Demand-Information-Policy Synergy Chain: A Study on Development Strategies for Driving the Silver Economy through the Smart Elderly Care Industry from a Four-Party Game Perspective

Authors: Li Zhetong, Peng Xiaoxian, Zhou Siyi, Li Zhetong

Date: 2025-05-17T00:00:00+00:00

Abstract

As China's population aged 60 and above exceeds 290 million, smart elderly care, as an important component of the silver economy, faces the practical dilemma of "hot policy, cold market, and technology spinning in vain." To investigate the impact of multi-agent collaborative mechanisms on the evolution of smart elderly care systems, this paper constructs a four-party evolutionary game model involving government, enterprises, platforms, and the elderly, integrating system dynamics methods to explore how multi-agent strategic games influence the evolution of the silver economy system. The model establishes key variables including government subsidy intensity, enterprise pricing strategy, platform data openness, and elderly acceptance willingness based on payoff-cost functions and replicator dynamic equations, quantifies behavioral evolution mechanisms, and employs Jacobian matrix analysis to examine strategic stability, thereby identifying strategic critical points and system stability conditions. Utilizing the Vensim platform, this paper designs six typical scenarios to simulate system evolution processes under different combinations of subsidy, pricing, and openness strategies. Simulation results demonstrate that platform openness and elderly acceptance constitute key feedback paths for system co-evolution; enterprise behavior is jointly influenced by platform incentives and user feedback, while the effectiveness of government subsidies depends on strategic alignment with market response. Under scenarios of elderly non-acceptance, system evolution exhibits strategic drift and non-equilibrium states, reflecting the core role of elderly group game decisions in system stability. Based on these findings, this paper proposes a "three-chain collaborative mechanism" and a "strategic response indicator system," providing quantitative foundations and policy references for optimizing multi-agent strategies in smart elderly care and promoting high-quality development of the silver economy.

Full Text

Demand-Information-Policy Collaboration Chain: A Study on Development Strategies for the Silver Economy Driven by the Smart Elderly Care Industry from a Four-Party Game Perspective

Li Zhetong[1],*, Peng Xiaoxian[1], Zhou Siyi[1]
[1](School of Mathematics and Statistics, Hunan University of Finance and Economics, Changsha 410205)

Abstract: As China' s population aged 60 and above exceeds 290 million, smart elderly care—an essential component of the silver economy—faces a practical dilemma of "hot policies, cold markets, and idle technology." To investigate how multi-agent collaboration mechanisms influence the evolution of smart elderly care systems, this paper constructs a four-party evolutionary game model involving government, enterprises, platforms, and the elderly, integrating system dynamics methods to explore the impact of multi-agent strategic gaming on silver economy system evolution. Based on benefit-cost functions and replicator dynamic equations, the model establishes key variables including government subsidy intensity, enterprise pricing strategies, platform data openness, and elderly acceptance willingness, quantifies behavioral evolution mechanisms, and employs Jacobian matrix analysis to examine strategy stability, identifying critical points and system stability conditions.

Using the Vensim platform, this paper designs six typical scenarios to simulate system evolution under different combinations of subsidies, pricing, and openness strategies. Simulation results reveal that platform openness and elderly acceptance constitute the key feedback path for system co-evolution; enterprise behavior is influenced by the linkage between platform incentives and user feedback, while government subsidy effectiveness depends on strategic alignment with market response. Under scenarios of elderly non-acceptance, system evolution exhibits strategy drift and non-equilibrium states, reflecting the core role of elderly group gaming decisions in system stability.

Based on these findings, this paper proposes a "three-chain collaboration mechanism" and a "strategic response indicator system," providing quantitative foundations and policy references for optimizing multi-agent strategies in smart elderly care and promoting high-quality silver economy development.

Keywords: Silver economy; Smart elderly care; Four-party game; Evolutionary game strategy; System dynamics

1. Introduction

As China' s population aging accelerates, the number of people aged 60 and above has surpassed 290 million, accounting for over 20% of the total population. This structural shift has spawned a vast silver economy market, with smart elderly care—empowering elderly services through information technology —viewed as a critical pathway to solving elderly care challenges and stimulating new economic momentum. However, current smart elderly care development faces a practical dilemma of "hot policies, cold markets, and idle technology" [1]: despite substantial policy support, enterprise participation willingness remains insufficient, technology applications misalign with elderly group needs, resulting in inefficient service supply and underutilized silver economy potential.

Existing research predominantly focuses on technological iteration or policy framework design for smart elderly care, yet exhibits significant limitations: First, insufficient attention to heterogeneous elderly needs and the dynamic impact of decision-making behaviors leads to ineffective supply-demand matching mechanisms. Second, silver economy research overemphasizes macro-scale estimation while lacking micro-level analysis of multi-agent collaborative evolution mechanisms. Third, game models are often confined to three agents or complete rationality assumptions, struggling to characterize complex interactions among government, enterprises, platforms, and elderly populations under information asymmetry. These theoretical gaps render existing strategies unable to resolve the governance dilemma of "growth without 获得感 (sense of gain)."

To address this, this paper innovatively constructs a "Demand-Information-Policy Collaboration Chain"framework [2], introducing a four-party evolutionary game model and system dynamics simulation methods to break traditional research paradigms. First, it integrates government subsidies, enterprise pricing, platform data openness, and elderly acceptance strategies to construct a dynamic game model under asymmetric information. Second, through differential equations and Jacobian matrix analysis, it examines strategy and strategy combination stability, revealing path dependence and critical conditions for multi-agent co-evolution. Third, via Vensim platform-based six-scenario simulation, it dynamically models the interaction mechanisms among policy intervention, market response, and technology empowerment, extracting leverage effects of key variables.

This study not only provides a theoretical model of "technology empowerment-policy incentive-market response" collaboration mechanisms for smart elderly care driving the silver economy, but also proposes actionable policy combinations such as "tiered subsidies" and "data openness incentives" to transform smart elderly care from "technology islands" to "ecological co-construction." The conclusions offer a China solution with both academic value and practical significance for aging society governance, facilitating the leap from "providing for the elderly" to "optimally providing for the elderly."

2. Literature Review

2.1 Smart Elderly Care Research

Smart elderly care, as a crucial strategy addressing population aging, has garnered widespread attention in recent years. By applying information technology and smart devices, smart elderly care aims to improve elderly quality of life and health levels. Zhang Chenggang et al. [1] propose that smart elderly care implementation can be achieved through dynamic interactions among technology, process, organization, and environment components. Chen Si et al. [3] further explore characteristics, connotation systems, and implementation paths for smart elderly care data intelligence transformation, emphasizing the strategic value of data elements in smart elderly care services.

Li Shanman et al. [4] analyze smart elderly care service contextualization from actor-network theory perspective, noting that logical pathways for smart elderly care services require multi-dimensional interactive service scenarios, structured data activities, and heterogeneous coupled actors as foundational architecture. These studies indicate that smart elderly care involves not only technology application but also policy support, market demand, and social participation. Smart elderly care service development requires interdisciplinary and cross-domain cooperation to achieve service optimization and innovation.

2.2 Silver Economy Research

The silver economy refers to economic activities targeting elderly populations, gradually becoming new economic development momentum as population aging intensifies. Zhang Yingxi et al. [5] explore the silver economy' s connotation, characteristics, and its role in promoting high-quality economic development, emphasizing the importance of government-market-technology synergy in driving silver economy development.

Zhu Chunhua et al. [6] analyze smart elderly care industry development from financial support perspective, identifying financial support as a key factor. Li Xiaotian [7] proposes that supporting high-quality silver economy development requires full-chain, full-cycle support for the silver economy including elderly care industries and services, systematically promoting industrial transformation and upgrading. Therefore, silver economy development can not only meet elderly needs but also promote related industries, driving economic structure optimization and upgrading. Its development requires joint action from policy guidance, market mechanisms, and technological innovation.

