# A Fast Prediction Method for Thermal Deformation of Large-Aperture Antennas (Postprint)

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**Date:** 2025-06-16T14:58:43+00:00

## Abstract

To address the problem of low computational efficiency in calculating thermally induced deformation of large reflector antennas, a method for rapidly predicting antenna thermal deformation is proposed. The method first establishes a thermal analysis model for a single antenna panel, then analyzes the antenna deformation patterns under different temperature differences—including those between the upper and lower surfaces of the panel, between the backup structure and the panel, and between the backup structure and the environment—through numerical calculations, and finally fits a linear regression relationship to enable rapid calculation of structural thermal deformation; using this regression relationship to predict antenna structural deformation, the predicted results show good agreement with simulated deformation results, while experimental measurement methods verify that the relationship between temperature difference and thermal deformation conforms to a linear relationship; application of this method to the thermal deformation calculation of a 110 m reflector antenna reveals that structural deformation under solar illumination is primarily dominated by backup structure deformation.

## Full Text

## A Fast Prediction Method for Thermal Deformation of Large Aperture Antennas

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### Abstract

To address the low computational efficiency in calculating thermal deformation of large reflector antennas, this paper proposes a method for rapidly predicting antenna thermal deformation. The method first establishes a thermal analysis model for a single antenna panel, then analyzes the antenna deformation law under different temperature differences between the panel' s upper and lower surfaces, between the back frame and panel, and between the back frame and environment through numerical calculations, and finally fits a linear regression relationship to enable rapid calculation of structural thermal deformation. Using this regression relationship to predict antenna structural deformation yields results that agree well with simulated deformation results. Additionally, the linear relationship between temperature difference and thermal deformation is verified through experimental measurements. The method is applied to thermal deformation calculations for a 110 m reflector antenna, revealing that structural deformation under solar irradiation is primarily dominated by back frame deformation.

**Keywords:** techniques: large-diameter antenna, Sun, thermal effect, methods: thermal deformation test, digital photogrammetric

## 1 Introduction

Reflector antennas are widely used in deep space exploration, manned spaceflight, radar communications, and other fields due to their high gain and narrow beam characteristics [**?**, **?**]. With the continuous advancement of deep space exploration and radio astronomy research, reflector antennas are gradually developing toward larger apertures and higher frequencies. Any minute deformation will cause performance degradation, and solar irradiation temperature, as one of the main loads on the antenna, causes structural deformation that significantly impacts the reflector surface accuracy [**?**, **?**, **?**].

Currently, domestic and international scholars have conducted relatively limited research on solar thermal deformation of large aperture antennas, focusing mainly on simulation and measurement analysis of solar deformation fields for medium and small aperture antennas. Yi Letian et al. performed thermo-mechanical coupling analysis on the Xinjiang Astronomical Observatory' s 25 m antenna, establishing a geometric relationship between the antenna mount displacement field and azimuth axis deviation, thereby enabling rapid calculation from measured temperature to mount deformation [**?**]. Li Peng et al. conducted electromechanical coupling analysis on the "Chang' e Project" 40 m antenna, pointing out that the impact of temperature on electrical performance is closely related to the distribution pattern of deformation [**?**]. Sun Jixian et al. studied the thermal deformation law of the Delingha 13.7 m antenna through experimental measurement, finding that solar heat caused irregular deformation of the antenna support leg structure, leading to periodic changes in azimuth axis tilt [**?**]. Fu Li et al. measured the temperature conditions of the Sheshan 25 m antenna at different times and evaluated its main surface deformation using holographic methods, with results showing that temperature significantly affects the accuracy of the antenna' s main reflector surface [**?**]. Li Wang et al. reconstructed the temperature field of the TM (Tianma) 65 m antenna back frame structure using inverse distance interpolation, discovering that temperature differences in different directions cause changes in antenna pointing error [?]. Internationally, Attoli et al. performed thermal simulation analysis on the Sardinia 32 m antenna, indicating that mount thermal deformation significantly affects reflector surface accuracy [?]. Greve et al. studied the IRAM (Institute for Radio Astronomy) 30 m antenna using a combination of dynamic and static methods, finding that deformation caused by temperature gradients significantly impacts antenna pointing accuracy [?]. Most of these scholars primarily calculate deformation by applying antenna temperature obtained through finite element simulation or temperature sensor measurement as loads on the structure. However, due to the large and complex structure of large aperture antennas, thermal deformation calculation consumes substantial computational time and suffers from low analysis efficiency. To save thermal deformation calculation time and improve computational efficiency, this paper proposes a fast method for predicting antenna thermal deformation. Based on the finite element numerical model of a single panel of the proposed QTT (QiTai Radio Telescope), this method extracts a regression relationship between structural deformation and temperature difference through numerical methods, then verifies the extracted mathematical relationship using finite element simulation and experimental measurement to confirm the accuracy of the regression relationship. Finally, the method is applied to QTT thermal deformation calculations, with the process flow shown in Figure 1 [Figure 1: see original paper].

