Postprint: Analysis Methods for Asteroid Impact Probability

Authors: Zheng Hang, Zhu Qingfeng, Song Yezhi, Li Xuzhi, Xu Xiaohui, Qiu Jinsheng, Zhao Junhan

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Abstract

In response to the current situation of inconsistent conclusions among existing asteroid collision monitoring systems, this study investigates the linear approximation method employed by line of variation collision monitoring systems, concluding that the deviation between the orbital distribution obtained through this method and the theoretical orbital distribution becomes progressively significant with increasing orbital propagation time. Using Monte Carlo methods, collision probabilities for six asteroid instances were calculated; comparison with existing Monte Carlo collision monitoring systems reveals a maximum discrepancy of 2.1 standard deviations. A detailed analysis of collision samples for asteroid 2020 VV during October 2056 was conducted, characterizing the temporal and spatial distribution patterns of these samples, which yielded conclusions consistent with existing collision monitoring systems. Regarding the comparison between different collision monitoring systems, the conclusion is that Monte Carlo collision monitoring systems and line of variation collision monitoring systems each present distinct advantages and disadvantages: the former does not introduce errors associated with linear approximation methods but incurs high computational cost; the latter, while introducing errors through its use of linear approximation methods, can identify low-probability virtual impactors that might be overlooked by the former, and maintains relatively lower computational cost.

Full Text

Preamble

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Research on the Analysis Method of Asteroid Impact Probability

ZHENG Hang[1,2] ZHU Qing-feng[1,2,3]† SONG Ye-zhi ‡ LI Xu-zhi[1,2] XU Xiao-hui[1,2]
QIU Jin-sheng[1,2] ZHAO Jun-han[1,2]

(1 CAS Key Laboratory for Research in Galaxies and Cosmology, Department of Astronomy, University of Science and Technology of China, Hefei 230026)
(2 School of Astronomy and Space Sciences, University of Science and Technology of China, Hefei 230026)
(3 Deep Space Exploration Laboratory, Hefei 230088)
(4 Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030)

Abstract

In view of the current situation of inconsistent conclusions between existing asteroid impact monitoring systems, we studied the linear approximation method used in the Line of Variations (LOV) impact monitoring system and concluded that the deviation of the orbit distribution obtained by this method relative to the theoretical orbit distribution gradually becomes significant as the orbit propagation time increases. We calculated the impact probability of six asteroid instances using the Monte Carlo method; compared with existing Monte Carlo impact monitoring systems, the maximum difference is 2.1 standard deviations. We conducted a detailed analysis of the impact samples for asteroid 2020 VV in October 2056, depicting the temporal and spatial distribution of impact samples, and obtained conclusions consistent with existing impact monitoring systems. Regarding the comparison between different impact monitoring systems, we conclude that the Monte Carlo impact monitoring system and the LOV impact monitoring system each have their own advantages and disadvantages: the former does not introduce errors from the linear approximation method, but has high computational cost; the latter’ s linear approximation method introduces errors, but can identify some virtual impact sources with low impact probability that might be missed by the former, and has relatively lower computational cost.

Key words minor planets, asteroids: near-Earth asteroids impact monitoring, astrometry and celestial mechanics, methods: data analysis

1 Introduction

Near-Earth Object (NEO) impact monitoring (IM) currently employs various analytical methods. In 1999, the CLOMON impact monitoring system at the University of Pisa used a one-dimensional space along the most elongated direction of the confidence region (CR), namely the Line of Variations (LOV), to analyze impact probability. Later, CLOMON2 improved upon the first-generation CLOMON by adopting a new astrometric error model. NASA’ s Jet Propulsion Laboratory (JPL) Sentry system used the same analytical method as CLOMON to calculate impact probability. Sentry-II improved upon Sentry by employing

Monte Carlo (MC) sampling in the six-dimensional orbital parameter space. To obtain high-precision results with acceptable computational resource consumption, Sentry-II used importance sampling to focus on regions of parameter space prone to collisions. Additionally, the European Space Agency (ESA) NEO Coordination Centre (NEOCC) operates AstOD, developed by SpaceDyS, which uses algorithms similar to CLOMON2 but with a different computational engine, suitable for different operating systems.

These three asteroid impact monitoring systems all use observations collected by the Minor Planet Center (MPC) for impact monitoring analysis. The warning information from CLOMON2, Sentry-II, and AstOD is published on NEODyS, the Center for NEO Studies (CNEOS), and NEOCC respectively; hereafter, the impact monitoring systems are referred to by the names of their information publication websites.