2.3 Game Models and Dynamic Simulation Research

Four-party game models analyze interactive relationships among multiple participants and have been widely applied in smart elderly care service system research. Sun Shiwei et al. [8] construct an integrated evolutionary game model covering government agencies, smart elderly care service providers, digital platform operators, and elderly user groups, revealing cooperation mechanisms among four parties in smart elderly care services through in-depth analysis of interaction dynamics and strategy selection.

Wang Yicheng et al. [9] examine smart elderly care information sharing evolution strategies from a three-party game perspective, analyzing dynamic replication processes. Hu Limi et al. [10] further explore four-party game evolution in elderly care issues under bounded rationality, using MATLAB simulation to analyze how different factors affect strategy selection among game participants.

These studies demonstrate that four-party game models and dynamic simulation are effective tools for analyzing multi-agent interactions in smart elderly care service systems, helping understand strategy selection and behavioral patterns, and providing theoretical support for smart elderly care service system optimization and innovation.

2.4 Literature Critique

In summary, existing literature exhibits significant limitations in systematic integration and strategy innovation: Smart elderly care research focuses on technological iteration and policy framework design but weakly analyzes differentiated elderly needs and social participation mechanisms, causing supply-demand misalignment. Silver economy research overemphasizes industrial scale measurement and economic benefit evaluation while insufficiently addressing elderly quality of life improvement and sustainable consumption market cultivation, failing to resolve the "growth without 获得感" dilemma. Game model construction is often limited to three agents (government, enterprise, family) and based on complete rationality assumptions, neglecting dynamic characteristics of information asymmetry and strategy evolution, thus limiting model explanatory power and practical guidance.

This paper breaks traditional research paradigms by innovatively constructing a "Demand-Technology-Policy Collaboration Chain" framework, deeply integrating four-party game models with system dynamics to propose systematic strategies for smart elderly care driving the silver economy. Distinguishing from traditional perspectives treating elderly groups as passive recipients, this paper incorporates them as independent game agents into the evolutionary model, directly quantifying how their demand preferences transmit effects to the silver economy through strategy sets. This "demand-embedded" mechanism compensates for existing research' s neglect of micro-level individual decision-making, providing theoretical foundations for precise supply-demand matching.

Addressing traditional game models' limitations in simplifying agent interactions, this paper 首次 integrates government, smart elderly care enterprises, digital platforms, and elderly populations as four agents, constructing an evolutionary game model under asymmetric information. It innovatively proposes a "technology empowerment-policy incentive-market response" collaboration chain. Theoretically, applying evolutionary game theory and system dynamics to the silver economy domain, it reveals leverage effects of key variables through parameter sensitivity analysis, filling methodological gaps between dynamic simulation and policy design. Practically, it proposes policy combinations such as "tiered subsidies," "data openness incentives," and "inclusive pricing guidance" to transform smart elderly care from "technology islands" to "ecological co-construction," providing an actionable China solution for aging society governance.

---

3. Four-Party Game Agent Analysis

3.1 Government: Policy Formulator and Incentive Mechanism Guide

In smart elderly care service systems, government plays the role of policy formulator and incentive mechanism guide. Its high or low subsidy strategies directly affect enterprise pricing and platform openness. For example, high subsidy policies can incentivize enterprises to reduce service prices, improve elderly acceptance, and encourage platforms to open data, enhancing service transparency. However, government subsidy policies are also constrained by fiscal pressure and regulatory costs, requiring balance between incentivizing market participation and controlling fiscal expenditure. Additionally, elderly satisfaction and acceptance 反向 influence government policy adjustments, forming dynamic feedback mechanisms. Sun Shiwei et al. (2023) note that government regulates enterprise behavior through policy formulation and supervision mechanisms, while smart elderly care service enterprises directly affect service quality and elderly group satisfaction [11].

3.2 Enterprise: Service Supplier and Market Response Regulator

As primary suppliers of smart elderly care services, enterprise strategy choices are significantly influenced by government policies, platform behavior, and elderly demand. Enterprise pricing strategies must balance profit maximization and market penetration. High pricing may generate higher profits but could reduce elderly acceptance, affecting government subsidy policies and platform cooperation willingness. Conversely, low pricing may increase market share, obtain more government subsidies and platform support, but may compress profit margins. Therefore, enterprises need to dynamically adjust strategies based on government subsidy policies, platform data openness, and elderly demand changes to achieve long-term sustainable development.

3.3 Platform: Information Hub and Collaboration Efficiency Promoter

Platforms serve as information hubs in smart elderly care service systems, connecting government, enterprises, and the elderly. Their strategy choices—such as open data or protected data—directly affect service collaboration efficiency and elderly trust. Open data can improve service transparency and collaboration efficiency, enhancing elderly trust, but may increase data security and privacy protection costs. Protecting data can maintain platform competitive advantages but may reduce service collaboration efficiency and elderly satisfaction. Platform strategy choices are influenced by government regulatory policies, enterprise cooperation willingness, and elderly demand feedback, requiring balance between data openness and protection to promote efficient system operation.

3.4 Elderly: Service Acceptance and Feedback Mechanism Key Participants

As end-users of smart elderly care services, elderly strategy choices significantly impact overall system operation. Elderly acceptance is influenced by multiple factors including service price, data security, service quality, and government subsidies. High acceptance can promote enterprise service supply and platform data openness, enhancing government policy effectiveness; low acceptance may reduce service supply and policy effectiveness. Therefore, elderly behavior is not only a reaction to the service system but also feedback to government, enterprise, and platform strategies, forming a bottom-up feedback mechanism that drives overall system optimization and adjustment.

The interaction mechanism among four agents presents as a dependent feedback network. First, government-enterprise feedback manifests as policy environment affecting enterprise behavior: government incentives such as subsidies or honor recognition increase enterprise enthusiasm for smart elderly care service investment; conversely, inadequate supervision or incentives may lead enterprises to adopt conservative strategies, causing insufficient service supply. Enterprise performance (e.g., service quality and coverage) 反过来 influences government decisions, prompting dynamic adjustments to supervision intensity or subsidy policies for strategy optimization. Second, government-platform interactions connect through data and supervision: government departments often rely on smart elderly care platforms for real-time data feedback to monitor service quality and elderly participation; platforms depend on government-established data standards and security norms for legal and compliant operation.

Evidently, strategy evolution exhibits path-dependent characteristics: one agent's strategy choices and payoff outcomes affect other agents' expectations and next-stage strategies through the platform hub, gradually converging toward dynamic equilibrium.

National policy attaches great importance to multi-agent collaboration in smart elderly care. The *Smart Health and Elderly Care Industry Development Action Plan (2021-2025)* explicitly proposes a "government guidance, multi-party linkage" collaboration approach [12], providing policy foundations for positioning government, enterprise, platform, and user gaming in the model. Practically, domestic provinces and cities have built smart elderly care platforms to integrate resources. For example, Taiyuan's smart elderly care digital platform has integrated over 500 community home-based elderly care service providers, covering hundreds of thousands of elderly people, and aggregated massive data on empty-nest and solitary elderly individuals along with sensor monitoring information, initially forming a data-driven elderly care service new model [13]. Authoritative research also indicates that developing smart elderly care must fully mobilize government, enterprise, and social entities' enthusiasm to achieve efficient resource allocation and information sharing through digital platforms, thereby meeting growing elderly care service demands [14].

