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Advances in Neutron External CT Reconstruction Methods

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Abstract

In the field of industrial non-destructive testing, accurate imaging of neutron external computed tomography (CT) can effectively overcome imaging blind spots and artifact problems in conventional CT caused by limited detector field-of-view or complex internal structures, which is of great significance for detecting defects such as cracks and corrosion in large-sized specimens. This paper first elucidates the artifact generation mechanism induced by projection truncation in neutron external CT from mathematical and physical fundamentals; secondly, reviews the technical evolution from traditional analytical inversion to modern constrained optimization methods; further identifies technical bottlenecks in current methods, including insufficient modeling of complex physical effects, algorithm parameter optimization relying on manual experience, and real-time computational resource consumption; finally, presents an outlook on the future development direction of neutron external CT imaging technology and discusses possible directions for technological innovation.

Full Text

Preamble

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A Review of Reconstruction Techniques for Neutron External CT

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Abstract

Neutron external computed tomography (CT) offers a promising solution to overcome imaging blind spots and artifacts inherent in conventional CT, which are often caused by limited detector coverage and complex internal structures. This capability is particularly critical for detecting defects such as cracks and corrosion in large-scale industrial components. This review begins with a theoretical analysis of the artifact formation mechanisms associated with projection truncation in neutron external CT. It then traces the development of reconstruction techniques from classical analytical approaches to modern constrained optimization methods. Key challenges are identified, including insufficient modeling of nonlinear physical effects, reliance on empirical parameter tuning, and high computational demands for real-time reconstruction. Finally, future research directions are discussed, with emphasis on physics-informed modeling, automated parameter optimization, and efficient computational frameworks.

Keywords: Neutron CT, Image Reconstruction Techniques, External CT, Projection Truncation

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X-rays and neutrons each offer distinct advantages in non-destructive testing [1]. X-rays primarily interact with extranuclear electrons and are sensitive to heavy metals [2], whereas neutrons interact with atomic nuclei through strong interaction and resonance effects, providing unique capabilities for detecting light elements, isotopes, and nuclear fuels [3]. As an important branch of non-destructive testing, computed tomography has expanded the applications of both modalities [4], but radiation dose and equipment limitations often lead to missing projection data, causing reconstruction artifacts [5]. Such reconstruction problems with incomplete projection data can be divided into two categories based on the target region: interior problem reconstruction and exterior problem reconstruction. Specifically, the interior problem requires acquiring projection data passing through the object' s interior and reconstructing only the internal region, while the exterior problem involves acquiring projection data passing through the object' s exterior and reconstructing only the external region [6]. Neutron external CT belongs to the latter category, with its core challenge being how to achieve high-precision reconstruction using limited external projection data while suppressing artifacts caused by data incompleteness [7].

The concept of external CT originated in medical imaging. In the early development of cardiovascular CT imaging [8], for instance, scan times exceeding the cardiac cycle caused unstable projection data, degrading image clarity. Researchers proposed a selective region reconstruction strategy that excluded projection data from the heart region and utilized only stable projection information from peripheral tissues for image reconstruction, thereby reducing motion artifacts and blurring. This breakthrough laid the methodological foundation for subsequent industrial external CT research. With advances in radiation imaging technology, external CT research gradually expanded to industrial non-destructive testing [9]. Due to neutrons' unique material penetration characteristics and nuclide discrimination capabilities, neutron CT has demonstrated significant value in industrial inspection. The necessity of external CT in practical applications manifests in three main aspects [6]:

1. **Priority detection of local features** [10]. In certain inspection tasks, the focus is on specific external regions. For example, in the non-destructive testing of cylindrical components, the bonding status of outer walls and surface defects (such as cracks and corrosion) are often the primary concerns, while internal structures may not be the target.
2. **Data truncation due to detector size limitations** [11]. In industrial applications, when the detector' s field of view is smaller than the sample size, incomplete projection data acquisition occurs, creating typical data truncation problems common in large objects and fuel rod inspections.
3. **Limited ray penetration due to high-absorption materials or complex structures** [12]. In inspections of complex structures or materials with high neutron absorption cross-sections (such as those containing lithium or boron), rays cannot penetrate the central region of the material, causing projection signals from that region to approach zero.

