# Principles of Quantum Mechanics from Statistical Constraints: Quantization, Indeterminacy, and the Incompatibility of Hidden–Variable Theories

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## Abstract

We argue that the existence of thermal phenomena, together with the phenomena of particle creation and annihilation, imposes fundamental constraints on the dynamics of physical systems. Specifically, the presence of these phenomena jointly requires that a finite system can only have a finite number of microscopic physical states, which naturally leads to the necessity of a quantization framework and an inherently probabilistic dynamic structure. We show that deterministic hidden-parameter theories, such as Bohmian mechanics, are incompatible with these requirements. Furthermore, we briefly explore why the gravitational wave field is more likely to be quantum in nature rather than classical.

## Full Text

## Preamble

**Principles of Quantum Mechanics from Statistical Constraints: Quantization, Indeterminacy, and the Incompatibility of Hidden-Variable Theories**

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## Abstract

We argue that the existence of thermal phenomena, together with the phenomena of particle creation and annihilation, imposes fundamental constraints on

the dynamics of physical systems. Specifically, the presence of these phenomena jointly requires that a finite system can only have a finite number of microscopic physical states, which naturally leads to the necessity of a quantization framework and an inherently probabilistic dynamic structure. We show that deterministic hidden-variable theories, such as Bohmian mechanics, are incompatible with these requirements. Furthermore, we briefly explore why it is more plausible for gravitational waves to be quantum rather than classical in nature.

Thermal phenomena—such as thermal equilibrium and phase transitions—are common and seemingly ordinary aspects of physical systems. Yet, their emergence is far from trivial when considered from first principles. Investigating these phenomena led to the development of statistical mechanics, a foundational and remarkably universal framework that introduces key concepts, such as statistical averages over all possible microscopic physical states. At thermal equilibrium, the probability assigned to each microscopic state universally depends on its energy through a Boltzmann factor.

In this statistical framework, the dynamical transitions between microscopic states, governed by exact dynamical laws, tend to play a negligible role. This fundamental simplification enables a powerful and efficient description of the thermal properties of physical systems. Notably, insights gained from statistical mechanics can also impose constraints on the underlying fundamental theories. A profound example is the ultraviolet catastrophe, where the failure of classical electromagnetic theory to predict a finite heat capacity at high frequencies was resolved by Planck's proposal [1]—ushering in the quantum era.

In this paper, we demonstrate that statistical considerations, together with the phenomena of particle creation and annihilation, impose fundamental constraints on the structure of (statistically) independent physical states in physical systems. Specifically, for any system confined to a finite volume and possessing finite energy, the set of microscopic physical states must be discrete and finite. The processes of particle creation and annihilation span a wide range of physical phenomena, including the formation and annihilation of particle-antiparticle pairs, transmutations among fundamental particles, molecular dissociation into constituents such as atoms, ions, electrons, or other molecules, and the formation or breaking of composite entities like Cooper pairs.

This seemingly simple yet profound fact—the finiteness of microscopic physical states in a finite system—highlights a fundamental distinction between quantum mechanics and classical mechanics. In quantum theory, the finiteness of microscopic physical states arises naturally from the principle of quantization, as partially reflected in the use of discrete quantum numbers to label states in finite systems. In contrast, classical mechanics assumes a continuum of possible states: distinct spatial configurations of individual particles correspond to distinct physical states, and because positions vary continuously, the classical state space forms an uncountable continuum—fundamentally distinct from a discrete set.

This insight also casts doubt on the Bohmian interpretation of quantum mechanics (see, e.g., [2]), which assumes an ontological framework in which particles follow well-defined, continuous trajectories. Such a view conflicts with the fundamentally finite and discrete nature of the quantum state space.

Moreover, the finiteness of microscopic physical states implies that any underlying physical theory must be inherently probabilistic. If the fundamental dynamics exhibit Markovian-like behavior—where the system's state at any given moment depends only on its immediate past, rather than on its full history—a probabilistic description naturally follows. In this light, the probabilistic structure of quantum mechanics, though often regarded as counterintuitive, emerges as an unavoidable consequence of these foundational principles.

### Bohmian Mechanics and Statistical Considerations

The Bohmian approach (or de Broglie-Bohm theory), originating with de Broglie and later developed by Bohm, asserts that a system's physical description involves both its wavefunction, which evolves according to the Schrödinger equation, and the definite positions of particles, determined by a "guiding equation" influenced by the wavefunction [3]. In this framework, even when the wavefunction is fixed—say, in a stationary state—different particle positions correspond to distinct physical states, resulting in a continuum of states associated with the same wavefunction.