---

4. Four-Party Game Analysis Among Government, Enterprise, Platform, and Elderly Population

[Figure 1: see original paper] Four-Party Game Model Logic Relationship Diagram

4.1 Model Basic Assumptions

Strategy choices among government, smart elderly care enterprises, digital platforms, and elderly populations can be viewed as dynamic gaming processes among four agents. However, differing from complete rationality characteristics in traditional game theory, real-world decisions are influenced by multiple practical conditions. Therefore, this paper' s four-party game is an evolutionary game where participants have incomplete information symmetry and make decisions under multi-factor influences that are not completely rational. Thus, this paper' s four-party game theory focuses on studying the optimal equilibrium point for co-creating the silver economy driven by smart elderly care. Based on this context, model basic assumptions are as follows:

Assumption 1: In this study, game agents are government (G), smart elderly care enterprise (E), digital information platform (T), and elderly group (O), all being information-asymmetric, boundedly rational game participants.

Assumption 2: Government strategy set is {high subsidy, low subsidy}. High subsidy means government participates in smart elderly care service systems, implementing high subsidy policies with fiscal intervention when necessary; low subsidy means government implements loose management systems, basically allowing market self-regulation with minimal subsidies when necessary. When government implements high subsidy policies, the fiscal risk borne is R_g.

Assumption 3: Smart elderly care enterprise strategy set is {high pricing, low pricing}. When implementing high pricing, enterprises obtain excess profit P_{he} but lose market, dissemination strength, and acceptance degree V_{he}; when implementing low pricing, enterprises lose partial profit P_{le} but gain additional market, enhancing dissemination strength and acceptance degree V_{le}. When enterprises implement low pricing and government provides high subsidy, they receive government fiscal subsidy S_e; when enterprises implement high pricing and government provides low subsidy, they face government fiscal regulation

F_e. When high pricing occurs, the partial silver economy market lost due to elderly non-acceptance is V_{ho}; when low pricing occurs, the sustained silver consumption market value obtained from elderly acceptance is V_{lo}, with the assumption $V_{lo} \geq V_{ho}$, otherwise enterprises would prefer "one-time" business, with choices trending toward high pricing.

Assumption 4: Digital platform strategy choices are {open data, protect data}. Open data means the platform provides information support to smart elderly care enterprises, selection assistance to elderly populations, and data assistance to government for policy formulation. When platforms protect data, they only provide necessary information to enterprises. When platforms choose open data and government provides high subsidy, platforms receive extra subsidy S_{ht}; when platforms choose open data and government provides low subsidy, they only receive basic subsidy S_{mt}; when platforms choose protect data and government provides high subsidy, government does not intervene, only receiving enterprise funding S_{lt}. The assumption $S_{ht} > S_{lt}$ exists, otherwise platform open data probability would approach 0. If platforms protect data and government provides low subsidy, they pay extra tax F_t. When opening data, the extra economic benefit brought to platforms by elderly acceptance is V_t.

Assumption 5: Elderly population strategy choices are {accept, not accept}. Elderly acceptance of smart elderly care products yields product potential value V_o. Under high government subsidy, elderly acceptance yields extra income C_o. Both digital platform open data and enterprise low pricing increase elderly acceptance decision probability. The assumption $C_o \geq 0$ holds, otherwise elderly acceptance probability would approach 0.

Assumption 6: Government implements high subsidy policy with probability x ($0 \leq x \leq 1$), thus low subsidy policy probability is $1-x$. Smart elderly care enterprise implements high pricing strategy with probability y ($0 \leq y \leq 1$), thus low pricing policy probability is $1-y$. Digital platform opens information with probability z ($0 \leq z \leq 1$), thus information protection probability is $1-z$. Elderly population accepts with probability v ($0 \leq v \leq 1$), thus non-acceptance probability is $1-v$.

4.2 Four-Party Game System Dynamics Model Establishment

Based on model assumptions and parameter settings, Table 1 presents the payoff matrix for strategy combinations after four-agent strategy selection, and Table 2 explains parameter settings and meanings for each game agent.

Four-Party Game Strategy Combination Payoff Matrix
Parameter Settings and Meanings for Each Game Agent

4.3 Strategy Stability Analysis for Four-Party Game Agents

4.3.1 Government Strategy Stability Analysis When government in the four-party game agents chooses "high subsidy," let decision expected payoff be

E_{x1}:

$$E_{x1} = v[yz(I_{gh}-C_{gh}-R_g-S_{ht}-C_o)+y(1-z)(I_{gh}-C_{gh}-R_g-C_o)+(1-y)z(I_{gh}-C_{gh}-R_g-S_e-S_{ht}-C_o)+(1-y)($$

When government chooses "low subsidy," let decision expected payoff be E_{x2}:

$$E_{x2} = yz(I_{gl}-C_{gl}+F_e-S_{mt})+y(1-z)(I_{gl}-C_{gl}+F_e+F_t)+(1-y)z(I_{gl}-C_{gl}-S_{mt})+(1-y)(1-z)(I_{gl}-C_{gl}+F_t)$$

From equations (1) and (2), government average expected payoff E_x is:

$$E_x = xE_{x1} + (1-x)E_{x2}$$

Following calculation methods in *Evolutionary Game Theory*, the replicator dynamic differential equation $F(x)$ for government strategy selection when encouraging smart elderly care development is:

$$F(x) = \frac{dx}{dt} = x(1-x)(E_{x1} - E_{x2})$$

By replicator dynamic differential equation stability principle, government game decision stability requires $F(x) = 0$ and $F'(x) < 0$. According to evolutionary stable strategy properties, evolutionary stable strategies are truly stable steady states resistant to interference—replicator dynamic equations restore stability even after accidental deviations by some game participants [15].

Proposition 1 (Proof in Appendix 1): When $y < y^*$, government choosing "low subsidy" is a robust strategy; when $y > y^*$, government choosing "high subsidy" is a robust strategy; when $y = y^*$, government robust strategy cannot be determined, where:

$$y^* = \frac{C_{gl} - F_t + I_{gh} - I_{gl} - R_g - S_e - S_{ht} - C_o v}{F_e - S_e + F_t - S_{mt}}$$

According to Proposition 1, in the dynamic process where smart elderly care enterprises make pricing decisions and digital platforms make data openness decisions, if enterprises choose low pricing, government gaming decisions will tend toward high subsidies to encourage enterprise development, promote technological innovation, and stimulate silver economy growth. If enterprises tend toward high pricing, considering that profit-seeking nature may harm consumer rights, government decisions will tend toward low subsidy policies to change this situation.

Further analysis of Proposition 1 shows that in Figure 2 [Figure 2: see original paper], C_{gl} is negatively correlated with y^*. When government low subsidy policy costs increase, y^* decreases, reducing the decision probability space for $y < y^*$ and pushing government decisions toward high subsidies. Similarly, I_{gh} is negatively correlated with y^*, while F_e, S_e, F_t, S_{mt}, C_o, and v are positively correlated with y^*. When other conditions remain unchanged, increases in I_{gh}, S_e, decreases in F_t, C_{gl}, R_g, S_e, and F_e all reduce y^*, shrinking the $y < y^*$ decision probability space and driving government decisions toward high subsidies.

[Figure 2: see original paper] Government Dynamic Game Decision Stability Evolution Diagram

4.3.2 Smart Elderly Care Enterprise Strategy Stability Analysis

When smart elderly care enterprise in four-party game agents chooses "high pricing," let decision expected payoff be E_{y1}:

$$E_{y1} = v[xz(V_e-C_{eh}+P_{he}-V_{he}-S_{ht})+x(1-z)(V_e-C_{eh}+P_{he}-V_{he}-S_{lt})+(1-x)z(V_e-C_{eh}+P_{he}-V_{he}-F_e-S_{mt})+$$

When enterprise chooses "low pricing," let decision expected payoff be E_{y2}:

$$E_{y2} = v[xz(V_e-C_{el}-P_{le}+V_{le}+S_e+V_{lo}-S_{ht})+x(1-z)(V_e-C_{el}-P_{le}+V_{le}+S_e+V_{lo}-S_{lt})+(1-x)z(V_e-C_{el}-P_{le}+V_l$$

From equations (6) and (7), smart elderly care enterprise average expected payoff E_y is:

$$E_y = yE_{y1} + (1-y)E_{y2}$$

From equations (6) and (7), the replicator dynamic equation $F(y)$ for enterprise pricing strategy selection is:

$$F(y) = \frac{dy}{dt} = y(1-y)(E_{y1} - E_{y2})$$

By stability principle, enterprise game decision stability requires $F(y) = 0$ and $F'(y) < 0$.