In these scenarios, conventional CT struggles to acquire complete projection data for full coverage. Although strategies such as offset scanning or second-generation CT can expand the field of view [13], their compensation effects are often limited by energy attenuation and data quality issues. While conventional CT can "selectively" acquire data to some extent, this approach cannot fully compensate for reconstruction defects caused by data incompleteness, often resulting in artifacts, blurring, and structural distortion. In contrast, neutron external CT technology is more targeted, with optimized scanning strategies focusing specifically on the sample' s external annular region rather than attempting full coverage. This approach leverages neutrons' superior penetration capabilities for certain materials and combines them with selective scanning to concentrate resources on data acquisition, thereby significantly improving image reconstruction quality while reducing unnecessary radiation dose and data redundancy.

1. Feature Analysis and Artifact Generation Mechanism in Neutron External CT

Neutron external CT imaging represents a typical inverse problem with incomplete projection data. Due to limited detector coverage, only edge projection information can be acquired, while critical internal structural information is missing, posing dual challenges for conventional reconstruction algorithms in terms of quality and artifact suppression [14].

From a mathematical perspective, the uniqueness of neutron external CT is manifested in three aspects:

1. **Incompleteness of projection data and ill-posedness of reconstruction** [15]. When neutron external CT projection data covers only the external region, the inverse Radon transform [16] cannot perform full integration, leading to three mathematical challenges. First, solution existence is limited because external projection data cannot uniquely map internal structures, forcing reconstruction solutions to be constrained to specific functional spaces with smoothness and rapid decay properties. Second, missing projection angles violate Hadamard' s uniqueness condition, as different internal structures may produce identical external projection data, resulting in solution multiplicity. Finally, the Radon transform as a compact operator causes rapid singular value decay, with high-frequency information loss making the reconstruction process extremely sensitive to noise. Small data perturbations are amplified during inversion, creating significant radial artifacts. Consequently, stable solution frameworks for ill-posed problems must be established through Tikhonov regularization [17] or sparse constraints.
2. **Spatiotemporal dynamic interaction mechanism of artifacts** [19]. Unlike limited-angle CT artifacts caused by global projection angle deficiency, the missing data range in neutron external CT varies dynamically with spatial position. Specifically, projection data for object edge regions is relatively complete, while data near the central region is severely missing. This non-uniform projection deficiency causes artifacts to distribute primarily along radial directions, with intensity gradually decaying as distance from the center increases. Mathematically, artifact generation can be viewed as a direct result of integral truncation in the inverse Radon transform [20], expressed as: f x y 表现为径向条纹伪影。伪影强度与中心距离 r 呈负相关，其衰减规律可通过 Bessel 函数 [21] 近似描述：2q 为有效投影角度边界。由于积分不完整，重建图像在对应缺失角度的频域方向上发生能量泄漏，J kr e a- 0J 为零阶 Bessel 函数；a 为衰减系数，反映了数据截断对高频分量的抑制作用。
3. **Microlocal analysis perspective on artifact generation mechanism**. Another key challenge in external CT is that although the reconstruction solution is theoretically unique due to projection data truncation, numerical calculation errors, data incompleteness, and noise cause "slope artifacts" [22] to frequently appear along radial edges in reconstructed images. The essence lies in the fact that the wavefront set corresponding to missing projections cannot be captured by detectors [23]. Let cW be the characteristic function of region W, with its wavefront set defined as a set describing the location and direction of function singularities: Û Î¶W Î xN is the normal vector space at point x. As shown in Figure 1 [Figure 1: see original paper], the wavefront set provides information about singular

point locations x Î ¶W where ¶W is the boundary of W, and singular directions xNzÎ.

Figure 1 Illustration of the relationship between the wavefront set of a function f and points and normal vectors on the boundary [7]. The left side shows the phantom; the right side shows the singular support of the phantom; xNzÎ denotes the normal vector at singular point x Î ¶W.

Equation (4) mathematically explains how projection data completeness affects the reconstruction of image singularities [24]: If the projection dataset contains measurements for a ray coinciding with a particular ray, then singular features in image f orthogonal to that ray (such as edges or gray-level discontinuities) can be stably reconstructed; conversely, if the singularity direction is inconsistent with ER f' s (0l, it leads to high-frequency information loss. Let f be a function defined on a compact support set, then there is a one-to-one correspondence between f and its H wavefrontset. Let 0s Î, , cos [0, 2] 1/2) wavefrontset ^ = - 0, sw be the position component, be the direction component, and the added component indicates this vector is a homogeneous coordinate; WF(0x represents the location of singular points in the image, 0v is a unit vector perpendicular to the direction (orthogonal direction), (0v^ is the projection distance of ray fa on function 0s in direction ,1 WF l 0v; (,1xv^- 0v^- is the projection on, 1 is attached to the wavefrontset in Sobolev space component H a × represents x in direction.

