution on mass ordering entropy through N-body numerical simulations. We assume each planetary system contains 20 planets orbiting a central star of solar mass in coplanar circular orbits. The innermost planet's semi-major axis is set to 0.1 au. Within each system, the distance between planets is set to be a multiple of their mutual Hill radius, with the semi-major axes of other planets selected according to the relation:

$$a_{i+1} - a_i = K \cdot r_H^{(i,i+1)}$$

where a_i is the semi-major axis of the i-th planet, M_* is the central star's mass (taken as one solar mass in this study), and K is a dimensionless quantity. A smaller K value indicates closer spacing between adjacent planets and greater system instability. Here, we assume K is drawn from a Gaussian distribution with a mean of 6 and a standard deviation of 3 to ensure that planetary systems exhibit unstable behavior during evolution [11], allowing us to study changes in mass ordering entropy during dynamical evolution. Notably, to avoid excessively small K values and potential negative values, we only consider $K > 1$. The mass of each planet in the system is drawn from a Gaussian distribution with a mean of 6 Earth masses and a standard deviation of 1 Earth mass, with a minimum mass of 0.1 Earth masses. Based on planetary masses, we use the Forecaster package from reference [12] to estimate planetary radii. We assume the initial mean anomalies of these 20 planets are randomly distributed between 0° and 360°.

Within the same system, the 20 planets can be arranged with masses increasing sequentially, decreasing sequentially, or randomly. We simulate 100 planetary systems for each of these three possibilities, designated as sets A, B, and C. We consider Newtonian gravitational forces between planets and the central star and between planets themselves, integrating these systems using the `mercurius` integrator in the Python package REBOUND [13, 14] for a duration of 10^6 yr. During this 10^6 yr period, when the distance between two planets becomes smaller than the sum of their radii, we consider a collision to have occurred. In our simulations, all collisions result in planetary mergers with conservation of orbital angular momentum, total mass, and momentum. Additionally, when a planet's distance from the central star exceeds 100 au, we consider it ejected from the planetary system. In subsequent analyses, we only count systems containing at least 4 planets at the end of integration.

The mass ordering entropy gradually increases for sets A and B. In set A, planetary masses initially increase from inside to outside, so the initial entropy is 0. We select one planetary system to analyze its mass ordering entropy evolution, as shown in Figure 2 [Figure 2: see original paper]. The left panel of Figure 2 shows that at the initial moment, planets in the system are arranged with increasing mass from inside to outside. Within 10^3 yr, the total number of planets decreases from 20 to 10, and some planets undergo positional exchanges, increasing the system's mass ordering entropy. Between 10^3 yr and 10^5 yr, the number of planets decreases from 10 to 6, reducing the system's mass ordering entropy, but F_T increases to 1. This indicates that although the system's entropy value decreases, its current arrangement represents the most probable state for that system. Throughout our simulation, the system's final configuration changes around 7.6×10^5 yr, leaving only 5 planets, which reduces the number of microstates and the system's entropy value, but the system remains in its most probable arrangement.

Figure 3 [Figure 3: see original paper] shows the evolution of F_T for the three simulation sets. The majority of systems in sets A and B show increasing F_T with evolutionary time, indicating that most systems gradually approach equilibrium. Meanwhile, set C maintains a consistently high F_T value. For sets A and B, which initially have $F_T = 0$, only 2% of systems maintain mass ordering from smallest to largest or largest to smallest after evolution. For set C, which initially has F_T close to 1, only 1% of systems end with random mass ordering.

4 Comparison with Observations

The Kepler satellite has detected 77 systems containing four or more planets (excluding Jupiter-containing systems). Here we consider only systems with four or more planets because these have relatively high observational completeness [15] and facilitate comparison with simulation results. Previous research [5] concluded that Kepler planetary systems exhibit relatively ordered mass sorting, treating all Kepler systems as an ensemble. They compared real Kepler planetary system mass ordering with random mass ordering and found that real Kepler systems show higher ordering. Building on this concept, we compare the average F_T of the 77 real Kepler systems (denoted as $\langle F_T \rangle$) with that of randomly mass-ordered systems. Specifically, we first use the mass-radius relation from reference [12] to estimate planetary masses based on their radii. We then take all planets from these 77 Kepler systems as a total sample and redistribute them back to the systems according to each system's original planet count. After redistribution, we recalculate F_T for all systems. Repeating this process 1,000 times yields 1,000 sets of $\langle F_T \rangle$, with results shown in Figure 4 [Figure 4: see original paper].

The upper panel of Figure 4 shows the distribution of $\langle F_T \rangle$ from random mass ordering versus the real value for Kepler systems, while the lower panel shows the distribution of F_T for Kepler multi-planet systems. The median $\langle F_T \rangle$ from random mass ordering is 0.911 (very close to the final median of 0.918 from random distribution numerical simulations), with a standard deviation of 0.027. The real Kepler systems have $\langle F_T \rangle = 0.798$. Therefore, the significance of ordered mass sorting in real Kepler systems is 4.1σ, slightly lower than that obtained in reference [5]. One possible reason is that the mass ordering parameter defined by equation (3) in reference [5] actually only compares the relative masses of the outermost and innermost planets in a system.

Among the 77 Kepler planetary systems, 53 have $F_T > 0.9$, indicating that these systems likely experienced relatively violent dynamical processes during their evolutionary history. Before gas disk dissipation, these systems were probably in a very compact configuration similar to our numerical simulations [7]. After gas disk dissipation, without the buffering effect of the gas disk, interactions between planets excited their eccentricities, making the entire system highly unstable. Mutual collisions, positional exchanges, or scattering among planets led to highly disordered mass arrangements.

Additionally, 13 systems have $F_T < 0.5$. These systems likely maintained their primordial mass ordering and probably did not experience frequent positional exchanges or collisions during their evolutionary history. The orbital configurations of these systems may not have been so compact before gas disk dissipation, or their planets may have been in relatively stable first-order resonant chains after disk migration. After gas disk dissipation, even without the buffering effect of the gas disk, interactions between planets would not cause frequent instability within the system. These systems may retain the mass distribution from the time of gas disk dispersal. The fraction of systems with $F_T < 0.5$ is $16.9\% \pm 4.7\%$. Observationally, the fraction of planetary systems with $F_T < 0.5$ is [missing value], where the error is Poissonian. In contrast, the random distribution numerical simulations from Section 3.2 show that only $1.0\% \pm 1.0\%$ of planetary systems have $F_T < 0.5$. Therefore, at the 3.3σ level, Kepler multi-planet systems are more ordered than randomly distributed simulated systems.

5 Conclusions

Based on previous research [8], this paper presents a definition of entropy related to planetary mass ordering in multi-planet systems and investigates, through Monte Carlo simulations and N-body dynamical simulations, the impact of planetary system dynamical evolution on mass ordering entropy.

Our results demonstrate that collisions, positional exchanges, and other behaviors during planetary system dynamical evolution can alter mass ordering entropy. Although system entropy may decrease in some cases, the ratio of the current system's mass ordering entropy to its maximum attainable mass ordering entropy consistently shows an increasing trend, indicating that the system gradually approaches equilibrium. Comparison with Kepler multi-planet systems reveals that approximately maintain high ordering, suggesting that these systems likely did not experience violent dynamical evolution after formation and retain their primordial mass ordering. This provides important clues for better understanding the dynamical evolution of exoplanetary systems.

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Note: Figure translations are in progress. See original paper for figures.