An Iteration Method for Astronomical Image Deconvolution and Reconstruction Postprint

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Abstract

Imaging serves as an important method for astronomical research. In practice, original images acquired by a telescope are often convolved and blurred by the point-spread function (PSF), which is highly detrimental to many scientific studies, including astronomy. This paper introduces a single-equation iterative method for solving complex linear equations, and this method can effectively deconvolve dirty images and eliminate the effects of the PSF. With different PSFs, this algorithm demonstrates excellent deconvolution performance. Furthermore, with a large PSF of aperture synthesis imaging, this algorithm improves the peak signal-to-noise ratio and structural similarity of the dirty images by 41.0% and 33.9% on average. In addition, this paper proves that the algorithm can deconvolve the dirty image by fully utilizing the information from each pixel in the image, even if the dirty image contains salt-and-pepper noise or even missing regions; through its excellent capability for flexible operation on individual pixels, all these adverse conditions can be addressed and the image successfully restored.

Full Text

Abstract

Imaging is a crucial method for astronomical research. In practice, images acquired by telescopes are inevitably convolved and blurred by the point-spread function (PSF), which severely hampers many scientific studies. This paper introduces a single-equation iterative method for solving complex linear equations that can effectively deconvolve dirty images and eliminate PSF effects. The algorithm demonstrates excellent deconvolution performance across different PSFs. For large PSFs characteristic of aperture synthesis imaging, the method improves the peak signal-to-noise ratio (PSNR) by 41.0% and structural similarity (SSIM) by 33.9% on average. Furthermore, we prove that the algorithm can deconvolve images even when they contain salt-and-pepper noise or missing

regions, by fully exploiting information from each pixel. Through its flexible pixel-level operations, the method successfully handles these challenging scenarios and restores the images.

1. Introduction

Image deconvolution has long been a vital research direction in astronomical image processing because telescope images are inevitably blurred by numerous factors, including limited resolution, bandwidth constraints, and the point-spread function (PSF). The primary culprit is the PSF, which convolves and blurs the image, obscuring details of astronomical objects. The PSF—literally causing point light sources to spread out—represents the impulse response of an imaging system. Its effect transforms the true light distribution into a degraded linear space. A larger main lobe or sidelobe reduces orthogonality between basis functions, causing the potential true image solution to spread across many locations and become difficult to recover.

Radio telescope arrays employing Very Long Baseline Interferometry (VLBI) face additional challenges. Since antenna sampling on the virtual objective lens cannot completely cover the entire observation surface, the virtual telescope's PSF develops non-negligible sidelobes that severely blur images (Faulkner & de Vaate 2015). Effective deconvolution algorithms are therefore essential for recovering true images with sufficient clarity to reveal details.

From a traditional signal processing perspective, a linear time-invariant system multiplies the original signal's spectrum by a system function in the frequency domain, which corresponds to convolution in the time domain. Consequently, deconvolution involves eliminating this system function (Rafael & Gonzalez 2017), requiring extension to two-dimensional discrete Fourier transforms. In practice, direct application is difficult, and preprocessing of dirty images is typically necessary (Dong et al. 2017; Rafael & Gonzalez 2017).

Alternative approaches based on image feature analysis have been explored extensively. These include dictionary-based sparse representation methods that match features in dirty images for restoration. Studies by Yang et al. (2010) and Hu et al. (2020) investigated image reconstruction methods based on such theories, while Lu et al. (2014), Dai et al. (2012), and Zhang et al. (2017) discussed sparse representation dictionary construction and optimization. Although these data-driven methods show promise, they share common limitations: unlike traditional methods, they cannot quantitatively and accurately analyze the convolution process, making them ill-suited for handling diverse convolution scenarios and producing reliable deconvolution results. Consequently, their application in astronomical imaging has been less than ideal.