## 2 Antenna Thermal Environment Model

Antenna structures primarily exchange heat with the surrounding environment through thermal radiation, thermal conduction, and thermal convection. These heat transfer methods are closely related to factors such as cloud thickness and wind speed. Under the combined effects of these factors, the thermal system composed of the antenna and surrounding environment exists in a transient thermal equilibrium state [?], with specific heat transfer pathways shown in Figure 2 [Figure 2: see original paper].

### 2.1 ASHRAE Clear Sky Model

Solar radiation, as the main source of thermal energy absorbed by the antenna, is affected by numerous factors including solar position, atmospheric mass, and antenna attitude. Solar radiation intensity is defined by various mathematical models, among which the representative ASHRAE clear sky model [?] describes three forms of solar radiation received by the antenna: direct radiation, reflected radiation, and scattered radiation.

As solar radiation passes through the atmosphere to reach the target object, some energy is dispersed and absorbed by gases, water vapor, clouds, and dust. The energy that reaches the object surface directly without being dispersed or absorbed is direct radiation. The energy reaching the target object after being scattered by the sky and ground is scattered radiation. The radiation reaching the object indirectly after reflection from surrounding objects is reflected radiation. These are defined as follows:

Direct radiation:

$$G_{ND} = A\exp(B/\sin(\phi_s))C_N\cos(\theta)$$

where $A$ is solar radiation intensity outside Earth's atmosphere (with zero atmospheric particle content); $C_N$ is atmospheric cleanliness coefficient; $\phi_s$ is solar altitude angle; $B$ is atmospheric extinction coefficient; $\theta$ is the angle between the solar incident beam and the normal of the inclined component surface.

Scattered radiation:

$$G_d(\phi) = 0.5CG_{ND}(1+\cos(\alpha_s))$$

where $C$ is scattered radiation coefficient, $\alpha_s$ is the angle between the object surface and horizontal ground.

Reflected radiation:

$$G_R = 0.5(\sin(\phi_s)+C)G_{ND}\rho_g(1+\cos(\alpha_s))$$

where $\rho_g$ is ground reflectivity, related to ground materials around the component.

The total solar radiation received by the antenna is:

$$G_t = \rho(G_{ND}+G_d(\phi)+G_R) = \rho[\max(\cos(\theta);0)+0.5C(1+\cos(\alpha_s))+0.5\rho_g(1+\cos(\alpha_s))]G_{ND}$$

where $\rho$ is the solar radiation absorption rate of the antenna surface. Since antenna surfaces are typically painted white, $\rho$ generally takes a value of 0.3 [?].

### 2.2 Convective Heat Transfer Model

Antennas in natural convective thermal environments exchange heat with the environment through convective heat transfer, which can be expressed using Newton' s cooling formula:

$$Q_c = h_c(T_w - T_f)$$

where $T_w$ is wall surface temperature; $T_f$ is surrounding fluid temperature; $h_c = 4.0v + 5.6$ is used for value determination [?], where $v$ is wind speed.

### 2.3 Net Longwave Radiation Model

Net longwave radiation refers to the radiative heat exchange between the radio telescope and the ground and distant sky. According to the Stefan-Boltzmann law, the net longwave radiation on the radio telescope surface is expressed as [?]:

$$Q_r = 0.5\epsilon\sigma[(1-\cos(\alpha_s))(T_g^4 - T_f^4) + (1+\cos(\alpha_s))(T_{sky}^4 - T_f^4)]$$

where $\sigma$ is the Stefan-Boltzmann constant with value $5.67 \times 10^{-8}\mathrm{W}/(\mathrm{m}^2 \cdot \mathrm{K}^4)$; $\epsilon$ is antenna emissivity; $T_{sky}$ is distant sky temperature; $T_g$ is ground temperature.

### 2.4 Transient Thermal Balance Equation

Structural thermal analysis of radio telescopes follows the law of energy conservation, with the energy balance equation for transient thermal analysis expressed as [?]:

$$C\dot{T} + KT = Q$$

where $T$ is the temperature vector to be solved; $\dot{T}$ is the time derivative of the temperature vector; $C$ is the specific heat matrix, related to structural mass distribution, material parameters, and specific heat capacity; $K$ is the thermal conductivity matrix, including thermal conduction parameters, convective heat transfer parameters, and thermal radiation parameters; $Q$ is the thermal input matrix, primarily related to environmental parameters such as solar radiation and thermal boundaries.