To illustrate the conflict between the Bohmian picture and standard quantum statistical mechanics, consider a pedagogical example of a single particle confined in a potential well. Setting aside subtleties related to thermalization in one-particle systems, we examine its thermal properties. The system's quantum states are taken as eigenstates of the Hamiltonian:

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V(r),$$

where $\hbar$ is the reduced Planck constant, $m$ is the mass of the particle, and $V(r)$ describes the potential well. We focus on the set of eigenstates with energies below a certain cutoff, as higher-energy states are exponentially suppressed by the Boltzmann factor $\propto e^{-E/kT}$ ($k$ is the Boltzmann constant). The number of relevant eigenstates is finite, denoted $N$, with eigenfunctions $\psi_i(r)$ and corresponding energies $E_i$, satisfying

$$\hat{H}\psi_i(r) = E_i\psi_i(r) \quad (i = 0, 1, 2, \dots, N-1).$$

In Bohmian mechanics, the specification of a physical state requires both the wavefunction $\psi(r,t)$ and the actual position of the particle, denoted by $x$. The position $x$ is constrained to regions where the wavefunction does not vanish. For stationary states of the form $\psi(r,t) = \psi_i(r)e^{-iE_i t/\hbar}$, the particle position $x$ remains static [2]. Consequently, Bohmian mechanics treats configurations like $(\psi_0(r), x_1)$ and $(\psi_0(r), x_2)$, where $x_1 \neq x_2$, as distinct physical states, even though the wavefunction component $\psi_0(r)$ is identical (see Fig. 1 [Figure 1: see original paper]).

In computing thermal averages within this framework, one must sum over all combinations of wavefunctions $\psi_i$ and particle positions $x$, yielding:

$$\bar{E}(T) = \frac{\sum_i \sum_x E_i e^{-E_i/kT}}{\sum_i \sum_x e^{-E_i/kT}}. \tag{2}$$

By contrast, in conventional quantum mechanics, physical states are determined solely by the wavefunction, and the thermal average energy takes the form:

$$\bar{E}(T) = \frac{\sum_i E_i e^{-E_i/kT}}{\sum_i e^{-E_i/kT}}. \tag{3}$$

To reconcile the Bohmian expression with the conventional result, the integration over particle positions $x$ in Eq. (2) must yield the same multiplicative factor for each eigenstate $\psi_i$. However, from first principles, integrating over $x$ reflects the spatial volume over which $\psi_i$ has non-negligible support. In realistic systems, particularly in condensed-matter contexts, different eigenstates can exhibit vastly different spatial extents—for example, between localized and extended orbitals—leading to significant variations in this volume factor.

Such variability implies that the Bohmian construction does not, in general, reproduce conventional quantum statistical predictions, exposing a fundamental inconsistency between Bohmian mechanics and the thermal behavior of quantum systems.

In the Bohmian framework, this inconsistency may be addressed through the quantum equilibrium hypothesis (QEH). The QEH postulates that the particle position $x$ is distributed according to the squared modulus of the wavefunction, $|\psi_i(x)|^2$. If one adopts this distribution and integrates accordingly, the expression for the average energy becomes:

$$\bar{E}(T) = \frac{\sum_i \int dx\, |\psi_i(x)|^2 E_i e^{-E_i/kT}}{\sum_i \int dx\, |\psi_i(x)|^2 e^{-E_i/kT}} = \frac{\sum_i E_i e^{-E_i/kT}}{\sum_i e^{-E_i/kT}},$$

where we have assumed the normalization condition $\int |\psi_i(r)|^2 dr = 1$ for each $\psi_i(r)$. This renders Eq. (2) formally equivalent to Eq. (3).

While the QEH provides a way to recover the correct statistical averages, it raises concerns from the standpoint of fundamental statistical principles. In the Bohmian interpretation, states such as $(\psi_0(r), x_1)$ and $(\psi_0(r), x_2)$ are regarded as distinct and independent. Since these states have the same energy, statistical mechanics requires that they should contribute equally to thermodynamic averages—as statistical weights are determined by conserved quantities like energy [4], not by position-dependent probability densities derived from the wavefunction. Thus, assigning unequal weights based on $|\psi_0(x)|^2$ lacks a fundamental justification. The need to impose such weighting suggests that $(\psi_0(r), x_1)$ and $(\psi_0(r), x_2)$ do not represent genuinely independent physical states, but rather are redundant representations of the same underlying state. This undermines the Bohmian interpretation's assertion of a continuous ensemble of distinct particle positions and reinforces the idea that only the wavefunction itself represents a complete physical state in quantum mechanics.

