A Cosmological Full-shape Power Spectra Analysis Using Pre- and Post-reconstructed Density Fields postprint

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Abstract

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Full Text

Preamble

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A Cosmological Full-shape Power Spectra Analysis Using Pre- and Post-reconstructed Density Fields

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Abstract

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Key words: cosmology: theory – (cosmology:) large-scale structure of universe – (cosmology:) cosmological parameters

1. Introduction

Large galaxy surveys such as the Dark Energy Spectroscopic Instrument (DESI; Aghamousa et al. 2016) provide vast datasets that are crucial for exploring cosmic large-scale structure. By extracting key cosmological probes, including baryon acoustic oscillations (BAO; Eisenstein & Hu 1998; Cole et al. 2005; Eisenstein et al. 2005) and redshift-space distortions (RSD; Kaiser 1987), these surveys enable tighter constraints on cosmological parameters, offer insights into the nature of dark energy, and provide a powerful framework for testing alternative theories of gravity.

The coupled photon–baryon fluid leaves a signature on the matter distribution after recombination, appearing as a localized peak in the correlation function or an oscillatory pattern in the power spectrum (Eisenstein et al. 1998a; Meiksin et al. 1999). The characteristic scale of BAO serves as a standard ruler for cosmological distance measurements (Eisenstein et al. 1998b). However, nonlinear structure formation driven by gravity broadens and shifts the BAO peak in the correlation function and dampens the oscillations in the power spectrum, leading to a loss of phase coherence and the blurring of BAO measurements (Eisenstein et al. 2007b; Crocce & Scoccimarro 2008; Seo et al. 2008; Smith et al. 2008).

Nonlinear evolution of BAO is primarily driven by large-scale bulk flows and gravitational clustering, effects that can be partially corrected via standard reconstruction techniques (Eisenstein et al. 2007a). These approaches estimate a displacement field based on the Zel'dovich approximation (Zel'dovich 1970) and use it to reposition both data and random particles. By separating long-wavelength displacements from the total displacement field, reconstruction effectively transfers crucial information to the reconstructed density field. As a result, standard reconstruction mitigates BAO damping and mode coupling caused by nonlinear evolution, thereby improving measurement precision and reducing systematic shifts (Seo et al. 2008, 2010; Padmanabhan et al. 2009).

Density-field reconstruction in BAO analysis has motivated deeper investigations into its underlying mechanisms for information recovery. During the process of restoring linear modes contaminated by nonlinear effects, reconstruction transfers higher-order statistical information from the unreconstructed density field, δ , to the reconstructed density field, δ (Schmittfull et al. 2015). In the absence of primordial non-Gaussianity, the higher-order N-point statistical information in the pre-reconstruction density field arises from gravitationally driven nonlinear evolution. Since reconstruction acts as an approximate inverse process to this evolution, it reduces the non-Gaussianity of the density field, resulting in a more linear and Gaussian post-reconstruction density field (Hikage et al. 2017, 2020b). Given these properties, density-field reconstruction can be extended beyond BAO analysis to a wide range of topics, including RSD (Zhu et al. 2018; Hikage et al. 2020a), neutrino properties (Wang et al. 2024b; Zang & Zhu 2024), and primordial non-Gaussianity (Shirasaki et al. 2021; Chen et al. 2024; Flöss & Meerburg 2024).

Although the post-reconstruction power spectrum, P , retains some of the higher-order information from the original unreconstructed density field beyond what is accessible in the pre-reconstruction power spectrum, P , incorporating the cross-power spectrum between the pre- and post-reconstruction fields, P , allows for a more comprehensive extraction of cosmological information. Jointly analyzing the three power spectra, referred to as P , effectively captures higher-order statistical information (Wang et al. 2024b). Density-field reconstruction enables the transformation of higher-order statistics into two-point statistics, allowing P and P to be interpreted in terms of specific higher-order statistics of δ (Schmittfull et al. 2015; Sugiyama 2024c; Wang et al. 2024b). Since these three power spectra reflect different levels of nonlinearity, they exhibit distinct dependencies on cosmological parameters and the higher-order statistics of δ . A joint analysis of P therefore helps in breaking degeneracies among cosmological parameters and small-scale clustering, substantially improving parameter constraints.