Proposition 2 (Proof in Appendix 2): When $v < v_1^*$, enterprise choosing "low pricing" is a robust strategy; when $v > v_1^*$, enterprise choosing "high pricing" is a robust strategy; when $v = v_1^*$, enterprise robust strategy cannot be determined, where:

$$v_1^* = \frac{C_{eh} - C_{el} + F_e - P_{he} - P_{le} + V_{he} + V_{ho} - V_{le} - F_e x + S_e x - V_{ho} z + V_{lo} z}{V_{lo} xz - V_{oxz} + V_{ho}\omega - V_o\omega - V_{lo} x\omega + V_{ox}\omega - V_{ho} z\omega + V_{oz}\omega}$$

According to Proposition 2, in dynamic decision processes among government, digital platforms, and elderly populations, if elderly groups choose non-acceptance, smart elderly care enterprises considering government policy orientation and long-term development will tend toward low pricing decisions. If elderly groups tend toward acceptance, enterprises will gradually increase pricing driven by "greed" principles seeking greater profits, pushing gaming decisions toward high pricing.

According to the smart elderly care enterprise decision phase diagram (Figure 3 [Figure 3: see original paper]) and equation (10), when other conditions remain unchanged, C_{eh} is positively correlated with $v_1^*(x,z)$. When high pricing costs increase for smart elderly care enterprises, $v_1^*(x,z)$ increases, expanding the $v < v_1^*$ decision probability space and pushing enterprise decisions toward low pricing. F_e, S_e, V_{le}, V_{lo} are also positively correlated with $v_1^*(x,z)$, while P_{he}, P_{le}, V_{he} are negatively correlated. When positive correlation indicators increase and negative correlation indicators decrease under unchanged conditions, the $v < v_1^*$ decision probability space expands, driving enterprise decisions toward low pricing.

[Figure 3: see original paper] Smart Elderly Care Enterprise Dynamic Game Decision Stability Evolution Diagram

4.3.3 Digital Platform Strategy Stability Analysis When digital platform in four-party game agents chooses "open data," let decision expected payoff be E_{z1}:

$$E_{z1} = x\omega(I_{to}-C_{to}+S_{ht}+V_t)+x(1-\omega)(I_{to}-C_{to}+S_{ht})+(1-x)\omega(I_{to}-C_{to}+S_{mt}+V_t)+(1-x)(1-\omega)(I_{to}-C_{to}+S_{mt})$$

When platform chooses "protect data," decision expected payoff E_{z2} is:

$$E_{z2} = x(I_{tc} - C_{tc} + S_{lt}) + (1 - x)(I_{tc} - C_{tc} - F_t)$$

From equations (11) and (12), digital platform average expected payoff E_z is:

$$E_z = zE_{z1} + (1 - z)E_{z2}$$

From equations (11) and (12), the replicator dynamic equation $F(z)$ for digital platform data strategy selection is:

$$F(z) = \frac{dz}{dt} = z(1 - z)(E_{z1} - E_{z2})$$

By stability principle, digital platform game decision stability requires $F(z) = 0$ and $F'(z) < 0$.

Proposition 3 (Proof in Appendix 3): When $v < v_2^*$, digital platform choosing "protect data" is a robust strategy; when $v > v_2^*$, government choosing "open data" is a robust strategy; when $v = v_2^*$, digital platform robust strategy cannot be determined, where:

$$v_2^* = \frac{C_{tc} - C_{to} + F_t - I_{tc} + I_{to} + S_{mt} - xF_t - xS_{ht} - xS_{lt} - S_{mt}x}{V_t}$$

According to Proposition 3, in dynamic decision processes among government, digital platforms, and elderly populations, if elderly groups choose non-acceptance, platforms will tend toward open data decisions to expand publicity. If elderly groups choose acceptance, digital platforms will consider various economic factors and avoid fiscal reputation risks from data security, tending toward data protection.

According to phase diagram 4 [Figure 4: see original paper] and equation (15), in $v_2^*(x)$, C_{tc} serves as the intercept term and is negatively correlated with $v_2^*(x)$. When platform data protection costs increase, $v_2^*(x)$ decreases, reducing the $v < v_2^*$ probability space and increasing the $v > v_2^*$ probability space, pushing platform decision probability toward open data. Similarly, S_{mt}, F_t, I_{tc} are negatively correlated with $v_2^*(x)$, while I_{to}, V_t are positively correlated. When other conditions remain unchanged, increases in F_t, I_{to}, S_{mt} and decreases in C_o, C_{tc} reduce the $v < v_2^*$ probability space, pushing platform decisions toward open data. Government decision x intervenes through slope coefficients F_t, S_{ht}, S_{lt}, S_{mt}.

[Figure 4: see original paper] Digital Platform Dynamic Game Decision Stability Evolution Diagram

4.3.4 Elderly Population Strategy Stability Analysis Let elderly population decision expected payoff when choosing "accept" be E_{v1}:

$$E_{v1} = xy(V_o+C_o-V_{ho})+x(1-y)(V_o+C_o-V_{lo})+(1-x)y(V_o-V_{ho})+(1-x)(1-y)(V_o-V_{lo})$$

Let elderly population decision expected payoff when choosing "not accept" be E_{v2}:

$$E_{v2} = -xyC_o$$

Let elderly population average expected payoff be E_v:

$$E_v = vE_{v1} + (1-v)E_{v2}$$

From equations (16) and (17), the replicator dynamic equation $F(v)$ for elderly population product selection is:

$$F(v) = \frac{dv}{dt} = v(1 - v)(E_{v1} - E_{v2})$$

By stability principle, elderly population game decision stability requires $F(v) = 0$ and $F'(v) < 0$.

Proposition 4 (Proof in Appendix 4): When $x < x^*$, elderly population choosing "not accept" is a robust strategy; when $x > x^*$, elderly population choosing "accept" is a robust strategy; when $x = x^*$, elderly population robust strategy cannot be determined, where:

$$x^* = \frac{V_{lo} - V_o}{C_o + V_{lo} - V_{ho}}$$

According to Proposition 4, in other agents' dynamic gaming processes, elderly population gaming stability is influenced by government decisions. When government decisions tend toward high subsidy, elderly populations are more willing to accept smart elderly care products; when government decisions tend toward low subsidy, elderly acceptance probability decreases. According to phase diagram 5 [Figure 5: see original paper] and equation (20), V_{lo} and V_o are negatively correlated with $x^*(y)$. When sustained silver consumption market value obtained by smart elderly care enterprises and product potential value obtained by elderly acceptance increase, $x^*(y)$ decreases, reducing the $x < x^*$ decision probability space and pushing elderly decisions toward acceptance. Other coefficients as slope terms adjust enterprise decision probability, thereby affecting elderly population stable probability space.

[Figure 5: see original paper] Elderly Population Dynamic Game Decision Stability Evolution Diagram

4.4 Combined Strategy Analysis in Four-Party Game

In the dynamic gaming process of silver economy development driven by smart elderly care industry, government, smart elderly care enterprises, digital platforms, and elderly groups as four game agents seek local stability of agent strategies. This study uses Lyapunov' s first method for discrimination.