### Systems with Variable Particle Numbers

This problem with the Bohmian picture becomes even more pronounced when considering systems with variable particle numbers, such as those involving particle creation or annihilation. For instance, consider a simple system in a finite volume where a particle A can decay into two particles B and C, and conversely, B and C can annihilate to form A:

$$A \rightleftharpoons B + C.$$

Restricting attention to physical states with total energy below a fixed cutoff, the system's Hilbert space can be partitioned into two sectors: one comprising one-particle states $\psi_i(r_a)$ with energies $E_i^a$ (for $i = 0, 1, 2, \dots, N_a - 1$), and the other comprising two-particle states $\phi_j(r_b, r_c)$ with energies $E_j^{bc}$ (for $j = 0, 1, 2, \dots, N_{bc} - 1$). At temperature $T$, the thermal probability that the system is in the one-particle sector is given by:

$$P_a(T) = \frac{\sum_i e^{-E_i^a/kT}}{\sum_i e^{-E_i^a/kT} + \sum_j e^{-E_j^{bc}/kT}}.$$

Because both $N_a$ and $N_{bc}$ are finite, there may generally exist a temperature range where $P_a(T)$ deviates significantly from both 0 and 1, implying a statistical coexistence of one-particle and two-particle states. This is a natural outcome of standard quantum statistical mechanics.

However, in the Bohmian picture, each physical state is specified not only by the wavefunction but also by definite particle positions. A one-particle state is described by $(\psi_i(r_a), x_a)$ while a two-particle state is described by $(\phi_j(r_b, r_c), x_b, x_c)$. When computing thermal probabilities, one must then sum over all such configurations:

$$P_a(T) = \frac{\sum_i \sum_{x_a} e^{-E_i^a/kT}}{\sum_i \sum_{x_a} e^{-E_i^a/kT} + \sum_j \sum_{x_b, x_c} e^{-E_j^{bc}/kT}}.$$

A critical issue arises in this context. The set of microscopic physical states in the two-particle sector corresponds roughly to the set of all points in a product space of $V \times V \times \{0, 1, 2, \dots, N_{bc} - 1\}$, where $V$ represents the set of all points in the spatial extent of the system. In contrast, the one-particle sector spans a lower-dimensional product space of $V \times \{0, 1, 2, \dots, N_a - 1\}$. Although both sets involve continuous spatial components, the ratio of their sizes—in a measure-theoretic sense—tends to zero: the measure of $V$ becomes negligible compared to that of the higher-dimensional space $V \times V$. As a result, the probability $P_a(T)$ vanishes, indicating that the system would almost never reach a state of thermal coexistence between the one- and two-particle sectors—an outcome that is inconsistent with certain physical observations.

This analysis highlights a broader principle: in systems with variable particle numbers, thermal coexistence between different particle-number sectors cannot be consistently achieved if the set of microscopic physical states is continuous, as would be the case when particles are assigned definite spatial positions. This, in turn, implies that classical mechanics—by preserving the notion of continuous particle positions—lacks a rigorous foundation for describing thermal coexistence involving particle transformations. By similar reasoning, one can argue that in order for thermal coexistence with variable particle numbers to be possible, the number of microscopic physical states—within a finite volume and bounded energy—shall necessarily be finite.

### Classical Statistical Mechanics and Discretization

In classical statistical mechanics, particularly as applied to systems like thermal atomic gases, the formalism implicitly introduces a discretization of the otherwise continuous physical state space. This is suggested by the appearance of a parameter with dimensions of $\hbar$, which defines a unit volume in the phase space for a particle undergoing one-dimensional motion. Typically, the formalism requires dividing the relevant phase space volume by an appropriate power of $\hbar$ to obtain a dimensionless quantity, corresponding to the effective number of microscopic physical states within that volume.

For example, in a system of $N$ particles, thermal quantities often involve an expression:

$$\frac{1}{(2\pi\hbar)^{3N}} \int dr_1 dp_1 dr_2 dp_2 \dots dr_N dp_N \dots,$$

where $r_k$ and $p_k$ represent the position and momentum of the $k$-th particle $(k = 1, 2, \dots, N)$.

This division effectively discretizes the continuous state space and thereby renders the number of microscopic physical states finite.

Such discretization is indispensable when extending classical statistical mechanics to systems that allow for particle creation and annihilation. Analogous to the role played by the quantum equilibrium hypothesis (QEH) in the Bohmian formulation of quantum mechanics, this discretization serves as a merely formal resolution to the tension between the foundational principles of statistical mechanics and the assumption of a continuous state space.