Recently, an emulator-based likelihood analysis using galaxy mocks further demonstrated the effectiveness of this approach, paving the way for its application to observational survey catalogs (Wang et al. 2024a). Besides emulator-based modeling, perturbation theory (PT) can also be employed for the joint analysis of P , although modeling smaller scales can be challenging. PT offers valuable insight into the physical underpinnings of this method. Numerous works have developed PT models for the pre-reconstruction power spectrum (Bernardeau et al. 2002), with the effective field theory (EFT) of large-scale structure (Baumann et al. 2012; Ivanov 2022) being widely applied in data analyses (e.g., D'Amico et al. 2020; Ivanov et al. 2020; Adame et al. 2024; Zhao et al. 2024). Furthermore, various studies have proposed PT-based models for P and P (e.g., Noh et al. 2009; Padmanabhan et al. 2009; White 2015; Seo et al. 2016; Hikage et al. 2017; Chen et al. 2019; Sugiyama 2024b; Zhang et al. 2025).

In this paper, we extend the perturbation-theory-based power spectrum framework of Hikage et al. (2020a) by introducing BAO parameters to account for the Alcock–Paczynski (AP) effect (Alcock & Paczynski 1979). We validate our theoretical models for P , P , and P using N-body simulation data at redshift $z = 1.02$. The joint analysis of P yields results consistent with those reported by Wang et al. (2024a). This paper is organized as follows. Section 2 reviews the theoretical model for the power spectra, Section 3 presents the joint analysis using P with simulation data, and Section 4 summarizes and discusses our main results.

2. The Modeling

In this section, we present one-loop models for the pre- and post-reconstruction, and cross-power spectra of the matter density field in redshift space. Our approach is built upon the EFT of large-scale structure and includes a leading-order counterterm for small-scale (ultraviolet, UV) physics. We also incorporate parameters associated with the AP effect to properly model geometric distortions.

To describe the power spectrum as an observable, we begin by defining the matter density contrast $\delta(x) = ((x) - \bar{\ })/\bar{\ }$, where x is the comoving coordinate, (x) is the local matter density, and $\bar{\ }$ is the mean density. Under the Newtonian approximation to general relativity, treating matter as a pressureless fluid, the density contrast δ and velocity field v evolve according to the continuity, Euler, and Poisson equations. Assuming an irrotational velocity field, we introduce the velocity divergence field = · v. This set of equations can be solved approximately using standard perturbation theory (SPT). In Fourier space, the nth order expansions of δ and take the form (e.g., Fry 1984; Goroff et al. 1986; Jain & Bertschinger 1994; Scoccimarro & Frieman 1996; Bernardeau et al. 2002; Matsubara 2008b):

$\delta^{(n)}(k) =$ d^3q_1...d^3q δ (q_1 ... - k) F (q_1,...,q) δ (q_1)...δ (q) $\tilde{}^{(n)}(k) =$ d^3q_1...d^3q δ (q_1 ... - k) G (q_1,...,q) δ (q_1)...δ (q)

where q_1 ... = q_1 + ... + q ; D(z) is the linear growth factor normalized to D(z = 0) = 1; f denotes the linear growth rate; δ is the linear density field at z = 0; and F , G are the nth order perturbation kernels for the matter density and velocity divergence fields, respectively. We adopt the Einstein–de Sitter approximation (Bernardeau et al. 2002), valid in near-ΛCDM cosmologies, so that $D^{(n)}(z)$ $D^n(z)$. For brevity, we write $\delta^{(n)}(k) = D^n(z)$ d^3q_1...d^3q δ (q_1 ... - k) F δ (q_1)...δ (q), where δ is the Dirac delta function.

To account for RSD under the distant-observer approximation, we relate real- and redshift-space positions according to the conservation condition. The resulting redshift-space density field is (Matsubara 2008b):

δ (k) = d^3x e^{-} $\delta(x)$ $[1 + f(\hat{z} \cdot\ )(v \cdot \hat{z})]^{-1}$

where $\hat{z}$ is the unit vector in the line-of-sight direction. Introducing the velocity divergence $\tilde{}(k)$ and expanding the exponential factor in a Taylor series leads to:

δ (k) = Σ $_1$^∞ (1/n!) d^3q_1...d^3q δ (q_1 ... - k) Z (q_1,...,q) δ (q_1)...δ (q)

Up to one-loop order, the pre-reconstruction power spectrum is:

P (k,) = $P_{11}(k)$ + $P_{13}(k,)$ + $P_{22}(k,)$

where $P_{11}(k) = D^2(z)P\ (k)$, and P_{13} and P_{22} represent one-loop corrections. By substituting the perturbative expansions of the density field δ and velocity divergence field into the redshift-space expression and performing a perturbative expansion, one can derive the nth order density fluctuation in redshift space as:

$\delta\ ^{(n)}(k) = D^n(z)$ d^3q_1...d^3q δ (q_1 ... - k) Z (q_1,...,q) δ (q_1)...δ (q)

where Z is the nth order redshift-space kernel (e.g., Heavens et al. 1998; Scoccimarro et al. 1999; Matsubara 2008a; Hikage et al. 2020a). In what follows, we omit the superscript "s."

We apply the standard reconstruction technique (Eisenstein et al. 2007a), which estimates a shift field $\tilde{s}$ from the smoothed nonlinear density field δ using the negative Zel'dovich approximation (Zel'dovich 1970):

$\tilde{s}(k) = (ik/k^2)\ (S(k)/b)\ \delta(k)$

with R denoting the smoothing scale used in reconstruction. This procedure mitigates nonlinear effects from bulk flows and cluster formation. Displacing both data and random particles by s yields the displaced and shifted fields δ_d and δ_s, whose difference defines the reconstructed density field:

δ_{rec}(k) = δ_d(k) - δ_s(k)

Because the matter sample has a sufficiently high number density in reconstruction (Sugiyama 2024c), discreteness effects are negligible.

An analogous perturbative expansion exists for δ_{rec}. The one-loop post-reconstruction power spectrum and cross-power spectrum can be expressed as:

P (k,) = P_{11}(k) + P_{13}^{post}(k,) + P_{22}^{post}(k,) P_{cross}(k,) = P_{11}(k) + P_{13}^{cross}(k,) + P_{22}^{cross}(k,)

where the second-order and third-order kernels (Z_2, Z_3) are replaced by their post- and cross-reconstruction counterparts (Zhang et al. 2025). Theoretical predictions are often expanded into Legendre multipoles:

P_ (k) = (2 +1)/2 $_1^1$ d P(k,) _ ( )

where _ is the Legendre polynomial of order . One-loop SPT alone does not fully capture nonlinear small-scale physics, so EFT introduces a counterterm to absorb UV contributions. For the one-loop pre-reconstruction spectrum, the counterterm is proportional to k^2P (k) (Senatore & Zaldarriaga 2014). We adopt a similar form for the post-reconstruction and cross-power spectra (Hikage et al. 2020a; Zhang et al. 2025):

P_ ^{EFT}(k) = P_ ^1-loop(k) + a_ k^2P (k)

Converting redshift to comoving distance introduces the AP effect (Alcock & Paczynski 1979), which distorts (k,) when the fiducial cosmology differs from the true one. Labeling true-cosmology coordinates by (k̃,ˆ), the full power spectrum with counterterms reads:

P_ ^{theory}(k) = (1/α $α^2$) P_ ^{EFT}(k̃,ˆ)

and we recover the 2D power spectrum from its multipoles for = 0, 2, 4. Higher-order multipoles are negligible for the scales of interest. The AP effect rescales k and according to:

k̃ = k/α [1 + ^{2}(α ^{2}/α ^{2} - 1)]$^{1\ 2}$ ˆ = (α /α) [1 + ^{2}(α ^{2}/α ^{2} - 1)]$^{-1\ 2}$

where α and α relate AP parameters to the Hubble function H and the angular diameter distance D_A at the effective redshift z_{eff}, and r_d is the sound horizon at the drag epoch. Note that the superscript f indicates fiducial values. Discrete k-binning in measurements is accounted for by averaging the theoretical predictions over each bin:

P_ ^{bin}(k_i) = (1/Δk) _{k_i-Δk/2}^{k_i+Δk/2} dk P_ ^{theory}(k)

where the integration includes the AP transformation. This procedure allows a more accurate comparison between theory and binned measurements in subsequent likelihood analyses.