Setting $F(x) = 0$, $F(y) = 0$, $F(z) = 0$, $F(v) = 0$ yields equilibrium points under different combined strategies. Ritzberger [21], Weibull [22], and Selten [23] note that in multi-population evolutionary games, strict Nash equilibria must be pure strategy points, and such game strategy stable solutions must be strict Nash equilibria—i.e., pure strategies. Therefore, this study focuses stability analysis on 16 pure strategy equilibrium points. When all eigenvalues of the Jacobian matrix corresponding to an equilibrium point have negative real parts, the combined strategy is locally asymptotically stable; conversely, if at least one eigenvalue has a strictly positive real part, the combined strategy is unstable.

Through four-party game agents' replicator dynamic equations, the Jacobian matrix is established as follows:

$$J = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} & \frac{\partial F(x)}{\partial z} & \frac{\partial F(x)}{\partial v} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} & \frac{\partial F(y)}{\partial z} & \frac{\partial F(y)}{\partial v} \\ \frac{\partial F(z)}{\partial x} & \frac{\partial F(z)}{\partial y} & \frac{\partial F(z)}{\partial z} & \frac{\partial F(z)}{\partial v} \\ \frac{\partial F(v)}{\partial x} & \frac{\partial F(v)}{\partial y} & \frac{\partial F(v)}{\partial z} & \frac{\partial F(v)}{\partial v} \end{bmatrix}$$

Substituting the 16 pure strategy equilibrium points of four-party game agents into this Jacobian matrix sequentially yields eigenvalue signs for stability judgment.

4.4.1 Strategy Combination Stability Under Elderly Acceptance

Strategy Combination Stability Analysis Under Elderly Acceptance

Stability analysis reveals (see Table 3) that four strategy combinations exhibit local asymptotic stability in long-term gaming:

1. **Low subsidy–Low pricing–Open data–Elderly acceptance (0,0,1,1):** Despite low government investment, enterprise price reduction, platform openness, and elderly willingness create a balanced situation of cost savings and service acceptance.
2. **Low subsidy–High pricing–Open data–Elderly acceptance (0,1,1,1):** Enterprises pursue profits but platforms fully open, maintaining elderly satisfaction–a market-driven strategy.
3. **High subsidy–Low pricing–Open data–Elderly acceptance (1,0,1,1):** Government incentives + enterprise concessions + platform synergy form a benign silver economy growth combination.
4. **High subsidy–High pricing–Open data–Elderly acceptance (1,1,1,1):** Applicable to high-quality supply scenarios with strong government intervention and high elderly payment willingness, possessing sustainability.

All stable strategy combinations share the common feature of "platform choosing open data ($z = 1$)"and "elderly group willing to accept ($v = 1$),"while government and enterprise strategies exhibit flexibility. That is, with platform openness and enhanced elderly perceived value, the system more easily reaches game equilibrium.

All other combinations show instability. In combinations like (0,0,0,1) and (1,1,0,1), platforms fail to open data, causing information matching obstacles despite elderly acceptance, leading to collaborative rupture and limited utility transmission. In combinations like (0,1,0,0) and (1,1,0,0), incentive mechanisms weaken or disappear, with high enterprise pricing, closed platforms, and elderly non-acceptance forming a reverse evolution path, trapping smart elderly care in a vicious cycle of "low supply—low demand—low service." These unstable combinations warn that platform openness and elderly acceptance are key elements triggering benign evolution, around which government subsidies and enterprise strategies should coordinate.

4.4.2 Strategy Combination Stability Under Elderly Non-Acceptance

Strategy Combination Stability Analysis Under Elderly Non-Acceptance

Under elderly non-acceptance gaming scenarios ($v = 0$), the overall system lacks locally asymptotically stable evolution paths. Table 4 analysis shows that among all 8 strategy combinations with $v = 0$, the Jacobian matrix has at least one eigenvalue with positive real part—i.e., all combinations fail stability conditions, exhibiting structural system imbalance. These combinations commonly feature enterprise profit-seeking, platform closure, or ineffective government subsidy transmission to service terminals, causing persistent elderly utility absence and ultimately forming feedback-broken vicious gaming paths. This demonstrates that in smart elderly care systems, elderly group behavioral choices are not only game outcome variables but also core links in system structure evolution. Any "top-down" policy intervention that cannot genuinely enhance elderly perceived value and service adaptability may cause system idling or even degradation.

---

5. Dynamic Simulation Experiments

In the dynamic gaming process of silver economy development driven by smart elderly care industry, this paper has analyzed stability of equilibrium points under strict Nash equilibria. Building on this theoretical foundation, this section uses Vensim for system dynamics numerical simulation experiments [16] to dynamically explore how government, smart elderly care enterprises, digital platforms, and elderly population gaming decisions change under parameter variations, establishing a system dynamics simulation model (Figure 6 [Figure 6: see original paper]):

[Figure 6: see original paper] SD Simulation Model for Silver Economy Development Gaming Driven by Smart Elderly Care Industry

According to Table 5 , since some equilibrium points have extremely 苛刻 equilibrium conditions that are difficult to realize in reality, this paper focuses on gaming evolution under equilibrium point (0,0,1,1) for social practical value significance. Its equilibrium conditions are: $C_{gh} + R_g + S_{ht} + C_o > I_{gh}$ and $S_{mt} > S_{lt}$. Based on previous research and real data fitting, parameters are assigned as follows:

Simulation Parameter Assignment

5.1 Evolution Analysis of Four-Party Gaming Economic Elements

5.1.1 Impact of Smart Elderly Care Product Potential Value on Evolution [Figure 9: see original paper] Impact of Smart Elderly Care Product Potential Value on Agent Payoff Expectations

In government payoff expectation (Figure 9), all curves peak near $t \approx 0.1$ then converge, finally stabilizing around 10, though initial fluctuations and peaks vary slightly. In enterprise payoff, higher V_o yields higher long-term returns: $V_o = 35$ reaches final payoff ~14, while $V_o = 5$ declines rapidly initially, ending below 6. This reflects that smart elderly care product intrinsic value is the core driver throughout the system behavior chain.

In platform payoff (Figure 9), curves for $V_o = 35$ and $V_o = 30$ rapidly rise to ~26 and stabilize, while $V_o = 5$ remains significantly lower, only stabilizing below 20, never catching up with high-V_o scenarios throughout. Platform response shows strong "value sensitivity," indicating platform payoff highly depends on data flow and interaction degree stimulated by product utility. Elderly population payoff curves show the largest differences: $V_o = 35$ user payoff rapidly jumps to 27 and stabilizes, while $V_o = 10$ curves are not only extremely low but overall approach 0.

5.1.2 Impact of Government Fiscal Risk Bearing on Evolution [Figure 10: see original paper] Impact of Government Fiscal Risk Bearing on Agent Payoff Expectations

In government payoff expectation (Figure 10), larger R_g values (e.g., $R_g = 30$) cause more dramatic initial fluctuations, even negative values around -6 to -8; while $R_g = 4$ shows smaller initial fluctuations, rapidly rising then stabilizing above 12. This indicates weaker government fiscal risk bearing capacity yields faster input results and more stable returns; conversely, long-term high-risk bearing causes system feedback delays and even initial input losses.

Enterprise and platform payoffs (Figure 10) are relatively less affected by R_g, with all curves basically overlapping, finally stabilizing around 14 for enterprises and 26 for platforms. Only in early stage $t < 0.2$, smaller R_g conditions yield faster enterprise and platform payoff growth, showing that smaller fiscal risk helps accelerate market strategy response speed but doesn' t change long-term payoff patterns.

Elderly population payoff (Figure 10) responds more nuancedly to R_g changes: smaller R_g yields higher initial user perceived payoff (~5.6), while $R_g = 30$ conditions yield minimum only ~4.8—a small but persistent gap. This shows appropriate government risk dispersion better enhances user trust and participation willingness, helping build benign interactive collaborative environments.