### Thermalization and State Space

There is yet another potential tension between thermalization and the notion of a continuous set of physical states. Since the number of such states is infinite, full thermalization may never occur within a finite time, as visiting all possible states could take an infinite duration. In contrast, conventional quantum mechanics avoids this issue through the quantization of physical states, inherently restricting a finite system to a finite set of states and thereby greatly enhancing the likelihood of thermalization within a finite time.

### Quantum Field Theory Perspective

Beyond these statistical concerns, the Bohmian approach also faces deeper incompatibilities with established physical reality. Consider the ground state of the hydrogen atom, denoted $|\Psi_{gs}\rangle$. When treated as a two-particle system (an electron and a proton), the state can be written as:

$$\int dr_e dr_p \Psi(r_e, r_p) |r_e\rangle |r_p\rangle,$$

where $r_e$ and $r_p$ represent the spatial positions of the electron and proton, respectively (the spin degrees of freedom are ignored for simplicity). In Bohmian mechanics, specifying the physical state in this case requires not only the wavefunction but also the actual positions of the particles, $x_e$ and $x_p$, as additional variables.

However, quantum electrodynamics reveals that this ground state is not strictly a two-particle state. Instead, it contains additional components involving electron-positron pairs, a feature confirmed through high-precision spectroscopic measurements of the hydrogen spectrum. A more accurate description of the ground state takes the form:

$$|\Psi_{gs}\rangle = C_{ep} \int dr_e dr_p \Psi(r_e, r_p) |r_e\rangle |r_p\rangle + \int \cdots C_{2ee^+p} \int \cdots \int dr_{e1} \cdots dr_p \Psi^\circ(r_{e1}, r_{e2}, r_{e^+}, r_p) |r_{e1}\rangle \cdots |r_p\rangle + \int \cdots C_{3e2e^+}$$

where $r_{e^+}, r_{e^+}$ represent the positions of the positrons in the system, and $C_{ep}$, $C_{2ee^+p}$, and $C_{3e2e^+p}$ are the amplitudes associated with each component. Importantly, the two-particle component does not carry the full probability weight, reflecting the fact that both the number and configuration of particles in this state are fundamentally probabilistic.

This presents a fundamental challenge for Bohmian mechanics, which posits that particles are real entities with well-defined trajectories. If the physical content of a state includes probabilistic superpositions of varying particle numbers, it becomes impossible to consistently assign definite particle configurations. Furthermore, the interactions between particles across these different configurations are inherently probabilistic, rendering the deterministic description of particle motion within the Bohmian framework untenable.

These observations underscore a deeper physical reality: the fundamental entities are not individual particles but quantum fields—such as the electron-positron field, among others—whose states are superpositions of components with varying particle numbers up to infinity. The intrinsically probabilistic character of the particle content, even in the ground state of the hydrogen atom, strongly suggests that the classical or Bohmian notion of definite particles is not capable of capturing the true nature of quantum reality. This perspective also illustrates that particles do not possess independent ontological status in reality; rather, they are specific manifestations of the energy inherent in an underlying quantum field—namely, the particle modes of that energy.

Furthermore, one can note that classical field theory itself cannot be extended—even heuristically—into a consistent thermal framework, as exemplified by the ultraviolet catastrophe in classical electromagnetism. The central issue arises from the fact that the phase space of a classical field, if it can be meaningfully conceptualized, would be infinite-dimensional, since any finite volume of a classical field contains a continuum of physical degrees of freedom. This makes it impractical to impose a consistent and reasonable discretization of phase space, even at a superficial level.

In contrast, quantum field theory introduces a natural discretization of microscopic physical states through quantization, enabling mathematically consistent and physically meaningful thermal descriptions.

### Markovian Dynamics and Probabilistic Transitions

We have seen that in conventional quantum mechanics, the finiteness of physical states in a finite system is essential for the emergence of thermal phenomena. An important implication of this finiteness is that the dynamics of the system must be probabilistic rather than deterministic. Such dynamics govern transitions among a finite set of physical states.

First, quantum transitions between these discrete states cannot be continuous in nature. Each physical state is characterized by a set of quantum numbers, which take on discrete values that reflect the intrinsic properties of the wavefunction. Consequently, the transitions between these states must also be discrete, manifesting as stepwise, rather than continuous, changes in the quantum numbers.

