Neutron scattering off even Gadolinium isotopes: Differential cross sections

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Abstract

This study investigates neutron differential scattering cross sections off even Gadolinium isotopes (Gd-156, Gd-158, Gd-160) at various energies (1.5, 2.5, 4.1 and 7 MeV), comparing theoretical calculations with evaluations from prominent nuclear data libraries (ENDF/B-VIII.0, JENDL-5 and EXFOR) and available experimental data. Analysis reveals a characteristic maximum for the forward angles and a deep minimum in the angular distributions, thus indicating a quasi-forbidden scattering angle range. Both theoretical and ENDF/B-VIII.0 results show excellent agreement in the forward direction at 1.5 MeV. However, discrepancies emerge at higher energies, particularly at 2.5 MeV, where neither ENDF/B-VIII.0 nor JENDL-5 accurately reproduce experimental data, especially for Gd-160. Notably, the rotational nature of Gd-160, as indicated by its low first excited state energy and E4+/E2+ ratio, is not consistently reflected in the ENDF/B-VIII.0 evaluations. For 4.1 MeV, JENDL-5 demonstrates superior agreement with experimental data compared to ENDF/B-VIII.0 and theoretical calculations. At 7 MeV, multiple minima are observed in both theoretical and ENDF/B-VIII.0 results, predicting specific angular intervals with near-zero scattering. While theoretical calculations generally align with experimental and evaluated data at forward angles, deviations occur at backward angles, likely due to increased compound nucleus contributions. Despite the limited experimental data, particularly the reliance on Bauge's team measurements, this study underscores the utility of theoretical models in providing structural insights into neutron scattering cross sections for deformed nuclei.

Full Text

Preamble

Neutron scattering off even Gadolinium isotopes: Differential cross sections Ayoub Abdessabour HAMZA,[1] Brahim SADALLAH,∗ and Brahim BELADEL†

[1]Materials Science and Informatics Laboratory, Djelfa, 17000, Algeria, Algeria This study investigates neutron differential scattering cross sections off even Gadolinium isotopes (Gd-156, Gd-158, Gd-160) at various energies (1.5, 2.5, 4.1, and 7 MeV), comparing theoretical calculations with evaluations from prominent nuclear data libraries (ENDF/B-VIII.0, JENDL-5 and EXFOR) and available experimental data. Analysis reveals a characteristic maximum for the forward angles and a deep minimum in the angular distributions, thus indicating quasi-forbidden scattering angle range. Both theoretical and ENDF/B-VIII.0 results show excellent agreement in the forward direction at 1.5 MeV. However, discrepancies emerge at higher energies, particularly at 2.5 MeV, where neither ENDF/B-VIII.0 nor JENDL-5 accurately reproduce experimental data, especially for Gd-160. Notably, the rotational nature of Gd-160, as indicated by its low first excited state energy and E4+ ratio, is not consistently reflected in the ENDF/B-VIII.0 evaluations. For 4.1 MeV, JENDL-5 demonstrates superior agreement with experimental data compared to ENDF/B-VIII.0 and theoretical calculations. At 7 MeV, multiple minima are observed in both theoretical and ENDF/B-VIII.0 results, predicting specific angular intervals with near-zero scattering. While theoretical calculations generally align with experimental and evaluated data at forward angles, deviations occur at backward angles, likely due to increased compound nucleus contributions. Despite the limited experimental data, particularly the reliance on Bauge's team measurements, this study underscores the utility of theoretical models in providing structural insights into neutron scattering cross sections for deformed nuclei.

Keywords: Neutron scattering, Cross section, even-even nuclei, Angular distribution

Introduction

Computer modeling and calculations in fields such as reactor design and safety require precise mathematical and analytical formulations of cross sections. Among these, the differential cross sections for elastic neutron scattering are particularly important, especially for deformed nuclei in the rare-earth region [1–9]. These data are indispensable not only for practical applications but also for advancing the understanding of fundamental neutron-nucleus interactions.

Over the decades, numerous evaluated and experimental nuclear databases have been developed to compile cross-section data, including total, differential, elastic, and inelastic scattering [10–12]. Databases such as ENDF/B-VIII.0, JENDL-5, and EXFOR [13–15] provide theoretical evaluations and experimental measurements across a wide range of targets and energy levels. Despite their value, these databases are subject to uncertainties and discrepancies, necessitating ongoing efforts to validate and refine their content.