In astronomical image processing, Högbom (1974) proposed the CLEAN algorithm in 1974. This method identifies peaks in dirty images one by one, subtracts a scaled convolution kernel, and finally restores the image using a Gaussian beam. Subsequent improvements led to the adaptive scale pixel decomposition algorithm (Asp-CLEAN) by Bhatnagar & Cornwell (2004), which minimizes an objective function to find optimal model components. Zhang et al. (2016) accelerated this approach with the Asp-Clean2016 algorithm. Later work (Zhang et al. 2018, 2019) combined Högbom-CLEAN and Asp-Clean2016 into the fused-clean algorithm, which ensures effective separation of emission from noise while accelerating deconvolution. Other scale-adaptive variants followed (Zhang et al. 2020).

Despite these achievements, CLEAN algorithms fundamentally do not solve the deconvolution problem—they merely eliminate complex sidelobes formally, making images appear cleaner without restoring substantial new information. The Richardson-Lucy algorithm (Richardson 1972; Lucy 1974) offers an alternative space-domain approach, iterating via Newton's method. While effective for simple problems, it lacks convergence guarantees and is sensitive to image defects (Fish et al. 1995; Laasmaa et al. 2011).

The method presented here is also a traditional space-domain deconvolution technique, conceptually similar to Richardson-Lucy but employing linear equation solvers rather than Newton's method. Historically, the Kaczmarz algorithm (Kaczmarz 1937) provides an efficient solution for massive linear systems, though it remains underrepresented in textbooks. Modern improvements by Strohmer & Vershynin (2009) and Needell (2010) introduced randomization and optimized sampling, accelerating convergence and improving solution quality—highly consistent with our programmatic improvements to iteration sequences. This method was introduced to computed tomography (CT) as the algebraic reconstruction technique (ART) by Gordon et al. (1970).

Applying this approach to astronomical image deconvolution offers several advantages. First, it operates at the pixel level rather than processing entire images or blocks simultaneously, providing exceptional flexibility for handling complex problems. Second, the algorithm guarantees convergence for deconvolution, enabling more iterations that yield results increasingly close to the original image.

2.1. The Mathematical Description of Imaging

In the spatial domain, a convolved dirty image $I_b(x, y)$ results from each light source $I_o(x_0, y_0)$ in the original image $I_o(x, y)$ passing through a linear system with response $h(x, y)$, creating a weighted diffused distribution:

$$I_b(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I_o(x_0, y_0) h(x - x_0, y - y_0) dx_{0dy}0$$

In the discrete case:

$$I_b(x,y) = \sum_{x_0} \sum_{y_0} I_o(x_0, y_0) h(x - x_0, y - y_0)$$

Conversely, given a dirty image, each point value $I_b(x_0, y_0)$ is determined by the brightness distribution centered at (x_0, y_0) in the original image according to a linear response $h^T(x, y)$:

$$I_b(x_0, y_0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I_o(x, y) h^T(x - x_0, y - y_0) dxdy$$

In discrete form:

$$I_b(x_0, y_0) = \sum_x \sum_y I_o(x, y) h^T(x - x_0, y - y_0)$$

where $h^T(x, y)$ is the centrally symmetric version of $h(x, y)$. This constitutes a linear equation that, given $I_b(x_0, y_0)$, can be represented as a hyperplane in n-dimensional or even infinite-dimensional space. Different brightness values in the dirty image constrain the light source distribution around (x_0, y_0) in the original image $I_o(x, y)$ to a hyperplane. Only light distributions within this hyperplane yield the observed dirty image after convolution.

Multiple values in the dirty image produce a series of hyperplanes constraining the image distribution $I_o(x, y)$ in n-dimensional space. Image restoration requires finding a distribution that satisfies all constraints as closely as possible. Even when exact solutions are non-unique or unattainable, the algorithm seeks the closest possible solution.

This can be framed as solving a matrix equation, though constructing it is extremely difficult. One must transform $h(x, y)$ into a large matrix and reshape the image into a vector. For large $h(x, y)$, this vector becomes prohibitively long, and the matrix equation typically has infinitely many solutions or may be unsolvable.