Traditional impact monitoring operates on a timescale of approximately 100 years. Overly long propagation times cause orbits to become so dispersed that accurate prediction becomes impossible. Fuentes-Muñoz et al. proposed methods for assessing asteroid collision risk on large timescales of millennia. Traditional impact monitoring systems consider targets with relatively abundant observational data, where high-precision orbits can be obtained through least squares (LS) orbit determination. This orbit determination information can be used to describe the asteroid orbit distribution for impact probability analysis. When target observational data is limited—for example, when the observational arc is insufficient—high-precision orbits and orbit distributions cannot be obtained, requiring modified analytical approaches. When least squares orbit determination can converge, the confidence region exhibits a two-dimensional disk-like characteristic, making it impossible to represent the orbit distribution confidence region with a line of variations; a two-dimensional line of variations approach can be used instead. When least squares orbit determination fails to converge, the orbit distribution can be analyzed using the Admissible Region (AR).

Impact monitoring systems for these special cases include JPL’ s Scout (information published on CNEOS), the University of Pisa’ s NEOScan (information published on NEODyS), and the University of Helsinki’ s NEORANGER.

Traditional impact monitoring systems produce inconsistent impact probability calculations: the three targets with the highest cumulative impact probability in CNEOS do not receive warnings in NEOCC, while for targets with warnings in both systems, the impact probabilities also differ (as of April 15, 2024). This paper addresses this situation by analyzing the errors introduced by the LOV impact monitoring system and using the Monte Carlo method to calculate impact probabilities for several asteroid instances to verify the results of existing impact monitoring systems. Section 2 introduces the asteroid impact probability calculation method and uncertainty analysis used in this paper. Section 3 analyzes the errors introduced by the linear approximation method used in LOV impact monitoring systems. Section 4 uses the Monte Carlo method to calculate impact probabilities for several asteroid instances and conducts a detailed analysis of the impact samples for asteroid 2020 VV in October 2056. Section 5 presents the conclusions of the entire paper.

2 Collision Probability Calculation Methods

High-risk asteroids can approach Earth at very close distances, a situation referred to as a close approach (CA), with a typical distance threshold of 0.05 AU. When a close approach occurs, it is necessary to establish an appropriate reference frame to study the relationship between the asteroid and Earth, generally based on the Target Plane (TP) and its coordinate system.

2.1 Target Plane

The target plane is a plane passing through Earth’ s center, with the target plane coordinate system centered at Earth’ s center. The normal to the target plane is parallel to the asteroid’ s geocentric velocity. More specifically, there are two definitions of the target plane: the first is called the B-plane, which is perpendicular to the direction of the asteroid’ s unperturbed geocentric relative velocity at infinity; the second is called the Modified Target Plane (MTP), which is perpendicular to the asteroid’ s velocity direction at closest approach to Earth. The geometric schematic is shown in [Figure 1: see original paper]. CA is the point on the asteroid’ s trajectory closest to Earth; the B-plane and MTP pass through Earth’ s center, with the B-plane perpendicular to V_∞ and the MTP perpendicular to V_{CA}. The main difference between these two target planes is that on the MTP, the deflection of the asteroid due to Earth’ s gravitational effect can be directly displayed because the intersection of the asteroid’ s trajectory with the target plane is the point of closest approach to Earth; whereas on the B-plane, information about gravitational effects is hidden. For close approach events with high relative velocity or large distance from Earth, the orbital deflection of the asteroid is very small, and the difference between the two target planes is negligible. This paper will use the MTP to analyze asteroid collisions.

2.2 Orbit Distribution and Confidence Region

Asteroid orbits are calculated from observational data. Since observational data contains uncertainties, asteroid orbits also have uncertainties, which can be estimated using a Gaussian distribution. Let the nominal orbit solution be $\mathbf{X}^*$, then this Gaussian distribution has $\mathbf{X}^*$ as its mean and   (the orbit determination covariance matrix) as its covariance matrix:

$$\mathbf{X} \sim \mathcal{N}(\mathbf{X}^*, \ ) \tag{1}$$

where $q(\mathbf{X})$ is the probability density distribution function, $\mathbf{X}$ is the orbital parameter, and $\mathcal{N}$ is the standard Gaussian distribution function. The confidence region in orbital parameter space is defined by the variation of the least squares orbit determination objective function and serves as the orbital parameter research space for asteroid impact analysis. The confidence region is defined as the set of parameters that make the objective function variation less than a certain threshold:

$$\Delta\mathbf{X}^{T\ -1}\Delta\mathbf{X} < m \tag{2}$$

where $\Delta\mathbf{X}$ is the difference between the orbital parameter vector and the nominal solution, and m is a 人为决定的限制置信区域大小的 parameter.