Quinto et al. systematically studied singularity distribution and artifact generation mechanisms in external CT reconstruction using microlocal analysis methods [25]. As shown in Figure 2 [Figure 2: see original paper], the left image is the original, and the right image is the reconstruction result. Let (, be the singular points (such as discontinuities or edges) in the image that exhibit different visibility characteristics during reconstruction: The edge direction at point A (arrow) is orthogonal to the direction of collectible projection data (solid line), so the singularity at this point is well recovered during reconstruction; however, the edge direction at point B is orthogonal to the direction of non-collectible projection data (dashed line), causing the singularity to be unstably reconstructed and diffuse to surrounding areas, forming obvious "slope artifacts."

Figure 2 Reconstruction results for the external problem: left -original image; right -reconstructed image [25]

2.1 Traditional Reconstruction Methods

Traditional reconstruction methods are primarily divided into analytical inversion and basic iterative approaches. Early research focused on projection data completion and local reconstruction strategies, but these methods exhibited deficiencies in noise robustness, artifact suppression, and computational efficiency.

Analytical Inversion Methods
The core of analytical inversion lies in directly solving the inverse problem

through mathematical transformations [26], but its application in neutron external CT is significantly limited by data incompleteness. Cormack [27] first derived the analytical inversion formula for the exterior problem in 1963, but numerical implementation suffered from severe instability, making it difficult to apply directly to practical problems. In 1964, Cormack proposed an improved algorithm based on Singular Value Decomposition (SVD) [28], which mitigated ill-posedness by decomposing singular functions of the Radon transform, laying a mathematical foundation for subsequent exterior problem research. To address data missing in external problems, Bates and Lewitt [29] proposed a projection data completion technique that projects truncated data onto the image space of the complete Radon transform and combines it with standard inversion algorithms for reconstruction.

Subsequently, in 1998, Quinto systematically constructed the singular value decomposition theory for the Radon transform in external problems [30], clearly distinguishing between the non-zero space components determined by actual acquired data and the zero-space components formed by data missing. Through SVD decomposition, projection data in external problems can be expressed as a linear combination of singular functions: ms are the corresponding singular values; m represents the index of singular values approaching zero, which corresponds to the zero space of the Radon transform; components with non-zero singular values correspond to stably reconstructible image components. Based on this theory, Quinto proposed an Exterior Reconstruction Algorithm (ERA) [31] and validated its practicality in industrial non-destructive testing. ERA' s basic process can be summarized as reconstruction of non-zero space components and bounded extrapolation of zero-space components. Using SVD decomposition, the image to be reconstructed f can be decomposed into non-zero space part range and zero-space part null range. The ERA reconstruction formula can be written as mg is the corresponding singular function in projection domain; f and zero-space part null range.

In 2006, Shi Yuanyuan' s team proposed an external problem correction algorithm based on parallel-beam filtered back-projection [32]. This algorithm designed weighting functions to compensate for high-frequency information loss caused by truncated projections and developed data extrapolation methods to extend the effective coverage of detectors, thereby achieving effective recovery of edge resolution in industrial component inspection. Subsequently, to address local feature extraction needs in external problems, Quinto [24] proposed in 2007 a method combining Lambda tomography with microlocal analysis. By applying data extrapolation and local filters, this method effectively enhanced boundary features perpendicular to detection lines, improving edge resolution. Experimental results on the Perceptics dataset demonstrated that the algorithm could significantly suppress blurring artifacts caused by limited angles, providing an effective solution for limited-angle external problems that enables fast reconstruction without complete data.