For radio interferometric imaging, deconvolution can bypass explicit convolution by directly setting up equations from phase and amplitude data. After appropriate selection and adjustment, solving these equations simultaneously yields the image.

2.2. Linear Equation Solving Strategy

As described, dirty image pixel values $I_b(x, y)$ constrain the light source distribution around points (x_i, y_i) in the original image $I_o(x, y)$ to multiple hyperplanes. If an initial distribution $I_c(x, y)$ lies outside these hyperplanes, a reasonable approach is to move directly along the hyperplane normal to ensure the new result resides within the hyperplane. This method, first proposed by Kaczmarz (1937) and applied to CT reconstruction by Gordon et al. (1970), can be adapted for astronomical image deconvolution.

In the discrete case, the normal direction of the hyperplane and the distance from the initial distribution $I_c(x, y)$ to the hyperplane are calculated by:

$$d = \frac{I_b(x_i, y_i) - \sum_x \sum_y I_c(x, y) h^T(x - x_i, y - y_i)}{\sum_x \sum_y h^T(x - x_i, y - y_i)^2}$$

Note the absence of absolute value in the numerator because distance must include direction relative to the normal vector. The original distribution $I_c(x, y)$ moves distance d along the normal to obtain a new distribution:

$$I_c'(x, y) = I_c(x, y) + d \cdot h^T(x - x_i, y - y_i)$$

Convolution kernels $h(x, y)$ typically have limited scope, with values outside a small central area being negligible and can be set to zero. This simplifies calculations, requiring updates to $I_o(x, y)$ only in a small region around each point. Multiple locations can be updated simultaneously, constituting a complete iteration when all dirty image pixels are processed.

[Figure 1: see original paper] illustrates the algorithm's process. $I_c(a_i, b_i)$ represents the distribution $I_c(x, y)$. In a simplified two-pixel, two-dimensional case, two constraint hyperplanes are generated. When the angle between hyperplanes is large (Figure 1(v)), the algorithm quickly converges to their common intersection, efficiently solving the two linear equations. When the angle is small (Figure 1(w)), many iterations may still leave the result far from the intersection. Despite this, convergence is guaranteed—even for unsolvable cases, the result converges to a particular solution or oscillates between hyperplanes without diverging.

In high-dimensional spaces, small angles between constraint hyperplanes cause slow convergence and accumulated errors that are difficult to eliminate. This inherent limitation can be mitigated through flexible iteration scanning design at the pixel level.

Strohmer & Vershynin (2009) randomized the Kaczmarz method, proving that randomization accelerates convergence and improves effectiveness. For image deconvolution, numerous traversal strategies exist for two-dimensional matrices. This paper optimized iteration scanning at the program level, significantly enhancing performance.

A simple left-to-right, top-to-bottom scan causes error accumulation due to high parallelism between adjacent convolution kernels. Alternative strategies—right-to-left, bottom-to-top, or combinations thereof—improve results without increasing computational cost per iteration.

Figure 2: see original paper shows the combined four-direction scanning approach adopted in our initial experiments. More complex traversal methods exist. Figure 2: see original paper demonstrates leapfrogging traversal, which

avoids high-parallelism regions and accelerates convergence. We introduced leapfrogging with random direction jumps, changing scan order after each full traversal.

Our experiments conclude that increasing traversal randomness substantially improves deconvolution quality—greater randomness yields better results.

2.3. Image Defects

Traditional deconvolution methods struggle with salt-and-pepper noise or damaged regions. Common approaches involve smoothing, filtering, or segmentation, but segmentation loses information near boundaries and fails to utilize all available data.

The single-equation iterative method handles these cases simply: when the scanning program detects invalid pixels, it skips them. For damaged regions, pixels can be marked invalid directly, requiring no major program modifications.

[Figure 3: see original paper] shows blurred images containing salt-and-pepper noise or missing parts.