3. Results

In this section, we present our analysis using a suite of high-resolution N-body simulations. This mock dataset was previously used in Hikage et al. (2020b) and Zhang et al. (2025). The input cosmological parameters for these simulations are based on the best-fit values from the Planck 2015 TT, TE, EE+lowP measurements: Ω_b = 0.0492, Ω_m = 0.3156, h = 0.6727, n_s = 0.9645, and $\sigma_8 = 0.831$ (Ade et al. 2016). The initial linear power spectrum is computed using CAMB (Lewis et al. 2000), which is also used to calculate our theoretical model predictions. The initial mass particle distribution is generated with second-order Lagrangian perturbation theory (2LPT) (Crocce et al. 2006; Nishimichi et al. 2009). We then perform N-body simulations using Gadget-2 (Springel 2005) to generate 4000 realizations, each with a box size of $L = 500\ h^{-1}$ Mpc, containing 512^3 particles at redshift z = 1.02. After standard reconstruction is applied to each realization, the pre-reconstruction, post-reconstruction, and cross-power spectra are measured from the simulation samples. Owing to the large measurement uncertainties associated with the hexadecapole, only monopole and quadrupole measurements are used in this work. Further details on the simulation data and reconstruction procedure can be found in Hikage et al. (2020b).

We measure the power spectrum multipole components from 4000 realizations of our simulations and use them to estimate the covariance matrix:

C_{ij} = (1/(N-1)) Σ_{n=1}^N [P_ ^n(k_i) - P̄_ (k_i)] [P_ '^n(k_j) - P̄_ '(k_j)]

where N = 4000 is the total number of realizations, and P̄_ (k) is the mean power spectrum multipole across all realizations. Due to the limited box size L $= 500\ h^{-1}$ Mpc, the mean power spectrum P̄_ exhibits a pronounced sawtooth pattern on large scales, especially for = 2. To mitigate this effect, we use an additional set of 8 realizations with a larger volume, $4\ h^{-1}$ Gpc, to correct the power spectrum measurements following Zhang et al. (2025):

P_ ^{corrected}(k) = P_ ^{500} Mpc(k) × [P_ ^4 Gpc(k)/P_ ^{500} Mpc(k)]_{smooth}

where the superscripts $4\ h^{-1}$ Gpc or $500\ h^{-1}$ Mpc indicate the side length of the simulation box. This "grid correction" ensures an accurate representation of large-scale modes, thereby improving our subsequent likelihood analysis.

The likelihood function is given by:

L exp(- ^{2}/2)

where the chi-square statistic takes the form:

^{2} = Σ_{i,j} [P_ ^{data}(k_i) - P_ ^{theory}(k_i)] C_{ij}^{-1} [P_ '^{data}(k_j) - P_ '^{theory}(k_j)]

Since the number of realizations N is finite, the inverse of the covariance matrix is rescaled by the Hartlap factor (N - N_{bin} - 2)/(N - 1) (Hartlap et al. 2007), where N_{bin} is the number of bins used in the fit.

In each power spectrum model, we treat five parameters as free: {α , α , f, c_0, c_2}. In the joint fit using three types of power spectra, parameters α , α , and f are shared across all three models, while each model retains its own counterterm parameters (c_0, c_2). This results in nine total free parameters. All parameters have uniform (flat) priors with ranges listed in Table 1 .

We construct our parameter estimation pipeline within the Cobaya framework (Torrado & Lewis 2021), using Markov Chain Monte Carlo (MCMC) sampling (Lewis & Bridle 2002) to explore the posterior distributions. The resulting MCMC chains are analyzed with GetDist (Lewis 2019), which yields marginalized posteriors and the corresponding contour plots. To ensure convergence, we require the Gelman–Rubin statistic to satisfy R - 1 < 0.001. In addition, we use iminuit (James & Roos 1975; Dembinski et al. 2020) to minimize ^{2} and obtain the best-fit parameter values.

In standard reconstruction, the choice of smoothing scale is essential. A larger R reduces the effectiveness of reconstruction, while a smaller R may enhance the gain compared to the pre-reconstructed case. However, excessively small values (e.g., R < 10 h^{-1} Mpc) can introduce large-scale nonlinearities due to reconstruction inaccuracy (Hikage et al. 2017), making theoretical predictions more challenging and potentially compromising the reliability of results. To ensure robustness, we adopt R = 15 h^{-1} Mpc in this work.