5.1.3 Impact of Enterprise Excess Profit on Evolution [Figure 11: see original paper] Impact of Enterprise Excess Profit on Agent Payoff Expectations

Figure 11 shows that as P_{he} increases, government expectation (E_x) peaks slightly rise, with $P_{he} = 20$ reaching maximum ~11 initially then gradually declining to stability. Enterprise payoff expectation (E_y) shows inverse characteristics from "high initial, low later" to "low initial, high later" with P_{he} changes. $P_{he} = 4$ has highest initial value (~13.5) but slowest growth, finally stabilizing around 13; while $P_{he} = 20$ starts at only 9 but grows fastest, finally stabilizing above 13. Intermediate parameters like $P_{he} = 8, 12, 16$ show smooth transitions with initial decline and later increase. This reflects that low-profit enterprises pursue short-term returns while high-profit enterprises adopt more long-term strategies, sacrificing short-term gains for system feedback-driven payoff amplification.

Platform payoff (E_z) remains relatively stable, with curves almost overlapping under different P_{he} conditions, finally stabilizing around 25, indicating low platform sensitivity to enterprise profit, relying more on user-side and data openness mechanisms. Elderly population payoff (E_v) declines overall with P_{he} increase, especially lowest at $P_{he} = 20$, only maintaining ~4, showing enterprise excess profit acquisition significantly compresses user 端获得感, inhibiting participation enthusiasm.

In summary, enterprise excess profit enhancement effectively improves its own payoff expectations while driving government payoff growth, but significantly compresses elderly 端获得感, with platforms basically unaffected. This indicates the system must guard against "enterprise payoff dominance" causing user 端 compression effects to avoid gaming imbalance affecting long-term evolutionary stability.

5.1.4 Evolutionary Impact of Government Subsidy Intensity for Silver Consumption Groups [Figure 12: see original paper] Impact of Subsidy Intensity on Agent Payoff Expectations

Figure 12 shows that as subsidy intensity C_o increases, payoff expectations of government, enterprise, platform, and elderly all change to varying degrees. Government payoff expectation (E_x) rapidly surges under high subsidies (e.g., $C_o = 16, 18$) but then declines, indicating high input yields quick short-term results but faces diminishing marginal returns. Enterprise payoff (E_y) grows stably, with larger subsidies yielding smoother growth, showing subsidies effectively reduce pricing and enhance market response.

Platform payoff (E_z) is most affected by subsidies, rapidly rising to high levels and maintaining under high C_o conditions, showing "high feedback elasticity." Elderly payoff (E_v) jumps initially then declines to stability, indicating strong perception response but lacking sustained growth, possibly requiring non-price factor support.

Overall, larger subsidy intensity yields faster system response, but different agents benefit through different mechanisms. Platforms and enterprises achieve stable long-term payoff growth, while elderly benefits are short-lived, requiring government to balance short-term incentives with long-term fiscal pressure to avoid resource inefficiency from "over-subsidization."

5.2 Evolution Analysis of Four-Party Gaming Decision Elements

5.2.1 Impact of Government Product Subsidies on Evolution [Figure 13: see original paper] Impact of Government Product Subsidies on Agent Decisions

Figure 13 (x-y-z) shows that as government subsidy levels increase, platform data openness probability z significantly rises, rapidly approaching high stable state $z \approx 0.95$ when $S_{ht} \geq 20$; enterprise high pricing strategy y gradually declines toward $y \approx 0.2$, reflecting that under subsidy pressure and market feedback, enterprises prefer lower prices to capture elderly markets. Government' s own strategy x converges to medium-high levels ($x \approx 0.55$) across different initial states, demonstrating rational equilibrium under "fiscal risk—collaborative payoff" gaming.

Figure 13 (x-y-v) shows that each subsidy intensity gradient increase ($S_{ht} + 5$) causes v' s final stable value to rise stepwise, approaching $v \approx 0.88$ at $S_{ht} = 30$, showing continuously enhanced subsidy intensity effectively improves user-side perceived value and strategy stickiness. Simultaneously, v' s increase further suppresses enterprise high pricing behavior (y decline), forming a virtuous cycle of "government incentive—enterprise concession—elderly acceptance."

Overall, government extra subsidies not only directly drive platform openness and elderly acceptance strategy evolution but also indirectly constrain enterprise behavior.

5.2.2 Impact of Smart Elderly Care Product Potential Value on Evolution [Figure 14: see original paper] Impact of Product Potential Value on Agent Decisions

This strategy probability evolution diagram (Figure 14) shows impacts of different product potential values $V_o = 5, 10, 15, 20, 25, 30, 35$ on system evolution trajectories under fixed strategy combination (0,0,1,1). In x-y-z simulation, all trajectories converge well, showing obvious asymptotic stability characteristics. As V_o increases, curves on z-axis show monotonic rise, finally converging to 1. Especially curves with $V_o \geq 25$ show rapid climb and stability, indicating high V_o can effectively drive system synergy and activate user feedback, achieving positive evolution even under low subsidy ($x = 0$).

In x-y-v, despite unchanged strategy combination, the system shows instability and structural distortion. High-value curves ($V_o = 25, 30, 35$) exhibit obvious "turning" or even "sinking" phenomena, especially showing decline trends in xoz projection, indicating system response to high V_o fails at some mid-stage, possibly due to interference from other parameters on system synergy mechanisms. Meanwhile, the black curve for $V_o = 5$ always remains in low acceptance range, showing that under extremely low product value conditions, even with platform openness and friendly pricing, it' s difficult to break demand-side cold start 困境.

5.3 Evolution Analysis of Elderly Population Acceptance Decisions

According to Table 4 combined strategy stability analysis, under elderly non-acceptance gaming scenarios ($v = 0$), the four-party game system lacks stable decision points, making elderly acceptance a key factor for silver economy development. To explore sensitive indicators of elderly acceptance decisions, SD simulation yields the following results.

[Figure 15: see original paper] Impact of Various Factors on Elderly Population Stable Decisions

Figure 15 shows evolution paths of enterprise high pricing strategy probability y as government high subsidy probability $x \in [0, 1]$ increases, under fixed elderly population strategy "non-acceptance" ($\omega \to 0$). Parameter combinations include elderly product potential value $V_o = 10, 15, 20, 25$, government user subsidy intensity $C_o = 4, 8, 12, 16$, and platform data openness payoff difference $V_t = 3, 6, 9$. Simulation results show that under elderly non-acceptance premise, even with favorable conditions ($V_o = 25, C_o = 16, V_t = 9$), enterprise strategy y still cannot stably converge, showing large fluctuations and obvious decision uncertainty and gaming structural instability. This demonstrates that when demand-side (elderly) exits the game, policy incentives and behavioral feedback mechanisms within the system are severely weakened, supply-side (enterprise and platform) cannot perceive effective market returns, and strategy evolution falls into ambiguity and non-deterministic states.

---

6. Conclusions and Recommendations

This study constructs a four-party evolutionary game model involving government, enterprises, platforms, and elderly from a "Demand–Information–Policy" collaboration chain perspective, combining system dynamics for multi-parameter simulation experiments to reveal key paths and strategic mechanisms for smart elderly care driving silver economy development. Findings show:

1. **Platform data openness and elderly acceptance willingness are core variables determining system stable evolution.** All stable strategy combinations are based on "platform choosing open data ($z = 1$)" and "elderly willing to accept ($v = 1$)," demonstrating that technology accessibility and user 获得感 are prerequisites for policy implementation and market activation.

2. **Government subsidies have significant phased effects.** High subsidies can initially stimulate enterprise and platform behavior, but excessive subsidies trigger fiscal risks with diminishing marginal returns. Therefore,

dynamically adjusting subsidy intensity to ensure a "subsidy—feedback—adjustment" mechanism loop is key to fiscal sustainability.