Although research into neutron scattering on deformed nuclei began over 70 years ago, the topic remains highly relevant [16]. The continuous accumulation of experimental data on nuclear interactions and the structures of deformed and superdeformed nuclei has driven the development of theoretical models and analytical formulations [17-20]. Such models are essential for addressing energy regions where experimental data are sparse or unavailable and for providing predictions for nuclei that have not been experimentally investigated due to practical or technological constraints, thereby filling gaps in current nuclear databases.

Investigating nuclear interactions can enhance understanding of monopole, dipole, and quadrupole excitations, which are essential for characterizing nuclear deformation and collective motions [21]. In addition, results from neutron scattering experiments can inform theoretical models of nuclear interactions and contribute to advancements in nuclear technology and materials science [22].

The development of accurate analytical formulations for the differential cross section, such as the one proposed in this study, offers several advantages. These formulations enhance our understanding of the fundamental mechanisms of neutron scattering and facilitate predictions of cross sections in under-explored energy regions. Moreover, elastic scattering contributions to total cross sections are often underestimated, highlighting the need for comprehensive investigations of these processes.

In a previous study published recently [23], we proposed a theoretical formulation for the angular distribution of neutrons scattered on doubly even nuclei. This formulation, rooted in the collective rigid rotor model and incorporating approximations such as the uniform distribution of nuclear matter and a Yukawa potential for neutron-nucleon interactions, demonstrated its utility in describing scattering phenomena.

The present work applies this formulation to the case of fast neutron scattering on even isotopes of Gadolinium, aiming to further evaluate and validate its applicability. Fast neutron scattering on gadolinium isotopes poses unique challenges and opportunities given their strong nuclear capture cross sections and relevance to reactor physics and shielding technologies. By comparing theoretical results with data from major nuclear databases such as ENDF/B-VIII.0, JENDL-5, and the experimental repository EXFOR, we aim to critically assess the predictive power of our formulation and identify potential discrepancies that warrant further investigation. Data from these prominent databases provide essential insights into these cross sections, facilitating accurate evaluations and comparisons across different nuclear reactions.

This study aims firstly to validate the proposed theoretical approach by identifying discrepancies or agreements with available data, thus contributing to a deeper understanding of elastic neutron scattering on nuclei in the rare earth element region. Secondly, it aims to provide theoretical values for these cross sections in energy ranges where no experimental measurements are available.

II. Methods and Techniques

In a recent study on neutron scattering on doubly even heavy deformed nuclei [23], an analytical expression for neutron scattering cross sections was introduced. In this paper, we apply that formula to determine the neutron differential scattering cross sections off three even isotopes of Gadolinium, which are classified as rotational within the Bohr-Mottelson Collective Model. This model describes these nuclei as permanently deformed surfaces of cylindrical symmetry (axially symmetric), where the rotational deformity is defined as $R = R_0[1 + \beta_2 Y_{2,0}(\Omega)]$ and characterized by a substantial quadrupole deformation parameter β_2.

For the theoretical calculation of the cross sections, we employed a straightforward approach. The Hill and Wheeler formalism [24] is followed to represent the whole neutron-nucleus interaction as if it were a single particle model problem. The neutron-nucleon interaction was modeled using the Yukawa potential, while the nuclear matter was approximated as having a uniform density within the nuclear volume.

$$V(\vec{r}_1, \vec{r}_2) = \sum V_n = V_0 \sum e^{-\rho_n/\lambda}$$

The general analytical formula giving the elastic scattering cross section of fast neutrons on doubly even nuclei used in this work is:

$$\frac{d\sigma(E;\Omega_{k'})_{\text{elastic}}}{d\Omega_{k'}} = \left(\frac{\hbar c}{2\pi\lambda_n(A+1)}\right)^2 \left[\frac{\cos(\theta) + \sqrt{\alpha^2 A^2 - \sin^2(\theta)}}{\sqrt{\alpha^2 A^2 - \sin^2(\theta)}}\right]^2 |\tilde{V}(q)|^2 \left|\sqrt{\pi}\int_0^\pi d\alpha \sin(\alpha)\frac{J_{3/2}(qR)}{(qR)^{3/2}}\right|^2$$

III. Results and Discussion

Despite their applied importance and physical interest, relatively little is known about the interaction of fast neutrons with gadolinium. Experimental data for fast neutron scattering cross sections on elemental gadolinium are very limited and old; those available date back more than 50 years. Elastic neutron scattering cross sections have been reported at selected incident energies primarily near 1.0 MeV [25][26].