Our likelihood analysis includes both individual and joint fits of the three power spectra. In individual fits, we set k_{max} = 0.20 h Mpc^{-1}, while also testing a more conservative choice of k_{max} = 0.16 h Mpc^{-1}. The corresponding results for this more conservative limit are presented in the Appendix. For the joint fit P , we initially attempted a combined fit using all data points with k (0.02, 0.20) h Mpc^{-1}. However, because all three power spectra closely resemble the linear power spectrum on large scales, their correlation coefficients (the correlation among P , P and P at the same k) approach unity, as seen in the left panel of Figure 1 [Figure 1: see original paper]. This means that including all three power spectra in the analysis is redundant, which can cause numerical instabilities when we invert the data covariance matrix. To address this issue, we remove part of the large-scale modes from the pre-reconstruction power spectrum, which effectively reduces the redundancy in the data vector. In practice, we set k_{min}^{pre} = 0.14 h Mpc^{-1}. The right panel of Figure 1 shows the correlation coefficients after this cut, whose absolute values are all well below unity, ensuring a stable numeric inversion in the likelihood analysis.

Figure 2 [Figure 2: see original paper] presents the resulting constraints on BAO and RSD parameters obtained from each individual power spectrum and their combined fit P . The best-fit results are all consistent with the fiducial parameter values within the 68% confidence region. Among the individual fits, P delivers the tightest constraints, with P performing comparably well. A complete summary of these results is provided in Table 2 . The joint fit P substantially tightens the parameter constraints relative to the post-reconstruction power spectrum alone, offering reductions in σ_p of up to 14%. The corresponding Figure of Merit (FoM) on the BAO and RSD parameters, defined as FoM = √det(C^{-1}) where C is the covariance matrix of {α , α , f} estimated from the MCMC samples, also shows a 10.5% enhancement when combining all three power spectra. This underscores the value of including all available spectra to achieve tighter cosmological parameter constraints.

Using the best-fit parameters obtained from each fit, we compute the corresponding theoretical monopole and quadrupole power spectra and compare them with the simulation data in Figure 3 [Figure 3: see original paper]. We decompose the linear power spectrum into wiggle and no-wiggle components via a polynomial-based method (Hinton et al. 2017), writing P (k) = P_w(k) + P_{nw}(k). Applying the fiducial linear growth rate f_{in} = 0.8796, we calculate the linear redshift-space power spectrum and its multipoles using the Kaiser formula (Kaiser 1987). For ease of comparison, both theoretical and simulation power spectra are divided by P_ ,nw. Figure 3 also shows the corresponding linear monopole and quadrupole; as expected, BAO wiggles become more prominent in the post-reconstruction and cross-power spectra than in the pre-reconstruction spectrum. Meanwhile, the cross-power spectrum exhibits a gradually decreasing amplitude, owing to the absence of infrared (IR) cancellation (Sugiyama 2024a).

As seen in Figure 3, the monopole and quadrupole from each individually fitted model agree closely with the simulation data, and the joint fit (P) also provides consistent results. Notably, for both post-reconstruction and cross-power spectra, the joint-fit curves are similar to those obtained in the individual fits. However, the BAO wiggles in P are not perfectly captured by the current PT approach. Incorporating IR resummation (Sugiyama 2024a, 2024b) could further improve the BAO modeling, and we plan to pursue this extension in future work.

4. Conclusion and Discussions

In this paper, we conduct a full-shape analysis for BAO and RSD parameters using the power spectra derived from both pre- and post-reconstruction density fields, while also including their cross-power spectrum. Our framework is developed at one-loop order in redshift space for the matter density field. We incorporate the AP effect into our models to constrain BAO parameters. To extend SPT to smaller scales, we introduce EFT counterterms to account for unmodeled UV physics. However, the counterterms may be partially degenerate with IR contributions and other modeling uncertainties. Our theoretical models for P , P , and P thus fit simulation data for the power spectrum multipoles, constraining the BAO parameters α , α and the linear growth rate f. Each of these three models provides unbiased estimates when assuming a reconstruction smoothing scale R = 15 h^{-1} Mpc and a covariance matrix estimated from 4000 N-body realizations at redshift z = 1.02, each with a volume of (500 h^{-1} Mpc)^3. The post-reconstruction power spectrum yields the tightest individual constraints, while P is intermediate between P and P , yet quite close to the latter in terms of constraining power.