The parallelism between hyperplanes depends on the angle between their normal vectors. For two hyperplanes in n-dimensional space with normal vectors $[x_1, x_2, \dots, x_n]$ and $[y_1, y_2, \dots, y_n]$, the angle θ between hyperplanes equals the angle between their normals. The cosecant of this angle reflects the relationship between equations and the interaction between noise and convolution.

First, calculate distance d from one normal vector to another hyperplane, then obtain $\sin\theta$:

$$\sin\theta = \frac{d}{\|[x_1, x_2, \dots, x_n]\|}$$

In practice, a preprocessing program scans all pixels, marking invalid pixels with a specific value (e.g., 1000) when conditions are met:

```
if condition:
    I(x, y) = 1000
```

The iteration solver skips pixels with this special value:

```
if I(x, y) == 1000:
    pass
```

If a bad pixel value is processed, the error won't affect the result, but this alone cannot reconstruct the image. Convolution's key property is that brightness information at a position spreads to surrounding pixels. This method changes the image distribution around a pixel when calculating each linear equation, providing an opportunity to recover lost information. While this may seem questionable from a linear algebra perspective (as these equations may be unsolvable), practice shows it generally suffices for image recovery.

2.4. The Interaction Between Noise and Convolution Kernel

Noise sensitivity to convolution kernels can be interpreted through hyperplane intersection shifts when pixel values in the dirty image are perturbed. The shift magnitude depends on the interrelationship between pixel equations.

Comparing Figure 4: see original paper and (v), when hyperplane angles are small, small pixel value changes cause large intersection shifts, indicating low stabilization ability. In n-dimensional space, with normal vectors $[x_1, x_2, \dots, x_n]$ and $[y_1, y_2, \dots, y_n]$, the cosecant $\csc\theta = 1/\sin\theta$ defines a parameter called shift parallelism. Calculating this for a convolution kernel $h(x, y)$ and its shifted version reveals the kernel's noise resistance:

$$\text{Shift Parallelism} = \csc\theta = \frac{\|h(x,y)\| \cdot \|h(x+\Delta x, y+\Delta y)\|}{\langle h(x,y), h(x+\Delta x, y+\Delta y)\rangle}$$

When shift parallelism equals 1, hyperplanes are perpendicular; a pixel offset ϵ shifts the intersection by ϵ. When shift parallelism exceeds 1, the same offset shifts the intersection by $\epsilon \cdot \csc\theta$. Since $\csc\theta$ is always greater than 1, convolution kernels always amplify noise. Large shift parallelism values cause small pixel shifts to produce large intersection displacements, rapidly destroying deconvolution results.

Shift parallelism depends on kernel size, shape, shift direction, and distance. It explains why image resolution cannot be increased indefinitely and guides adjustments to the single-pixel deconvolution method. Convergence speed also depends on shift parallelism; low shift parallelism in large sidelobes can create persistent shadow patterns because shadow solutions become very close to the true solution.

An important conclusion emerges: when convolution is unavoidable but controllable, kernels with low shift parallelism in specific directions and distances should be selected to preserve information. Resolution can be sacrificed to combat noise—by ensuring sufficient scanning intervals and low shift parallelism between scanned pixels, images can be restored despite reduced resolution. This reveals fundamental relationships between noise, convolution, and resolution in the spatial domain.

For radio interferometric imaging, the virtual image contains no pixel value shifts but rather phase and amplitude measurement errors, which impact results differently and less severely than thermal noise. The analysis remains fundamentally similar, viewing noise as affecting the PSF.

2.5. The Optical Basics of This Method Compared with CLEAN Algorithm

The classical Högbom CLEAN algorithm assumes the true sky distribution comprises a finite number of point sources with mostly empty surrounding regions. These sources are gradually identified and removed from dirty images.