2.3 Monte Carlo Method

The dynamical model of asteroids involves the N-body problem, and orbits cannot be expressed analytically. It is impossible to directly obtain the future states of all orbits within the confidence region through analytical methods; only numerical integration can be used. The Monte Carlo method can be employed to analyze asteroid impact probability.

As described above, when observational data is relatively abundant, the asteroid orbit can be obtained through least squares orbit determination to obtain a nominal solution, thereby determining the orbit distribution. Let the region in orbital parameter space where collisions can occur be $\mathcal{F}$. Region $\mathcal{F}$ is also called the Virtual Impactor (VI), defined as the subset of orbital parameter space where collisions can occur. The impact probability is:

$$P_{\text{Impact}} = \int_{\mathcal{F}} q(\mathbf{X})d\mathbf{X} \tag{3}$$

This is the exact expression for impact probability. Sampling within the confidence region $\mathcal{F}$ yields samples called Virtual Asteroids (VAs). Each sample is then propagated through orbital dynamics to determine whether it can collide with Earth, which is equivalent to judging whether the virtual asteroid sample is in region $\mathcal{F}$. Through:

$$\hat{P}_{\text{Impact}} = \frac{N_{\text{VI}}}{N_{\text{VA}}} \tag{4}$$

we obtain an estimate of the impact probability. In equation (4), N_{VA} is the number of samples, and N_{VI} is the number of VAs in $\mathcal{F}$. Following reference [15], the Monte Carlo sampling used in this paper belongs to repeated sampling of random variables from a binomial distribution, using samples to estimate distribution parameters:

$$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i; \quad \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 \tag{5}$$

where n is the number of observations, X_i are the samples, and $\bar{X}$ is the sample mean. For this paper, collision samples X_i take the value 1, and non-collision samples take 0. With this convention, the impact probability estimate can also be expressed as $\hat{P}_{\text{Impact}} = \bar{X}$. According to the description of the Central Limit Theorem in reference [15], the distribution of the mean $\bar{X}$ of independent and identically distributed variables will approach a Gaussian distribution as the sample size increases:

$$\lim_{n\to\infty} P\left(\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \leq x\right) = \Phi(x) \tag{6}$$

where μ and σ are the expectation and standard deviation of the variable, and Φ is the standard Gaussian cumulative distribution function. Using equation (5) to estimate μ and σ, then substituting into equation (6) to obtain the Gaussian distribution that the sample mean $\bar{X}$ approaches, we can estimate $\sigma_{\bar{X}}$:

$$\sigma_{\bar{X}} = \sqrt{\frac{\hat{\sigma}^2}{n}} = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(X_i-\bar{X})^2} \tag{7}$$

Equations (6) and (8) are the expressions for estimating asteroid impact probability and its uncertainty, respectively.

3 LOV Collision Monitoring System Errors

The NEOCC and NEODyS impact monitoring systems use the LOV method to analyze impact probability, sampling on a two-dimensional LOV space to analyze the close approach distance between LOV samples and Earth, searching for minima of the close approach distance function on the LOV. The orbit distribution at the approach epoch is estimated for each local minimum point through linear approximation:

$$(t) = \ (t,t_0)\ (t_0)\ ^T(t,t_0) \tag{8}$$

where is the state transition matrix, and t and t_0 are the time variable and initial time, respectively. Following the orbit distribution calculation method described previously, the orbit covariance matrix at the approach epoch is used as the parameter for the orbit distribution at that epoch, thereby obtaining the orbit distribution at the approach epoch. Then, through a linear relationship, the mapping between the asteroid' s position on the target plane and the orbital parameters at the approach epoch is obtained, and the distribution of the impact point is derived. Integrating the distribution of impact points within the collision cross-section on the target plane yields the asteroid impact probability.

The contour surfaces of the distribution function can be used to describe the distribution profile, where every point on this geometric object has the same probability density value. [Figure 2: see original paper] shows the contour surfaces of the orbit distribution for asteroid 2020 VV obtained by the linear approximation method and the distribution of Monte Carlo samples in space, displaying the orbit distribution 10, 20, and 50 years after the orbit determination epoch. The plane in the image is the orbital plane of asteroid 2020 VV, with the origin at the heliocenter and the x' direction being the ascending node direction of the orbit on the ecliptic plane. The velocity in the Cartesian-type orbital parameters is taken as the expected value of the distribution. Monte Carlo samples are obtained through numerical integration, and their distribution can be considered the theoretical orbit distribution. For a short time after the orbit determination epoch, such as 10 years, the orbit distribution obtained by the linear approximation method highly coincides with the theoretical orbit distribution. As the orbital dynamics propagation time increases, the difference between the orbit distribution obtained by the linear approximation method and the theoretical orbit distribution gradually becomes apparent; after 50 years, the linear approximation result has clearly deviated from the theoretical orbit distribution.