We carry out a joint fit of all three power spectra, P , to further tighten parameter constraints. Because these spectra are strongly correlated on large scales, especially P and P , numerical instability in the covariance matrix can arise when using limited-volume simulations and a finite number of realizations. In addition, small-scale nonlinearities may also bias the inferred parameters if not fully captured by the PT. We employ a pragmatic remedy in this work: increasing the minimum wavenumber k_{min} for P to 0.14 h Mpc^{-1}, which significantly reduces the bias in f. We find that raising k_{min} for P also yields similar but milder benefits. Ultimately, we opt to remove large-scale modes of P alone, given its stronger correlation with P and its weaker individual constraining power.

With these adjustments, the joint fit P delivers unbiased best-fit parameter values while improving constraints relative to P alone, yielding uncertainty reductions of about 11%, 16%, and 14% in α , α , and f, respectively. Additionally, the FoM for the joint fit is enhanced by 10.5%. These results corroborate earlier findings (Wang et al. 2024a, 2024b) that the three spectra together retain complementary information about the pre-reconstruction density field, enabling greater precision than any individual spectrum can achieve.

We confirm that each power spectrum model separately reproduces the simulation results well, barring some mismatch in the BAO wiggles for P . Likewise, the joint analysis accurately recovers both simulation and theoretical predictions. In this study, we focus on a relatively high redshift slice at z = 1.02. However, it would be valuable to assess our model and methodology at other redshifts in future studies, especially at lower redshifts. At lower redshifts, nonlinear effects are more pronounced, and the improvements from reconstruction tend to be more significant (e.g., Hikage et al. 2020b; Wang et al. 2024b). Therefore, we expect that the joint analysis method will be even more effective at lower redshifts, although it is more challenging to apply PT-based models to smaller scales at those redshifts.

Looking ahead to applications in real surveys such as DESI, we plan to improve the theoretical model by recognizing the mismatches between assumed and true cosmological parameters for the BAO reconstruction (Sherwin & White 2019), incorporating IR resummation (Sugiyama 2024a, 2024b), introducing galaxy bias and other observational effects, and evaluating the approach with halo or galaxy catalogs. We will also explore the FFTLog technique (Hamilton 2000) for loop integrals in SPT, and accelerate the evaluation of theory-based emulators (Donald-McCann et al. 2022) to further improve the efficiency of the theoretical predictions.

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Appendix

Supplementary Test Results for Different Scale Ranges

In this section, we provide supplementary results that explore how the choice of scale range affects our analysis. Specifically, we consider a more conservative maximum wavenumber of k_{max} = 0.16 h Mpc^{-1}, complementing our primary results at k_{max} = 0.20 h Mpc^{-1} in the main text.

Figure A1 [FIGURE:A1] shows the constraints on α , α , and f obtained from the individual fits of P , P , and P , as well as the joint fit P , at k_{max} = 0.16 h Mpc^{-1}. The cross-power spectrum performs similarly to P . The combined fit P continues to yield the tightest constraints—particularly on the AP parameters—but offers only a marginal improvement in f compared to P . This is in contrast to the more substantial gain in f noted in the main text at k_{max} = 0.20 h Mpc^{-1}. The detailed results are presented in Table A1 [TABLE:A1], which also compares the FoM across different scenarios. Specifically, P reduces the uncertainties in α , α , and f by 17%, 17%, and 8%, respectively, relative to P , while increasing the FoM by 8.3%.

In Figure A2 [FIGURE:A2], we compare the theoretical predictions (using best-fit parameters corresponding to Figure A1) against the measured multipoles. We again find good agreement between theory and data for both individual fits and the joint fit P .

In Section 3 of the main text, we stressed the importance of cutting large-scale modes to ensure robust joint fits of P . Here, we expand on that discussion by fixing k_{max} = 0.20 h Mpc^{-1} and exploring how varying k_{min} influences the resulting parameter constraints. As shown in Figure A3 [FIGURE:A3], taking k_{min} = 0.02 h Mpc^{-1} for all spectra produces a best-fit linear growth rate f that is noticeably smaller than its fiducial value, reflecting the strong correlations among the three power spectra on large scales and the ensuing instability in the inverse covariance matrix. The left panel of Figure A3 demonstrates that this bias in f decreases as k_{min}^{pre} is increased. Similarly, the right panel shows that raising k_{min}^{cross}, k_{min}^{post}, or both mitigates the bias, although to a slightly lesser degree. Based on these tests, in the main text we chose k_{min}^{pre} = 0.14 h Mpc^{-1} for the joint fits, as this approach most effectively reduces biases while retaining sufficient constraining power.

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Note: Figure translations are in progress. See original paper for figures.