## 3 Investigation of the Relationship Between Antenna Structural Thermal Deformation and Temperature Difference

Large aperture reflector antennas are composed of thousands of precision panels. Since each panel contains numerous elements, calculating the thermal deformation of the entire reflector structure using traditional methods consumes substantial computational time and suffers from low analysis efficiency. To better investigate the relationship between structural deformation and solar irradiation while improving computational efficiency, this section analyzes the relationship between panel deformation and temperature difference under thermal loads using a single panel of the proposed QTT as an example.

### 3.1 QTT Single Panel Model

When a single panel is affected by solar irradiation, panel buckling occurs, primarily manifested as deformation in the vertical direction (Z-direction), while deformation in the horizontal directions (X and Y) is minimal. The time scale is typically one day, specifically showing outward expansion of the panel during daytime and inward concavity at night, as illustrated in Figure 3 [Figure 3: see original paper]. Point A represents a reference position on the panel in its initial state, while point A' represents the new position of this reference point after moving under temperature gradient effects. The causes of panel buckling mainly include material differences in components, temperature difference between the upper and lower surfaces of the panel, and temperature difference between the back frame and panel [?].

Based on the above theory, a single panel of QTT is now analyzed. Its finite element model is shown in Figure 4 [Figure 4: see original paper], mainly including the reflector panel and back frame components. This model contains 7,318 elements and 3,483 nodes. The panel material is aluminum, and the back frame material is steel, with thermal and physical parameters listed in Table 1 . The connection between the back frame and the four bolts is simulated through direct fixation, while full constraints are applied at location B, and rotational and Z-direction displacement constraints are applied at locations C, D, and E.

#### 3.2.1 Single-Factor Temperature Difference Analysis

The solar deformation field distribution of reflector antennas is influenced by numerous factors, among which the temperature difference between the back frame and panel, the temperature difference between the upper and lower surfaces of the panel, and the temperature difference between the back frame and environment are particularly critical. The single-variable method is now employed to analyze the influence of each temperature difference on the antenna panel deformation field.

**(a) Panel and Back Frame Uniform Temperature Change-Induced Panel Buckling (Temperature Difference Between Back Frame and Environment)**

Figure 5 [Figure 5: see original paper] shows the panel deformation distribution cloud map after applying identical uniform temperature loads to both the panel and back frame. Analysis of the deformation cloud map reveals that when uniform temperature loads are applied, the panel bulges outward, with the structural deformation field showing a distribution trend of high in the middle and low around the edges, due to the fixed constraints applied at the four contact positions between the back frame and bolts. Simultaneously, the panel buckling amount continuously increases with rising temperature loads.

From the above analysis results, the deformation at the panel center is much greater than at other locations under uniform temperature loading. Therefore, this section primarily investigates the relationship between temperature difference and deformation at the panel center node. Based on the deformation data shown in the figure, the relationship between uniform temperature load and panel buckling amount is fitted as equation (8), showing that as temperature load increases, panel buckling amount also increases, with both satisfying a linear relationship:

$$\Delta Z = 0.03875\Delta T_1 + 0.00065$$

where $\Delta T_1$ is the temperature difference between the back frame and environment (°C), and $\Delta Z$ is the Z-direction deformation amount (mm).

**(b) Panel Buckling Caused by Temperature Difference Between Back Frame and Panel**

For antennas in long-term outdoor environments, the panel and back frame exhibit different temperatures when affected by solar irradiation due to their different material properties. To investigate the influence law of temperature difference between them on panel structural deformation, different uniform temperature loads are applied to the back frame and panel respectively. The structural deformation conditions are shown in Table 2 .

Analysis of the deformation data in the table shows that the greater the temperature difference between the panel and back frame, the larger the panel buckling amount. The fitted relationship is shown in equation (9), and analysis of this relationship reveals a linear relationship between the temperature difference between panel and back frame and panel buckling amount:

$$\Delta Z = 0.03879\Delta T_1 + 0.03778\Delta T_2$$

where $\Delta T_2$ is the temperature difference between the panel and back frame (°C).

**(c) Panel Buckling Caused by Temperature Difference Between Upper and Lower Panel Surfaces**

Since the panel has a certain thickness, its upper and lower surfaces exhibit different temperature distributions when subjected to solar radiation, which may cause panel bending. Different uniform temperature loads are applied to the upper and lower surfaces of the panel. To eliminate the influence of temperature difference between the back frame and panel on the results, the temperature of the back frame and the lower surface of the panel are set to be consistent. The panel structural deformation conditions are shown in Table 3 , with the upper and lower surface temperatures denoted as STP (Surface Temperature of the Panel) and USTP (Underside Surface Temperature of the Panel), respectively.