Second, the time evolution of such a system should obey Markovian-like dynamics—where the future state of the system depends only on its immediate past, not on its full history. This Markovian-like requirement is fundamentally incompatible with deterministic behavior. To see this, consider a simple system with two quantum states, $|\phi_0\rangle$ and $|\phi_1\rangle$. Suppose the system starts in $|\phi_0\rangle$ at $t = 0$. Under a deterministic evolution, one must specify a precise time $t = \tau_1$ at which the system suddenly transitions to $|\phi_1\rangle$. However, if the dynamics is Markovian-like, there is no justification for why the transition must occur exactly at $\tau_1$ rather than at an earlier time $\tau_0$ (where $0 < \tau_0 < \tau_1$), since in both cases the system's immediate past is the same: the state $|\phi_0\rangle$. The Markovian-like property would then imply that the system should remain in $|\phi_0\rangle$ at both $t = \tau_0$ and $t = \tau_1$ under deterministic evolution, which contradicts the assumed sudden jump at $t = \tau_1$. Therefore, to comply with Markovian-like behavior, one needs to accept that the dynamics are fundamentally probabilistic.

An interesting analogy can be drawn with a particle decay process in a classical context. Suppose classical mechanics could somehow be extended to describe the decay of a single particle A, initially at rest, into two particles B and C. To preserve Markovian-like behavior in such a process, the dynamics must necessarily be probabilistic. There is no deterministic way to specify the direction of motion of particle B after the decay (see Fig. 2 [Figure 2: see original paper]). Mathematically, there is no continuous trajectory from the state of particle A to the state of two outgoing particles B and C moving in opposite directions. The transmutation inherently involves a discontinuous 'jump' in the specification of the system's state. Under the constraint of Markovian-like dynamics, this jump must be governed probabilistically. This indeterminacy is independent of whether the system is thermalizable; it arises solely from the nature of particle transmutation and the Markovian constraint. On the other hand, the quantization principle can be viewed as a framework for consistently accommodating both particle transmutation and thermalizability.

### Implications for Hidden-Variable Theories

Due to the inherent finiteness of microscopic physical states in any finite system, the plausibility of hidden-variable theories of quantum mechanics beyond Bohmian mechanics can also be questioned. These theories typically attribute the apparent randomness of quantum mechanics to the ignorance of underlying variables—so-called hidden variables—which are generally assumed to take continuous values, thereby implying a continuum of physical states. The continuity of hidden variables is essential for any attempt to recover deterministic dynamics. If, instead, the hidden variables were discrete, the dynamics would still be subject to probabilistic constraints under the requirement of Markovian-like behavior, as indicated by the discussions of the preceding paragraphs.

### Gravitational Waves and Quantum Nature

An intriguing question—one that can be at least partially addressed—is whether gravitational waves (GWs) are fundamentally quantum or classical in nature. Experimental observations establish that GWs carry energy and interact with astrophysical objects, such as binary neutron stars [5–8]. If GWs were classical in nature, they would exhibit unusual properties from the viewpoint of statistical physics. First, the gravitational wave field would not be thermalizable, similar to the case of the classical electromagnetic field. Second, when gravitational waves interact with quantum matter, the total energy of the combined system would tend to flow into the classical gravitational field, which, possessing a continuum of microscopic physical states, could absorb energy without bound.

Sustained interactions between the classical gravitational field and quantum matter would thus lead to the progressive cooling of the quantum sector. This outcome, however, may appear inconsistent with astrophysical observations. The universe is known to have undergone a hot and dense epoch, as described by the Big Bang theory (see, e.g., [9, 10]). Had strong interactions between classical gravitational wave fields and quantum matter during that epoch caused substantial energy depletion in the quantum sector, one would expect to observe distinctive signatures—markedly different from those predicted in the absence of such depletion.

Detecting the graviton—the quantum of the gravitational field—has long been regarded as an extraordinary experimental challenge. Early theoretical analyses suggested that atomic absorption of a graviton would be impractical [11, 12]. Various proposals have been made, including the use of resonant mass detectors, optomechanical systems, and quantum decoherence effects as indirect signatures [13–16]. A recent study proposed detecting single gravitons via a gravito-phononic analog of the photoelectric effect, using a solid-state sample cooled to near absolute zero [17]. More recently, we have proposed a new method for direct graviton absorption based on a natural quantum enhancement mechanism [18–20]. In this scheme, an ultraweak graviton-induced atomic transition is collectively amplified through a correlated absorption process involving multiple photons and multiple atoms, thereby significantly boosting the effective absorption rate. This approach is readily implementable with current technologies and may offer a practical route to resolving the quantum nature of gravitational waves within a relatively short time frame.

## Conclusion

In conclusion, we have argued that the thermalizability of physical systems fundamentally relies on the finiteness of microscopic physical states in a finite system. This, in turn, necessitates both a quantized framework and a probabilistic form of dynamics. Hidden-variable theories are inconsistent with these requirements and therefore incompatible with quantum realities.

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*Note: Figure translations are in progress. See original paper for figures.*