3. **Enterprise pricing strategies highly depend on elderly acceptance levels and government guidance.** Low pricing enables long-term market expansion and policy synergy, while high pricing brings short-term profits but weakens user participation, blocking system evolution pathways.
4. **Smart elderly care product intrinsic value (V_o) has nonlinear impacts on all four agents' behavior.** When $V_o \geq 25$, platform payoff, user perception, and enterprise profit rise synchronously with obvious system synergy jumps; when $V_o \leq 10$, the system falls into "supply-demand mismatch" states, showing that technology aging-friendliness and product value are foundations for closing supply-demand loops.

Based on these conclusions, the following policy recommendations are proposed:

First, construct a "tiered subsidy + user feedback" mechanism to evolve government subsidies from single expenditures to feedback-based adjustments, improving fiscal leverage efficiency.

Second, advance platform data hierarchical openness alongside privacy protection, establishing a data governance standard system of "bounded openness, controllable usage."

Third, guide enterprises to implement "inclusive pricing + service differentiation" strategies, exploring value-added service profit spaces while ensuring basic service 普及.

Finally, encourage product design and service supply to be deeply aging-friendly, enhancing elderly populations' actual 获得感 and acceptance willingness, strengthening demand-side driving effects.

In conclusion, smart elderly care is not single-point technology stacking but a systemically collaborative social 工程. Achieving high-quality silver economy development requires breaking agent 孤岛, reshaping collaboration mechanisms, and building a positive evolution path of government guidance, enterprise response, platform synergy, and user driving to release future industrial momentum amid aging challenges.

References

[1] Zhang Chenggang, Ning Xuesi. The Generative Logic and Practical Path of Smart Elderly Care in Grassroots Communities—Case Exploration Based on TOE Framework[J/OL]. Journal of Public Management, 2025: 1-15
[2] Li Xiaotian. The Internal Logic and Path of Pension Finance Supporting High-Quality Silver Economy Development[J]. Jiangsu Social Sciences, 2025(2):

45-53.
[3] Chen Si, Li Jinhao, Zhao Yuxiang, et al. "Data Factor ×" Effect on Smart Elderly Care Service Digital Intelligence Transformation: Characteristics, Connotation, and Path[J/OL]. Information Studies: Theory & Application, 2025: 1-11
[4] Li Shanman, Yao Leye. Data-Driven Smart Elderly Care Service Contextualization Analysis—Using Actor-Network Theory as Framework[J]. Information and Documentation Services, 2025, 46(1): 40-49.
[5] Zhang Yingxi, Jin Shuxing, Chen Ting. Mechanism and Path of Government-Market-Technology Synergy Driving High-Quality Silver Economy Development[J/OL]. Price: Theory & Practice, 2025: 1-6
[6] Zhu Chunhua. Research on Financial Support Strategies for China' s Smart Elderly Care Industry[J]. Southwest Finance, 2025(1): 15-25.
[7] Li Xiaotian. The Internal Logic and Path of Pension Finance Supporting High-Quality Silver Economy Development[J/OL]. Jiangsu Social Sciences, 2025: 1-9
[8] Sun Shiwei, Li Xiahe, Li Yaoyao, et al. Research on Evolutionary Game Model of Smart Elderly Care Services Under Four-Party Participation[J/OL]. Journal of Systems Science and Mathematical Science, 2025: 1-23
[9] Wang Yicheng, Liu Junhui. Research on Evolutionary Strategies for Smart Elderly Care Information Sharing from Three-Party Game Perspective[J]. Information Research, 2024(12): 1-7.
[10] Hu Limi, Qiu Xiaoling. Research on Four-Party Game Evolutionary Dynamics Model for Elderly Care Issues Under Bounded Rationality[J]. Mathematics in Practice and Theory, 2023, 53(5): 48-66.
[11] Sun Shiwei, Li Xiahe, Li Yaoyao, et al. Research on Evolutionary Game Model of Smart Elderly Care Services Under Four-Party Participation[J]. Journal of Systems Science and Mathematical Science, 2023, 43(6): 1034-1048.
[12] Ministry of Industry and Information Technology, Ministry of Civil Affairs, National Health Commission. Smart Health and Elderly Care Industry Development Action Plan (2021-2025)[Z]. Beijing: Ministry of Industry and Information Technology, 2021.
[13] National Data Administration. Taiyuan: Smart Elderly Care Digital Platform Opens New Path for Full-Industry Smart Services (No. 49 in Urban Digital Transformation Typical Case Series)[R]. Beijing: National Data Administration, 2025.
[14] Wang Jue, Ren Guozheng. Latest Trends and Policy Recommendations for China' s Smart Elderly Care Industry[EB/OL]. IIGF Perspectives, International Institute of Green Finance, Central University of Finance and Economics, 2024-05-09
[15] Song Yan, Chen Sai, Zhang Ming. Local Government Heterogeneity and Regional Environmental Cooperative Governance—Evolutionary Game Analysis Based on Chinese-Style Decentralization[J]. Chinese Journal of Management Science, 2020, 28(1): 201-211.
[16] Wang Qifan. System Dynamics[M]. Beijing: Tsinghua University Press, 1988: 272.

[17] Hu Limi, Qiu Xiaoling. Research on Four-Party Game Evolutionary Dynamics Model for Elderly Care Issues Under Bounded Rationality[J]. Mathematics in Practice and Theory, 2023, 53(5): 48-66.
[18] Sun Shiwei, Li Xiahe, Li Yaoyao, et al. Research on Evolutionary Game Model of Smart Elderly Care Services Under Four-Party Participation[J]. Journal of Systems Science and Mathematical Science, 2025, 45(1): 113-128.
[19] Wang Yicheng, Liu Junhui. Research on Evolutionary Strategies for Smart Elderly Care Information Sharing from Three-Party Game Perspective[J]. Information Research, 2024(12): 1-7.
[20] Shen Junxin, Liu Yating, Wang Xiaohan. Research on Tripartite Win-Win Cooperation Mechanism for Elderly Care PPP Projects Based on System Dynamics Evolutionary Game[J]. Journal of Hebei University of Economics and Business, 2019, 40(2): 100-109.
[21] RITZBERGER K, WEIBULL J W. Evolutionary selection in normal-form games[J]. Econometrica, 1995, 63(6): 1371-1399.
[22] WEIBULL J W. Evolutionary Game Theory[M]. WANG Yongqin, trans. Shanghai: Gezhi Press, 2015: 242.
[23] SELTEN R. A note on evolutionarily stable strategies in asymmetric animal conflicts[J]. Journal of Theoretical Biology, 1980, 84(1): 93-101.

Appendix 1: Proof of Proposition 1

Let $P(y) = C_{gl} - F_t + I_{gh} - I_{gl} - R_g - S_e - S_{ht} - C_o v - F_e y + S_e y + F_t z - S_{mt} z$. $P(y)$ is an increasing function of y. When $y < y^*$, $P(y) < 0$, then $x = 1$ is stable. When $y > y^*$, $P(y) > 0$, then $x = 0$ is stable. When $y = y^*$, stability cannot be determined. Any $x \in [0, 1]$ can stabilize the game.

Appendix 2: Proof of Proposition 2

Let $G(v) = -C_{eh} + C_{el} - F_e + P_{he} + P_{le} - V_{he} - V_{ho} + V_{le} + F_e x - S_e x + V_{ho} z - V_{lo} z + V_{lo} xz - V_{oxz} + V_{ho}\omega - V_o\omega - V_{lo} x\omega + V_{ox}\omega - V_{ho} z\omega + V_{oz}\omega$. $G(v)$ is an increasing function of v. When $v < v_1^*$, $G(v) < 0$, then $y = 1$ is stable. When $v > v_1^*$, $G(v) > 0$, then $y = 0$ is stable. When $v = v_1^*$, stability cannot be determined. Any $y \in [0, 1]$ can stabilize the game.