Analysis of the data in the table shows that the greater the temperature difference between the upper and lower surfaces of the panel, the larger the panel buckling amount. The fitted relationship between them is:

$$\Delta Z = 0.01554\Delta T_3 + 0.03877\Delta T_4$$

where $\Delta T_3$ is the temperature difference between the upper and lower surfaces of the panel (°C), and $\Delta T_4$ is the temperature difference between the lower surface of the panel and the environment (°C).

Based on the analysis results of the above three cases, panel buckling amount satisfies a linear relationship with each temperature difference, meaning that greater temperature difference leads to larger panel deformation. Therefore, panel deformation results can be estimated through changes in temperature difference to achieve rapid prediction of panel deformation.

### 3.2.2 Multi-Factor Temperature Difference Analysis

To reduce result errors from single-factor analysis and improve analysis accuracy and rationality, the orthogonal experimental design method [?, ?] is now employed to investigate the relationship between multiple factors and the target. Based on the previous analysis results, the temperature difference between the upper and lower surfaces of the panel, the temperature difference between the panel and back frame, and the temperature difference between the back frame and environment are selected as experimental factors, with the Z-direction deformation amount at the panel center as the analysis index. The factors and levels are shown in Table 4 , with specific design schemes and calculation results shown in Table 5 .

Based on the orthogonal experimental calculation results, the regression relationship between the above three factors and the analysis target is fitted as follows:

$$\Delta Z = 0.0388\Delta T_1 + 0.0379\Delta T_2 + 0.0157\Delta T_3$$

Analysis of this equation shows that the antenna solar thermal deformation satisfies a linear regression relationship with each temperature difference. Therefore, this relationship can be used to achieve rapid calculation of antenna structural deformation. However, since large aperture reflector antennas use different materials and structures, the relationship between temperature difference and deformation needs to be re-derived for application to the entire structure.

## 3.3 Physical Model Simulation

To verify the accuracy of the relationship between antenna structural deformation and temperature difference derived from the mathematical model, the working condition of a clear, cloudless day on June 21 (summer solstice) with a wind speed of 3 m/s is selected for full-day temperature field and deformation field analysis of a single QTT panel. Figure 6 [Figure 6: see original paper] shows the structural temperature field distribution cloud maps at 8:00, 10:00, 12:00, and 16:00 local time. Analysis of the temperature cloud maps reveals that the solar temperature field of the antenna structure shows a radial gradient distribution around the solar direct radiation point, characterized by non-uniformity and time-varying properties.

Under the condition of considering only temperature loads, the structural deformation under the above temperature fields at different times is solved using the constraint method shown in Figure 4. Figure 7 [Figure 7: see original paper] shows the structural thermal deformation distribution cloud maps. Analysis of these cloud maps reveals that thermal deformation caused by solar irradiation is mainly concentrated at the panel center, with relatively small deformation at other locations. Simultaneously, as solar radiation intensity increases, panel buckling amount also increases.

Several locations shown in Figure 4 are randomly selected to verify the panel deformation results from both the mathematical model and physical model at the above four moments. The deformation comparison results are shown in Figure 8 [Figure 8: see original paper]. Analysis of these results shows that the panel deformation results obtained through mathematical model calculation (MMC) and physical model simulation (PMS) exhibit good overall consistency, with minimal difference between the two calculation results and a maximum error of only 7.4%, demonstrating that calculating structural deformation through temperature difference relationships is feasible.

### 3.4 Experimental Model Validation

This section employs a single-camera digital photogrammetric system to conduct deformation field numerical simulation experiments on a simplified aluminum plate (composed of an 800 mm × 800 mm × 2 mm aluminum plate, bolts, and ring beam) to verify the relationship between temperature difference and structural deformation derived from the mathematical model.

The experimental instruments mainly include a wireless temperature collector, temperature data receiver, Nikon SB-400 (Speedlight Bounce) camera, circular retroreflective targets (target points), coded targets, and carbon fiber reference scales. The temperature collector probes need to be coated with silicone grease and attached to the back of the test object using aluminum foil tape, primarily achieving wireless collection and data transmission of temperature data through RF communication with the receiver. Retroreflective targets are attached to the test object surface, mainly reflecting light to achieve disappearance of the measured target. Coded targets and carbon fiber reference scales are used to achieve automatic orientation of photography and automatic matching of measurement points. The measurement accuracy obtained through formula calculation is 0.01 mm [**?**]. The target point position distribution and back structure are shown in Figure 9 [Figure 9: see original paper], with the experimental site shown in Figure 10 [Figure 10: see original paper].