The excited structure of a number of gadolinium isotopes has been extensively studied through Coulomb excitation, charge particle, and neutron capture gamma ray measurements [27][28]. However, general understanding of the structure is incomplete, and this, coupled with uncertainties in reaction mechanisms and associated potentials, makes quantitative calculation of neutron cross sections difficult. Data from prominent databases such as ENDF/B-VIII.0, JENDL-5, and EXFOR provide essential insights into these cross sections, facilitating accurate evaluations and comparisons across different nuclear reactions.

Figure 1 [Figure 1: see original paper] shows the elastic differential scattering cross sections from two evaluated nuclear data libraries (ENDF/B-VIII.0 and JENDL-5) and the corresponding analytical results for the even-even isotopes of gadolinium at 1.5 MeV. Unfortunately, there are no experimental measurements available at this energy. To facilitate representation of data in the same frame, we multiplied ENDF/B data by 10 and JENDL data by 100. We note that the angular distributions are identical, especially for forward angles 60°. Regarding the ENDF/B-VIII.0 database, we note the non-existence of a lower limit for the isotope 160. This is surprising because this isotope is more "rotational" than its two homologs. The Gd-160 isotope is more "rotational" than the others, as can be seen directly from the low value of the energy of the first excited level compared to the other two isotopes and can also be deduced from the application of the rule $E_{4+} = 3.33$ [29]. Therefore, we find the behavior of the ENDF/B-VIII.0 database inconsistent with the nuclei' s "rotationality" according to Bohr' s collective nuclear model. Regarding the comparison between theoretical calculations and ENDF/B-VIII.0 evaluations, the curves generally agree at forward angles, with the presence of a very deep minimum in the theoretical calculation, which suggests the presence of a "quasi-forbidden" angular interval [80°, 87°] for neutron scattering at this energy for all three isotopes.

Figure 2 [Figure 2: see original paper] shows elastic angular distributions at 2.5 MeV for the three isotopes. For the same reason as in Figure 1, we multiplied the 156Gd isotope data by 10^6 and the 158Gd data by 10^3. Both databases ENDF/B-VIII.0 and JENDL-5 fail to reproduce the experimental data, unlike what occurs at 4.1 MeV, as mentioned in the next paragraph, especially for the 160 isotope. Beyond the 90-degree angle, we notice that the theoretical results underestimate the cross section relative to their experimental counterparts. This can be attributed to the fact that in this angular range and at this energy, the contribution of the compound nucleus reaction is more important and larger than the contribution of direct interaction.

At 4.1 MeV, contrary to the theoretical calculation, we notice that the ENDF/B-VIII.0 database does not clearly indicate the presence of the first minimum around 40 degrees, with a slight exception in the case of the 160Gd isotope, as shown in Figure 3 [Figure 3: see original paper]. Notably, the JENDL-5 database fits the experimental data exactly, as if it were a pure interpolation. To investigate the behavior of the cross sections with increasing energy, we plotted Figure 4 [Figure 4: see original paper], which shows the differential cross sections at 7 MeV. In this figure, three minima appear for all isotopes, whether for the theoretical calculation or the ENDF/B-VIII.0 database. As a result, three angular intervals appear where neutron scattering can be predicted to be almost zero, namely [37°, 43°], [71°, 77°], and [112°, 123°].

As shown in Figures 3 and 4, analytical calculations reproduce the data very well out to the first minima within the estimated experimental uncertainty. It is just at backward angles that they deviate appreciably from the experiment. Fortunately, 90% of neutrons scatter within these forward angles.

Overall, the analysis of various calculated cross sections for a wide range of deformed even-even nuclei [23] demonstrates that this relatively simple model can yield valuable structural insights into neutron scattering cross sections that are otherwise difficult to obtain.

IV. Conclusion

The limited availability of experimental neutron scattering data on Gadolinium, especially relying on the single measurements of Bauge' s team at only two energies (2.5 and 4.1 MeV), presents a challenge to comprehensive analysis. Nevertheless, this study reveals significant insights: neutron differential scattering cross sections on even-even Gadolinium isotopes exhibit (a) a characteristic forward-angle maximum and a deep minimum at larger angles; (b) our theoretical calculations demonstrate qualitative agreement with experimental and evaluated results, confirming that there is a systematic tendency for the largest experimental results to occur towards forward scattering angles.

This relatively simple model can yield valuable structural insights into neutron scattering cross sections that are otherwise difficult to obtain.

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Note: Figure translations are in progress. See original paper for figures.