Appendix 3: Proof of Proposition 3

Let $H(v) = C_{tc} - C_{to} + F_t - I_{tc} + I_{to} + S_{mt} - xF_t - xS_{ht} - xS_{lt} - S_{mt} x + V_t \omega$. $H(v)$ is an increasing function of v. When $v < v_2^*$, $H(v) < 0$, then $z = 1$ is stable. When $v > v_2^*$, $H(v) > 0$, then $z = 0$ is stable. When $v = v_2^*$, stability cannot be determined. Any $z \in [0, 1]$ can stabilize the game.

Appendix 4: Proof of Proposition 4

Let $Q(x) = V_o + C_o x + V_{lo}(1-y) - V_{ho}y + C_o xy$. $Q(x)$ is an increasing function of x. When $x < x^*$, $Q(x) < 0$, then $v = 1$ is stable. When $x > x^*$, $Q(x) > 0$, then $v = 0$ is stable. When $x = x^*$, stability cannot be determined. Any $v \in [0, 1]$ can stabilize the game.

Appendix 5: Agent Strategy Stability Calculation Code

Implementation software: Wolfram Mathematica
Code function: Calculate stability strategy differential equations for each agent

```
(* Government *)
Ex1 = $\omega$*(y*z*(Igh - Cgh - Rg - Sht - Co) + y*(1 - z)*(Igh - Cgh - Rg - Co) + (1 - y)*
Ex2 = y*z*(Igl - Cgl + Fe - Smt) + y*(1 - z)*(Igl - Cgl + Fe + Ft) + (1 - y)*z*(Igl - Cgl -
Simplify[Ex1 - Ex2]
P[y_] := Cgl - Ft + Igh - Igl - Rg - Se - Sht - Co*w - Fe*y + Se*y + Ft*z - Smt*z
D[P[y], y]
y1 = Solve[P[y] == 0, y]
P[w_] := Cgl - Ft + Igh - Igl - Rg - Se - Sht - Co*w - Fe*y + Se*y + Ft*z - Smt*z
D[P[w], w]
y1 = Solve[P[w] == 0, w]
P[z_] := Cgl - Ft + Igh - Igl - Rg - Se - Sht - Co*w - Fe*y + Se*y + Ft*z - Smt*z
D[P[z], z]
y1 = Solve[P[z] == 0, z]

(* Smart Elderly Care Enterprise *)
Ey1 = $\omega$*(x*z*(Ve - Ceh + Phe - Vhe - Sht) + x*(1 - z)*(Ve - Ceh + Phe - Vhe - Slt) +
Ey2 = $\omega$*(x*z*(Ve - Cel - Ple + Vle + Se + Vlo - Sht) + x*(1 - z)*(Ve - Cel - Ple + Vl
Simplify[Ey1 - Ey2]
P := -Ceh + Cel - Fe + Phe + Ple - Vhe - Vho - Vle + Fe*x - Se*x + Vho*z - Vlo*z + Vlo*x*z -
DP = D[P, x] // Simplify
x1 = Solve[P == 0, x] // Simplify
DP = D[P, z] // Simplify
z1 = Solve[P == 0, z] // Simplify
DP = D[P, $\omega$] // Simplify
$\omega$1 = Solve[P == 0, $\omega$] // Simplify

(* Digital Platform *)
Ez1 = x*$\omega$*((Ito - Cto + Sht + Vt)) + x*(1 - $\omega$)*(Ito - Cto + Sht) + (1 - x)*$\o
Ez2 = x*(Itc - Ctc + Slt) + (1 - x)*(Itc - Ctc - Ft);
Simplify[Ez1 - Ez2]
```

```
H[x_] := Ctc - Cto + Ft - Itc + Ito + Smt - x*Ft - x*Sht - x*Slt - Smt*x + Vt*$\omega$
D[H[x], x] // Simplify
x1 = Solve[H[x] == 0, x]
H[$\omega$_] := Ctc - Cto + Ft - Itc + Ito + Smt - x*Ft - x*Sht - x*Slt - Smt*x + Vt*$\omega
D[H[$\omega$], $\omega$] // Simplify
w1 = Solve[H[$\omega$] == 0, $\omega$]

(* Elderly Population *)
Ew1 = x*y*(Vo + Co - Vho) + x*(1 - y)*(Vo + Co - Vlo) + (1 - x)*y*(Vo - Vho) + (1 - x)*(1 -
Ew2 = -x*y*Co;
Simplify[Ew1 - Ew2]
Q[x_] := Vo + Co*x + Vlo*(1 - y) - Vho*y + Co*x*y
DQ = D[Q[x], x] // Simplify
x1 = Solve[Q[x] == 0, x]
Q[y_] := Vo + Co*x + Vlo*(1 - y) - Vho*y + Co*x*y
DQ = D[Q[y], y] // Simplify
y1 = Solve[Q[y] == 0, y]
```

Appendix 6: Jacobian Matrix Eigenvalue Calculation Code

Implementation software: Wolfram Mathematica
Code function: Calculate Jacobian matrix eigenvalues under each combined strategy equilibrium point

```
(* Government *)
Ex1 = $\omega$*(y*z*(Igh - Cgh - Rg - Sht - Co) + y*(1 - z)*(Igh - Cgh - Rg - Co) + (1 - y)*
Ex2 = y*z*(Igl - Cgl + Fe - Smt) + y*(1 - z)*(Igl - Cgl + Fe + Ft) + (1 - y)*z*(Igl - Cgl -

(* Smart Elderly Care Enterprise *)
Ey1 = $\omega$*(x*z*(Ve - Ceh + Phe - Vhe - Sht) + x*(1 - z)*(Ve - Ceh + Phe - Vhe - Slt) +
Ey2 = $\omega$*(x*z*(Ve - Cel - Ple + Vle + Se + Vlo - Sht) + x*(1 - z)*(Ve - Cel - Ple + Vl

(* Digital Platform *)
Ez1 = x*$\omega$*((Ito - Cto + Sht + Vt)) + x*(1 - $\omega$)*(Ito - Cto + Sht) + (1 - x)*$\o
Ez2 = x*(Itc - Ctc + Slt) + (1 - x)*(Itc - Ctc - Ft);

(* Elderly Population *)
E$\omega$1 = x*y*(Vo + Co - Vho) + x*(1 - y)*(Vo + Co - Vlo) + (1 - x)*y*(Vo - Vho) + (1 - x
E$\omega$2 = -x*y*Co;

(* Replicator Dynamic Equations *)
dx := x (1 - x) (Ex1 - Ex2);
dy := y (1 - y) (Ey1 - Ey2);
dz := z (1 - z) (Ez1 - Ez2);
d$\omega$ := $\omega$ (1 - $\omega$) (E$\omega$1 - E$\omega$2);
```

```
(* Jacobian Matrix *)
jacobian = {
  {D[dx, x]}, {D[dx, y]}, {D[dx, z]}, {D[dx, $\omega$]},
  {D[dy, x]}, {D[dy, y]}, {D[dy, z]}, {D[dy, $\omega$]},
  {D[dz, x]}, {D[dz, y]}, {D[dz, z]}, {D[dz, $\omega$]},
  {D[d$\omega$, x]}, {D[d$\omega$, y]}, {D[d$\omega$, z]}, {D[d$\omega$, $\omega$]}
};

(* Calculate Jacobian at equilibrium point (1,1,1,1) *)
jacobianAtEquilibrium = jacobian /. {x -> 1, y -> 1, z -> 1, $\omega$ -> 0};

(* Calculate eigenvalues *)
eigenvalues = Eigenvalues[jacobianAtEquilibrium];
Print["Eigenvalues: ", eigenvalues]
```

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv —Machine translation. Verify with original.