Basic Iterative Methods

Against the backdrop of filtered back-projection methods struggling with incomplete and noisy data, Gordon, Bender, and Herman [33] introduced the Algebraic Reconstruction Technique (ART) in 1970, pioneering iterative thinking in CT reconstruction. Iterative reconstruction algorithms gradually became important solutions for external CT due to their advantages in handling incomplete or noisy data [34]. Their core is based on a discretized CT imaging model, essentially solving the following linear system [35]:

where A represents the projection matrix with dimension M×N; Λ is the image to be estimated; represents the measured projection data. Within this framework, various iterative methods have different focuses: ART [36] and Simultaneous Algebraic Reconstruction Technique (SART) [37,38] achieve data consistency through pixel-by-pixel correction. Statistical iterative algorithms [39] (such as Maximum-Likelihood Expectation-Maximization, ML-EM) employ probabilistic modeling to optimize the reconstruction process, theoretically improving handling of low-dose data. In external CT, Fu et al. [40] systematically evaluated ML-EM performance under three typical data truncation scenarios (peripheral ROI, internal occlusion, and external truncation), innovatively introducing system matrix null-space mapping technology and confirming that when projections satisfy the two-dimensional Hilbert transform data sufficiency condition, peripheral ROI reconstruction error is significantly lower than in conventional interior problems, revealing the spatial correlation between system null-space distribution and reconstruction artifacts. Projection-backprojection iterative algorithms [41] utilize virtual detectors to complete missing projection data, forming a closed-loop optimization process of "reconstruction-reprojection." For external CT, Xiong Liang' s team [42] in 2007 compared three initialization strategies for virtual detector data (zero-filling, mean-filling, and linear attenuation filling) for radial data missing problems in tangential CT geometry, validating the importance of prior knowledge for missing data compensation, though reconstruction quality remained limited by accumulated high-frequency oscillation artifacts during iteration.

Although traditional iterative methods provide a basic mathematical framework for solving data incompleteness problems, they generally fail to fully utilize spatial characteristics in neutron external CT (such as radial artifact distribution and non-uniform projection angle coverage) and relevant prior information (such as edge structures and material attenuation characteristics). Consequently, researchers gradually attempted to introduce regularization constraints into the iterative optimization process to construct a collaborative reconstruction model of "data-driven + prior guidance," thereby further improving reconstruction quality.

2.2 Regularization Methods

Regularization methods transform the originally ill-posed inverse problem into a better-conditioned optimization problem by introducing prior constraints (such as smoothness, sparsity, low-rankness, etc.) into the reconstruction model, thereby stabilizing the solution and suppressing artifacts and noise. These methods demonstrate unique advantages in CT exterior problems [43].

Basic TV Regularization
Early research focused on utilizing single prior information to stabilize the solution of ill-posed problems in CT reconstruction, among which Total Variation (TV) regularization [44] was widely applied due to its effective characterization of image gradient sparsity. TV regularization can not only suppress noise while preserving image edge information but also alleviate ill-posedness caused by incomplete projection data. Sidky et al. pioneered the TV minimization model [45], whose basic idea is to balance the data fidelity term and gradient sparsity constraint term, achieving TV minimization using gradient descent methods under constraints of projection data and image non-negativity. Mathematically, it is expressed as:

where f is the image vector to be reconstructed, M is the system matrix, g is the measured projection data, and 0 if ^{3} is the image non-negativity constraint.

To efficiently solve this model, Sidky et al. proposed the TVM-POCS (Total Variation Minimization-Projection onto Convex Sets) iterative algorithm [46]. In 2013, Chen et al. applied TVM-POCS to external CT reconstruction [47], with results showing that under projection truncation conditions, the method could effectively suppress noise and reduce artifacts along the polar angle direction. However, since TV regularization relies only on first-order gradient information, the algorithm still tends to produce staircase effects in smooth regions and has limited ability to suppress gradual artifacts caused by truncated projections, making it difficult to completely eliminate progressive distortion.

High-Order and Directionally Weighted Regularization
Although TVM-POCS can effectively suppress noise and reduce polar angle artifacts in external CT problems, its reliance on first-order gradient information makes it difficult to completely eliminate radially distributed "slope artifacts." To address this, researchers introduced high-order and directionally weighted regularization, which assigns differentiated weights to derivatives of different directions or orders to further enhance artifact suppression and edge preservation capabilities. Jin et al. [48] proposed in 2010 an anisotropic total variation (ATV) reconstruction method based on parameterized gradient magnitude, which adjusts gradient weights in vertical and horizontal directions to reduce regularization constraint strength in data-missing directions, thereby avoiding excessive penalization of structures with low recoverability.