Our algorithm differs fundamentally in its assumptions. While also assuming a sky composed of point sources, it allows sources everywhere. For an imaging system, smaller point sources in adjacent tiny regions can be approximated as point (far-field) diffraction coherent sources due to Fraunhofer diffraction (Eugene 2021). Thus, the imaging region can be pre-divided into finite small grid regions.

Assuming one coherent source per region, the number of small regions defines the pre-assumed image resolution. The intensity in each region becomes a linear combination of coherent source intensities in surrounding divided regions (optical details omitted but results equivalent). Under this assumption, deconvolution becomes solving linear equations to restore true coherent source intensities. Unlike CLEAN, it simultaneously solves all sources, avoiding interference problems when searching for nearby point sources.

Both methods are traditional, non-data-based algorithms, but differ in basic assumptions and operation. Our algorithm offers better flexibility and adaptability for complex surface light sources.

3.1. Experiment Settings

Four astronomical images (A, B, C, D) shown in Figure 5: see original paper were used to test the algorithm. All images have a unified 512$×$512 resolution and are monochrome. Images A (nebula) and B (Sun) contain complex distributed spontaneous light sources to test deconvolution of complex point sources. Images C and D (lunar surface) contain large-area reflection sources to test performance on low-contrast extended emission.

Two convolution kernels shown in Figure 5: see original paper were used: an all-ones matrix and a mixed Gaussian function combining three lobes. These enable simple general effectiveness tests.

Iteration numbers were determined through pre-testing: 200–300 iterations generally achieved desired sharpness for simple cases. Convergence speed depends on scanning mode and PSF size/shape. The iteration count can be set dynamically by generating a convolved reconstruction, calculating its difference from the dirty image, and stopping at a threshold.

Two metrics evaluate deconvolution: PSNR and SSIM. PSNR represents the ratio of maximum possible signal power to destructive noise power (Welstead 1999). For monochrome images I and K, mean square error is:

$$\text{MSE} = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2$$

PSNR is defined as:

$$\text{PSNR} = 20\log_{10}\left(\frac{\text{MAX}_I}{\sqrt{\text{MSE}}}\right)$$

where MAX_I is the maximum pixel value. SSIM measures image similarity (Wang et al. 2004):

$$\text{SSIM}(x,y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)}$$

where μ denotes mean, σ denotes variance and covariance, and c are constants.

3.2. Deconvolution Verification of Different Convolution Kernels

Four images were convolved with both kernels and deconvolved using multi-direction traversal. Results show good restoration after 200–300 iterations. [Figure 6: see original paper] shows the results, with PSNR and SSIM values in .

Visually, the algorithm achieves excellent restoration for all dirty images, improving PSNR by 43.2% and SSIM by 22.1% on average. Counterintuitively, the complex mixed Gaussian kernel yields better deconvolution because its lower shift parallelism better preserves original image information in dirty image pixels. The program successfully deconvolves all test images with remarkable effectiveness.

3.3. Deconvolution Verification of Radio Interferometry Imaging

To test radio interferometry imaging deconvolution, we used the Radio Astronomy Simulation, Calibration, and Imaging Library (RASCIL) to generate a convolution kernel. The kernel's properties depend on antenna array configuration, frequency, and observation mode. Using the SKA-Low LOWBD2 standard configuration ($r_{\max} = 750$), snapshot mode, 100 MHz center frequency, 1 MHz bandwidth, R.A. +15°, decl. −45°, and J2000 precession reference date, we obtained u, v coverage. A Bézier function fitted this coverage to approximate the aperture illumination distribution. Its two-dimensional Fourier transform yields the interferometry imaging kernel—a 256$×$256 distribution shown in [Figure 7: see original paper].

Analysis reveals a main lobe width of ~25 pixels with numerous large, energetic sidelobes, creating a highly dispersed energy distribution that severely damages images and makes restoration difficult. The experiment uses the same four astronomical images (512$\times 512 monochrome)convolvedwiththis256\times$256 kernel.