The LOV impact monitoring system introduces additional errors after obtaining the orbit distribution at the approach epoch through linear approximation, and then obtaining the mapping between the asteroid’ s impact point on the target plane and the orbital parameters at that moment through a linear relationship. This part of the error is determined by the geometric characteristics of the asteroid’s orbit distribution and varies among different orbits. The LOV method also introduces another error: during the initial search for virtual impact sources on the LOV, samples are selected on the LOV, and using the LOV to replace the confidence region is also an approximation. When asteroid observational data is relatively limited and orbit determination precision is low, the confidence region becomes wider, and the error introduced by replacing the confidence region with the LOV becomes higher.

4 Collision Probability Analysis Examples

We selected six asteroids with the highest cumulative impact probability on the NEOCC risk list that also have warnings in CNEOS (as of June 27, 2023) for impact probability calculation. Following the Monte Carlo method described previously, the calculation process is divided into three parts: orbit determination, sampling, sample orbital dynamics propagation, and impact probability estimation. The orbital calculation tool used in this section is the open-source asteroid orbit calculation software OrbFit.

We obtained observational data records for these six asteroids from the Minor Planet Center for orbit determination. We define the orbital parameter difference indicator:

$$\delta_{12} = \sqrt{\sum_{i=1}^{6}(X_{1i} - X_{2i})^2} \tag{9}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are orbital parameter vectors. The orbital parameter vectors require dimensionless processing, with distance dimension in AU and angular dimension in radians. shows the orbit determination results for the six asteroids and their difference indicators compared with the Minor Planet Center and JPL' s Horizon system, where a, e, i, Ω, ω, M are orbital parameters, and MJD is the orbit determination epoch. The differences between orbit determination results are very small, with the largest difference indicator on the order of 10^{-3}. [Figure 3: see original paper] shows the distribution of orbit determination residuals versus MJD for the six asteroids. All asteroid orbit determination residuals are within a few arcseconds. Based on these comparisons with authoritative results and the residual distribution, the orbit determination results are considered reliable.

Through orbit determination information, we obtain the asteroid orbit distribution, which is the multidimensional Gaussian distribution given by equation (1). Sampling from this distribution yields virtual asteroids, with 10^6 samples for each asteroid. The orbits of the virtual asteroid samples are propagated for 100 years (to January 1, 2124) to determine whether they will collide with Earth, recording collision samples. The impact probability and its uncertainty are calculated. Combined with warning information for relevant targets from CNEOS and NEOCC, the results are shown in , where the MC and Uncertainty columns are the impact probabilities and uncertainties calculated in this paper, and the CNEOS and NEOCC columns are the impact probabilities from the two existing impact detection systems. Comparing the three results, for all targets, the results from this paper and CNEOS are relatively close, with the largest difference for target 2006 JY26 being 2.1σ. CNEOS uses a variant of the Monte Carlo method, which is essentially also a Monte Carlo method, so similarity with our results is expected. NEOCC is an LOV impact monitoring system, and compared with the results from the other two Monte Carlo methods, the differences for 2017 WT28 and 2020 VV are significant, while other targets are relatively close.

We selected asteroid 2020 VV for further detailed analysis, increasing the sample size to 10^7 and recalculating. The resulting impact probability is 1.49×10^{-4}, with an uncertainty of 4.0×10^{-5}, differing from the CNEOS result by 0.2σ. To study the characteristics of the impact samples of asteroid 2020 VV, we created a distribution diagram of the close approach dates of impact samples for asteroid 2020 VV in [Figure 4: see original paper], with the horizontal axis showing dates and the histogram binned by month. The figure shows that impactable virtual asteroid samples exist in 37 different months, with the month having the most impact samples being October 2056.