Inspired by directional constraint strength distribution, Chen et al. proposed another ATV formulation [49], which decomposes traditional isotropic TV into a weighted fusion of multi-directional one-dimensional total variations, adjusting regularization weights for each direction based on projection angle coverage. Guo et al. [50] applied ATV' s core idea to external CT problems in 2017, constructing a Weighted Directional Total Variation (WDTV) model that applies different weights in radial and tangential directions, improving image detail recovery capability in external CT reconstruction and effectively addressing the shortcomings of traditional TVM algorithms in edge preservation. Lin Qiang’ s team [5] further enhanced robustness to noise and incomplete projections by introducing multi-directional difference combinations and neighborhood grayscale equilibrium mechanisms based on the above model. Experiments demonstrated that this improved model exhibited excellent artifact suppression capability and edge preservation characteristics in cold neutron gear detection experiments, validating the applicability of directional weighting strategies in complex imaging scenarios. The Root Mean Square Error (RMSE) comparison of different methods is shown in Figure 3 [Figure 3: see original paper], and local magnified reconstruction results from different algorithms are shown in Figure 4 [Figure 4: see original paper].

Figure 3 RMSE curves obtained using different reconstruction algorithms [5]

Figure 4 Local magnified views of reconstructed images using different algorithms [5]: (a) signature; (b) FBP algorithm; (c) SART-TV algorithm; (d) WDTV algorithm; (e) Improved WDTV algorithm

To further address noise and artifact problems, Deng et al. [51] proposed a model using generalized shrinkage operators as regularization terms based on visible and invisible boundary theory, which effectively suppresses noise and artifacts by using “visible edges” as anchors to restore image structure. Xu et al. proposed the Alternating Edge-preserving Diffusion and Smoothing (AEDS) algorithm [52], which effectively improves reconstruction quality by introducing horizontal and vertical regularization terms. In external CT, this model adjusts direction-aware regularization weights to prioritize edge recovery consistent with projection direction tangents. Building on this, Shi and Qu et al. [53] innovatively combined weighted preprocessing methods with AEDS for ultra-limited-angle imaging problems with scanning angle ranges less than $\pi/2$, proposing a Reweighted AEDS (Rw-AEDS) reconstruction model based on condition number optimization. In 2018, Xu et al. [54] extended AEDS to polar coordinates and proposed a Polar-AEDS algorithm, which alternately applies “edge-preserving diffusion” and “smoothing” constraints in radial and tangential directions. This model alleviates “curved edge effects” through polar coordinate transformation, demonstrating advantages in pipeline wall defect detection. In 2023, Qin et al. [55] introduced a polychromatic X-ray projection model based on P-AEDS, jointly suppressing external artifacts and metal artifacts through Newton linearization of nonlinear beam hardening equations.

In limited-angle CT reconstruction, projection data truncation and limited scanning angles lead to edge information loss and structural reconstruction defects, which are also the core challenges of external CT. To address these challenges, researchers have gradually proposed improvements from perspectives of anisotropic constraints and direction-adaptive optimization. For example, Zeng et al. [56] proposed a new reconstruction method based on Anisotropic Relative Total Variation (ARTV), whose core idea is to dynamically balance gradient penalty strengths in different directions by adaptively utilizing local structural information, thereby suppressing artifacts while improving image quality and edge recovery. To further enhance directional awareness, Gong et al. [57] introduced an adaptive weighting mechanism based on ARTV, combining limited-angle imaging geometry with image directional features to propose the AWATV (Adaptive Weighted Anisotropic Total Variation) model. This model dynamically adjusts gradient weights in different directions by analyzing the correlation between projection angle distribution and image gradient direction, thereby more accurately preserving anisotropic edges and achieving significant improvements in both artifact suppression and structure restoration. To adapt to the special geometric characteristics of external CT problems, Guo et al. [58] extended ARTV to polar coordinates, proposing the P-ARTV (Polar-coordinate Anisotropic Regularized Total Variation) model. This model innovatively applies differentiated weights in radial and tangential directions, strengthening radiation-direction edge preservation capability while suppressing tangential artifact diffusion through polar coordinate transformation. Experimental results show that compared with existing TV, WDTV, and RTV algorithms, this method has significant advantages in edge preservation and artifact suppression, as shown in Figures 5 [Figure 5: see original paper] and 6 [Figure 6: see original paper].