The Z-direction deformation of the obtained target points is interpolated to obtain the entire aluminum plate deformation field distribution, as shown in Figure 11 [Figure 11: see original paper] at 14:28, 14:33, 14:39, and 14:55 local time. Analysis of the deformation field cloud maps reveals that under solar radiation, the panel deformation field shows a distribution trend similar to simulation results (high in the middle and low around the edges), with panel buckling amount decreasing as solar radiation weakens. The measured temperature and deformation results are shown in Table 6 .

Based on the above experimental results, the relationship between temperature difference and deformation is fitted as:

$$\Delta Z = -0.03479\Delta T_1 + 0.02434\Delta T_2 + 0.1535$$

Analysis of this equation shows that under actual measurement conditions, panel structural deformation satisfies a linear relationship with each temperature difference. Comparing the magnitude of fitted coefficients reveals that the temperature difference between the back frame and environment is the key factor affecting panel deformation. Combined with the analysis results of equation (11), both mathematical calculation and experimental simulation yield regression equations that satisfy linear relationships, demonstrating that calculating antenna thermal deformation through linear regression equations is feasible. However, due to the simplified aluminum plate structure used in experimental simulation, differences exist between the regression relationships obtained from the two methods.

## 4 Case Application

A 110 m aperture reflector antenna is now used as the application object. Thermal analysis software is employed to obtain its structural temperature field, and the linear regression relationship derived earlier between antenna structural deformation and temperature difference is used to achieve rapid calculation of the main reflector deformation field. The specific calculation process is shown in Figure 12 [Figure 12: see original paper].

The antenna model is established with an elevation angle of 90° and an azimuth angle of 0°, and full-day temperature field time-history analysis is performed on this model. Figure 13 [Figure 13: see original paper] shows a version of the QTT finite element model, mainly composed of the main reflector panels (PMR) and back-up structure (BUS), using SHELL63 elements and BEAM188 elements respectively.

Figures 14 [Figure 14: see original paper]–16 [Figure 16: see original paper] show the temperature field distribution cloud maps of the main reflector and upper chord nodes of the back frame at different times under a wind speed of 3 m/s. The temperature data in these figures can be used to obtain the temperature difference between each panel component and the environment, which is then substituted into the linear regression equation (11) to calculate the deformation data for each panel. Finally, the entire main reflector deformation field distribution is obtained through the Barnes interpolation algorithm.

Figures 17 [Figure 17: see original paper]–19 [Figure 19: see original paper] show the reflector thermal deformation cloud maps at different times. Analysis of the deformation results reveals that structural deformation caused by the reflector' s own temperature field under solar irradiation is mainly local deformation, primarily manifested as bulging and 凹陷 of some panels. The overall deformation shows a "yuanbao-shaped" distribution gradually outward along the radial direction, with torsional deformation mainly occurring near the edges.

Figure 20 [Figure 20: see original paper] shows the surface accuracy distribution curves of the reflector at an elevation angle of 90° under different wind speeds (1.5 m/s, 3 m/s, 4.5 m/s). Analysis of these curves shows that at the same elevation angle, RMS (Root Mean Square) curves under different wind speeds show similar variation trends, all presenting a "saddle-shaped" distribution. Near sunrise/sunset (5:00/19:00), when sunlight illuminates the antenna from the side, RMS values begin to change dramatically. As wind speed increases, RMS values decrease because increased wind speed reduces the overall temperature difference of the structure, thereby decreasing non-uniform deformation of the back frame and improving reflector surface accuracy.

## 5 Conclusion

To address the low computational efficiency of existing large reflector antenna deformation field calculations, this paper proposes a simple, efficient, and rapid method for predicting thermal deformation of large aperture reflector antennas by combining finite element simulation and experimental measurement. The main conclusions are:

1. Antenna structural deformation can be described using three temperature differences: between the back frame and panel, between the upper and lower surfaces of the panel, and between the back frame and environment, enabling rapid prediction of the antenna panel deformation field. The mathematical modeling method' s extracted regression relationship is verified through physical simulation and digital photogrammetric experiments.

2. Applying this method to thermal deformation calculations for a 110 m reflector antenna reveals that structural deformation under solar irradiation is primarily dominated by back frame deformation, while structural deformation caused by the reflector' s own temperature field is mainly local deformation.

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*Note: Figure translations are in progress. See original paper for figures.*