[Figure 8: see original paper] shows the damage exceeds that of simple test kernels. While lunar surface features remained somewhat distinguishable with simple kernels, they become nearly invisible with this kernel. Such extended, low-contrast sources are nearly hopeless for traditional CLEAN algorithms.

This experiment employed leapfrogging and random-direction scanning. Results in [Figure 8: see original paper] demonstrate that even with such a massive kernel, the algorithm performs well. Though flaws remain, the results are usable for astronomical observations. Restoring such heavily damaged images is inherently difficult. PSNR and SSIM values appear in , showing 41.0% PSNR improvement and 33.9% SSIM improvement on average—excellent results for a non-data-based traditional method.

Processing large 256$\times$256 kernels is time-consuming (hours vs. minutes for smaller kernels), but the effectiveness justifies the cost. The algorithm has not been parallelized; multicore or GPU implementation would significantly accelerate it. Memory consumption remains below 1 GB even for large PSFs, making it suitable for modern desktop computers.

For comparison, we deconvolved dirty image C using the Högbom CLEAN algorithm with the same 256$\times$256 SKA PSF. The best result appears in [Figure 9: see original paper]. While CLEAN successfully identified point sources and convolved them with a clean beam (a simple Gaussian), the result poorly resembles the original compared to our algorithm, losing almost all true details. In low-contrast cases, Högbom CLEAN cannot correctly identify true sources because surrounding brightness confuses the algorithm.

3.4. Reconstruction of Image Defects

The first experiment added 5% salt-and-pepper noise to four dirty images, applied four-direction combined scanning, and deconvolved them. Results appear in [Figure 10: see original paper], with metrics in .

The noise has minimal impact on this pixel-iteration-based method, with negligible PSNR and SSIM degradation. Visual inspection shows slight sharpness reduction compared to noise-free deconvolution, but this is not a major issue.

For larger lost blocks, performance varies. Dirty image D was tested with missing blocks at regular intervals: (a) 3$\times 3 blocksat6-pixelintervals,(b)10\times 10 blocksat20-pixelintervals,(c)6\times 6 blocksat8-pixelintervals,and(d)16\times$16 blocks at 40-pixel intervals. Preprocessing set missing pixels to 1000.

The deconvolution algorithm processed these defective images, with results in

[Figure 11: see original paper] and metrics in , including reference values for defect-free restoration.

Missing blocks significantly impact deconvolution, but the algorithm remains robust. Figure 12: see original paper shows that by leveraging information from slender surrounding regions, the algorithm successfully restores complete images with good details. Figure 12: see original paper demonstrates that for sporadic small-area losses, restoration is nearly perfect, as if the losses never existed, proving the information was redundant.

The experiments confirm that the algorithm efficiently utilizes information to restore images without introducing false structures like data-driven methods. This suggests an intriguing possibility: when deconvolution matures, the convolution process could be used in reverse to disperse light information across many locations, maximizing sensor capacity and redundancy.

4. Conclusion

To address complex deconvolution problems in radio interferometry imaging, this paper reanalyzed general image deconvolution from the dirty image perspective. We proposed that pixel brightness in dirty images constrains surrounding light distributions to hyperplanes, and that deconvolution requires solving these brightness linear equations.

We introduced the Kaczmarz method—an iterative algorithm that steps along hyperplane normals—particularly suitable for uniform equations like image deconvolution. By adding varied scan directions, the algorithm achieves excellent results, improving PSNR by 43.2% and SSIM by 22.1% on average.

We further improved the scan iteration method with leapfrogging and random-direction scanning, applying it to radio interferometry baseline imaging. The resulting large, complex convolution kernel severely damages images, but our improved algorithm restores them to recognizable quality, improving PSNR by 41.0% and SSIM by 33.9% on average. Experiments verify that by skipping invalid pixels, the algorithm fully utilizes dirty image information without destroying structure, demonstrating strong adaptability through pixel-level operations.

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Note: Figure translations are in progress. See original paper for figures.