Figure 5 Reconstruction results of the cracked pipeline model under Poisson noise [58]

Figure 6 Seven-magnification of the region of interest in Figure 5 [58]

Additionally, Gong et al. [59] reduced sampling data correlation and improved artifact suppression and structural fidelity by introducing a Segmented Limited-Angle (SLA) sampling strategy and image structural priors. Similarly, Gong and Liu et al. [60] constructed a new limited-angle CT reconstruction model based on Weighted Relative Structure (WRS) determined by image gradients, and further reduced artifacts while improving structural recovery performance through efficient optimization algorithms. Effectively utilizing prior information to alleviate artifacts caused by data missing is a key challenge. Gong and Song et al. [61] innovatively solved the problem of inconsistency between prior and target image structures through a coupled relative structure method with bidirectional structural constraints. This method introduces mutual guidance between prior and target images in the regularization term, forcing them to converge to a common structure during iteration while preserving target-specific features through projection data fidelity.

Sparse Representation and Dictionary Learning

In external CT, sparse representation and dictionary learning provide another approach to compensating for incomplete projections and suppressing artifacts. Early research focused on gradient or direction-based regularization but still had insufficient suppression of gradual artifacts. To address this, Shen et al. proposed an optimization model based on Wavelet Tight Frame (WTF) and 0l quasi-norm regularization [62], which utilizes the multi-resolution characteristics of wavelet transforms to capture image details and enhances coefficient sparsity through 0l norm constraints, thereby achieving effective suppression of gradual artifacts and edge enhancement in external CT reconstruction. It can be expressed in the following mathematical form:

0l WTF arg min

where f is the image vector to be reconstructed, A is the system matrix, p is the measured projection data vector, W is the wavelet tight frame transform, il is the regularization parameter, and g is the quadratic stabilization term coefficient.

Building on the above transform domain sparse model, Zeng et al. [63] further proposed using image gradient 1l norm as a regularization term for limited-angle reconstruction and proved that 0l norm has better performance in edge preservation [64]. In recent years, Gong et al. [65] further proposed an anisotropic self-guided filtering method for orthogonal limited-angle scanning scenarios, reducing projection data correlation through a dual-orthogonal scanning strategy and innovatively constructing a direction-adaptive gradient weight function: setting lower constraint weights in directions perpendicular to scanning angle distribution to protect real edges, while applying strong regularization in artifact diffusion directions to suppress stripe artifacts. On the other hand, to address simultaneous projection truncation and limited-angle artifact problems, Cao et al. [66] introduced dictionary learning regularization into external CT reconstruction and proposed an analytical-iterative hybrid framework: first using the ATRACT algorithm for two-dimensional filtering and extrapolation of truncated projections to obtain an initial image; then adopting a dual dictionary learning strategy to alternately optimize the image and two over-complete dictionaries during iteration—one learning real structural features and the other learning limited-angle artifact patterns—making sparse representation and reconstruction mutual priors. Simulation experiments show that this hybrid method outperforms single regularization in suppressing both truncation and limited-angle artifacts, particularly in outer ring crack and thin-layer structure recovery.

Mixed Regularization Frameworks

To address reconstruction difficulties caused by the superposition of projection truncation and artifacts in external CT, researchers combined global regularization with local segmentation strategies to form multi-stage hybrid frameworks. Specifically, Liu Baodong' s team [67] pioneered a Subregion Averaged TV Minimization (SA-TVM) algorithm framework for truncated projection problems. This algorithm achieves image reconstruction through three-stage iteration: first using Simultaneous Algebraic Reconstruction Technique (SART) for initial reconstruction and pixel value range constraints; second performing total variation minimization to improve global smoothness; and finally introducing a segmentation strategy based on the Chan-Vese (C-V) active contour model to divide the image into homogeneous subregions and perform regional grayscale mean correction, thereby achieving clear boundary recovery. Zeng Li et al. successfully applied this algorithm to fan-beam external CT reconstruction, achieving clear boundary reconstruction through precise identification of uniform regions using the C-V model [68]. The research team subsequently extended the SA-

TVM framework to cone-beam and helical scanning CT systems, effectively suppressing artifacts under complex scanning trajectories through improved three-dimensional segmentation methods [69]. Meanwhile, to overcome the limitations of the traditional C-V model in regions with uneven grayscale, Guo Yumeng et al. further proposed the Region-Scalable Fitting (RSF) model [70], which optimizes region division through adaptive weighting functions and demonstrates good performance in suppressing gradual artifacts and beam hardening artifacts. To address the core bottleneck of regularization parameter dependence on manual experience adjustment in multi-stage optimization, Wang' s team [71] proposed an interactive limited-angle CT reconstruction method based on multi-objective optimization, using Structural Similarity Index (SSIM) as a value function to guide optimization direction and dynamically balancing conflicting objectives of artifact suppression and structure preservation through simulated annealing algorithms, achieving adaptive parameter control. Its SSIM-driven mechanism significantly improves consistency between reconstructed images and human visual characteristics.