Analyzing the October 2056 impact samples, [Figure 5: see original paper] shows the impact time distribution of asteroid 2020 VV' s impact samples in October 2056, with a time span of within 3 days and a sample number of 1261. The two vertical dashed lines represent the mean MJD of samples in the two clustered regions. The time distribution clearly divides into two regions, with the left region having a mean MJD of 72279.72 and the right region having a mean MJD of 72282.01. These are the two impact event warnings given for October 2056 in this paper. CNEOS also provides two impact event warnings for October 2056, with MJDs of 72279.65 and 72282.00, respectively. The differences in occurrence time between the two impact events are 0.07 days and 0.01 days, respectively, which can be considered consistent with our results.

Theoretically, these two impact events belong to discontinuous virtual impact sources in orbital parameter space, so their impact results are also discontinuous. [Figure 6: see original paper] shows the distribution of virtual asteroids in the target plane coordinate system using two mutually perpendicular planes: the left panel is the target plane, and the right panel is the plane perpendicular to the target plane. The coordinate system is for the epoch MJD 72281.99, which is the average impact epoch of all October 2056 impact samples. This coordinate system is the target plane coordinate system of the nominal orbit, with the origin at Earth' s center. The normal direction $\hat{\zeta}$ of the target plane is the direction of the nominal orbit' s geocentric velocity, the vertical direction $\hat{\eta}$ is the opposite direction of Earth' s heliocentric velocity projected onto the target plane, and $\hat{\xi}$ forms a right-handed system. The left panel plane is the target plane. The figure shows three strip-like concentrated distributions of orbits, each labeled with a number corresponding to three different virtual impact sources. According to the definition of the target plane, the direction of motion of virtual asteroids relative to Earth in the image is roughly perpendicular to the target plane, meaning there is a significant trend of motion parallel to the right panel plane toward the left. As time changes, Earth passes through the three concentrated orbital distributions, resulting in three impact events. However, this paper and CNEOS only found two virtual impact sources. The earlier occurring one is Virtual Impact Source 1, which is relatively sparse, indicating a relatively low probability of occurrence, corresponding to the left concentrated region in [Figure 5: see original paper]; the later occurring one is Virtual Impact Source 2, corresponding to the right concentrated region in [Figure 5: see original paper]. Virtual Impact Source 3 was not found to have impact samples.

summarizes the virtual impact source information for asteroid 2020 VV in October 2056. Unlike the Monte Carlo impact monitoring systems of this paper and CNEOS, the two LOV impact monitoring systems, NEOCC and NEODyS, can find all three virtual impact sources. LOV impact monitoring systems analyze the close approach distance function of LOV samples with Earth, finding virtual impact sources by searching for extrema of the function, which is a process of analyzing continuous functions, so virtual impact sources will not be missed due to insufficient sample numbers. In contrast, Monte Carlo impact monitoring systems may miss low-probability virtual impact sources due to insufficient sample numbers.

5 Conclusions

By analyzing the linear approximation method used in LOV impact monitoring systems, we conclude that the deviation of the orbit distribution obtained by this method relative to the theoretical orbit distribution gradually becomes significant as the orbit propagation time increases. This paper conducted impact analysis on several asteroid instances: we selected six asteroids with the highest cumulative impact probability on the NEOCC risk list that also had warnings in CNEOS (as of June 27, 2023), obtained orbital information through least squares orbit determination, and determined the orbit distribution. Using the Monte Carlo method, we calculated the impact probability and its uncertainty. Compared with CNEOS, the maximum difference in impact probability for the targets was 2.1σ. We selected asteroid 2020 VV, a target with significant differences between the two impact monitoring systems, for further detailed analysis. With the sample size increased tenfold, the calculated impact probability differed from the CNEOS result by 0.2σ. We analyzed the temporal and spatial distribution of impact samples for asteroid 2020 VV in October 2056, providing impact event warnings consistent with CNEOS. Additionally, we found that LOV impact monitoring systems can identify virtual impact sources that should exist but were not found by Monte Carlo impact monitoring systems. Comparing the principles of different asteroid impact probability analysis methods reveals that the Monte Carlo method has high computational resource requirements; theoretically, the resolution of the calculated impact probability is the inverse of the number of samples. To obtain high-resolution results, extensive sampling and orbital propagation are required. The LOV method has much lower computational requirements, needing only relatively small numbers of orbital propagation calculations during the process of finding virtual impact sources. Combining the research content of this paper, we conclude that Monte Carlo impact monitoring systems and LOV impact monitoring systems each have their own advantages and disadvantages: the former does not introduce errors from the linear approximation method but has high computational cost; the latter's linear approximation method introduces errors but can identify some low-probability virtual impact sources that would be missed by the former, with relatively lower computational cost.

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Note: Figure translations are in progress. See original paper for figures.