Regularization methods systematically alleviate high-frequency loss, noise amplification, and artifact problems caused by incomplete projection data in neutron external CT by introducing prior information such as gradient sparsity, transform domain priors, and anisotropic constraints. Starting from basic TV regularization, research has progressively built more robust models by suppressing staircase effects through high-order derivatives, optimizing edge preservation through directional weighting, adapting geometric characteristics through polar coordinate transformation, and compensating data missing through sparse representation. Mixed regularization frameworks further integrate global optimization with local segmentation strategies, improving complex structure reconstruction quality through multi-stage collaboration. However, existing methods are still limited by empirical parameter dependence, insufficient nonlinear projection modeling, and computational efficiency bottlenecks. Future work needs to integrate physical mechanisms with intelligent optimization theory to achieve breakthroughs in both accuracy and efficiency for ill-posed reconstruction.

3. Summary and Outlook

This review systematically examines the physical causes, mathematical modeling challenges, and recent algorithmic research progress in neutron CT exterior reconstruction problems. Under conditions of limited imaging geometry and spatial projection data truncation, external CT reconstruction still faces severe structural information loss and artifact interference, which is particularly pronounced in neutron imaging due to lower signal-to-noise ratios. In recent years, researchers have conducted in-depth explorations in regularization modeling, polar coordinate optimization, and structural prior fusion. Methods such as Anisotropic Total Variation (ATV), Weighted Directional Total Variation (WDTV), Polar-coordinate Anisotropic Regularized Total Variation (P-ARTV), and dictionary learning have achieved significant progress in improving edge preservation and artifact suppression. Additionally, structure-prior-guided hybrid frameworks and limited-angle optimization strategies have also demonstrated strong adaptability and expansion potential. Nevertheless, neutron CT exterior reconstruction still faces three key challenges:

1. **Insufficient modeling of projection nonlinear errors**: Existing linear attenuation-based models struggle to accurately fit the projection process due to physical effects such as scattering and neutron moderation.

2. **Regularization parameter dependence on empirical adjustment**: Current methods mostly rely on manual setting of regularization weights, lacking a unified adaptive parameter control mechanism that can cause over-smoothing or structural bias.

3. **Difficulty in real-time deployment of high-quality methods**: Many high-precision models (such as iterative-optimization coupling types) have high computational complexity, limiting their widespread application in industrial real-time inspection scenarios.

Future research can prioritize exploration and breakthroughs in two directions: First, **regularization model construction based on physical mechanisms**. Current regularization terms are mostly mathematical prior constraints that do not fully utilize physical characteristics of neutron scattering and absorption in materials. Explicitly introducing physical models of neutron propagation processes into regularization terms can more accurately characterize the relationship between object structural features and projection data, improving physical consistency and adaptability of reconstruction models. Second, **deep learning-assisted parameter estimation and artifact suppression**. Regularization methods often rely on fine-tuning of multiple hyper-parameters in application, while traditional parameter tuning is inefficient and lacks universality. With deep neural networks, predictive models for automatic regularization parameter estimation can be trained on large amounts of simulated data, improving algorithm intelligence levels. Simultaneously, networks can be used to learn distribution characteristics of projection artifacts, enabling joint optimization of artifact suppression and structure enhancement during iterative reconstruction, thereby improving reconstruction quality in complex boundary regions. With continuous improvements in neutron sources, detector precision, and algorithmic capabilities, neutron CT exterior reconstruction technology will play an increasingly important role in aerospace, high-end equipment, nuclear engineering, and new energy material evaluation, providing efficient and precise solutions for non-destructive testing of complex structures.

Author Contributions

WANG Qibiao proposed the writing ideas and performed the main writing work; ZHAN Pengfei and YANG Hongchao were responsible for writing and revision; LI Pengcheng and LIANG Guohua conducted literature research and classification; DENG Chao and XIE Bo performed in-depth literature analysis and theoretical context 梳理; YANG Jianbo and WU Yadong extracted key viewpoints; TUO Xianguo was responsible for article structure arrangement.

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