Probing Hadron-quark Transition Through Binary Neutron Star Merger (Postprint)

Authors: Ling-Jun Guo, Wen-Cong Yang, Yong-Liang Ma and Yue-Liang Wu

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Abstract

The cores of massive neutron stars offer a unique environment for the nuclear matter at intermediate density in the universe. The global characteristics of a neutron star, as well as the gravitational waves emitted from the mergers of two neutron stars, offer valuable insights into dense nuclear matter. In this paper, we comprehensively investigate the effect of the potential hadron-quark transition on the properties of neutron stars and the signals of the gravitational waves stemming from the merger of binary neutron stars, including waveforms, frequency evolutions as well as the spectrum curves, utilizing the equations of state constructed from the Maxwell ansatz, Gibbs ansatz and, the crossover scenario. We explicitly construct the equations of state in such a way that they converge at low and high densities therefore the differences are only from the scenarios of the transitions and the locations—or the parameters in the equation of state. Using such constructed equations of state, we simulate the signals of the gravitational wave (GW) and analyze their differences due to locations of the transition, the scenarios of the transition, and the masses of the component stars. We find that (1) in both the Maxwell ansatz and Gibbs ansatz, GW signals are sensitive to the location and the latent heat of the phase transition, (2) in the post-merger phase, the frequency of GW increases with the evolution in Maxwell type transition but is stable in the other two types of transitions and, (3) the amount of radiated energy is the biggest in Gibbs construction (GC) type transition and the smallest in the crossover construction (CC) type transition. By combining our findings with the expected detection of gravitational waves around (2–4) kHz from binary neutron star mergers and their associated electromagnetic signals, we expect to uncover some key characteristics of dense nuclear matter.

Full Text

Preamble

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Probing Hadron-quark Transition Through Binary Neutron Star Merger

Ling-Jun Guo[1],[2], Wen-Cong Yang[1],[2], Yong-Liang Ma[2],[3],[4],[5], and Yue-Liang Wu[4],[5],[6]

[1] Center for Theoretical Physics and College of Physics, Jilin University, Changchun 130012, China
[2] School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China; ylma@nju.edu.cn
[3] School of Frontier Sciences, Nanjing University, Suzhou 215163, China
[4] TaiJi Laboratory for Gravitational Wave Universe (Beijing/Hangzhou), University of Chinese Academy of Sciences, Beijing 100049, China; ylwu@ucas.ac.cn
[6] Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

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Abstract

The cores of massive neutron stars offer a unique environment for studying nuclear matter at intermediate densities in the universe. The global characteristics of neutron stars, as well as the gravitational waves emitted from binary neutron star mergers, provide valuable insights into dense nuclear matter. In this paper, we comprehensively investigate the effect of potential hadron-quark transitions on neutron star properties and gravitational wave signals from binary neutron star mergers, including waveforms, frequency evolutions, and spectrum curves. We utilize equations of state constructed from the Maxwell ansatz, Gibbs ansatz, and crossover scenario. We explicitly construct the equations of state such that they converge at low and high densities, ensuring that differences arise only from the transition scenarios, their locations, or the parameters in the equation of state. Using these constructed equations of state, we simulate gravitational wave signals and analyze their differences due to transition location, transition scenario, and component star masses. We find that: (1) in both the Maxwell and Gibbs ansatzes, GW signals are sensitive to the location and latent heat of the phase transition; (2) in the post-merger phase, the GW frequency increases with evolution in Maxwell-type transitions but remains stable in the other two types; and (3) the amount of radiated energy is largest in Gibbs construction (GC) type transitions and smallest in crossover construction (CC) type transitions. By combining our findings with expected detections of gravitational waves around (2-4) kHz from binary neutron star mergers and their associated electromagnetic signals, we anticipate uncovering key characteristics of dense nuclear matter.

Key words: gravitational waves -equation of state -dense matter -stars: neutron

1. Introduction

Despite decades of study, numerous fundamental questions regarding dense nuclear matter at intermediate densities remain to be clarified [?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. Its properties at densities $n \gtrsim 2n_0$, with $n_0 \approx 0.16$ fm^{-3} being the saturation density, cannot be accessed by terrestrial experiments or lattice QCD simulations. Neutron stars (NS) provide a unique gravitational environment for understanding cold nuclear matter at densities ranging from $\approx 2n_0$ to approximately $10n_0$ in the universe.

Observations of massive pulsars [?, ?, ?, ?, ?, ?, ?, ?] have effectively ruled out soft equations of state (EoSs) of nuclear matter, which are unable to produce neutron stars with masses $\gtrsim 2.0M_\odot$ (where $M_\odot$ is the solar mass). Joint constraints from the global properties of massive neutron stars and heavy-ion collision data have further narrowed the bounds on the EoS [?, ?]. Recently, the detection of gravitational waves (GWs) emitted during the merger of binary neutron stars (BNSs) by the LIGO-Virgo collaboration in GW170817 has provided a novel means for investigating dense nuclear matter [?, ?], supplementing electromagnetic probes. GW170817 has provided additional constraints on the nuclear matter EoS through the extracted tidal deformability—a quantity sensitive to the EoS that describes how much a star is deformed in the presence of a tidal field [?, ?]. Furthermore, the evolution of GWs emitted from merger processes carries information about dense nuclear matter, as the merger inevitably involves matter exchange.

In the low-density region up to $\sim 2.0n_0$, the EoS of nuclear matter can be well described by pure hadronic models and constrained by terrestrial experiments. At super-high densities $\gtrsim 100n_0$, the EoS can be reliably calculated using the first principles of strong interaction—quantum chromodynamics (QCD) [?]. However, within the intermediate density region relevant to compact star cores up to $\sim 10n_0$, the EoS presents a complex and challenging scenario [?, ?, ?].

Typically, compact star matter is described in terms of pure hadrons, quark-hadron transitions, quarkyonic matter, hyperons, or topological objects [?, ?, ?, ?, ?, ?, ?], with no definitive resolution among these models yet emerging. Although the inclusion of additional degrees of freedom, such as quarks and hyperons, softens the EoS, their presence cannot be definitively excluded and remains consistent with observations of massive neutron stars with masses $\gtrsim$

$2.0M_\odot$ [?, ?, ?, ?, ?, ?]. In the literature, BNS merger processes have been extensively explored using EoSs derived from models with only hadrons [?, ?, ?, ?], topology change effects [?], and hyperons [?, ?]. These studies clearly show that GWs emitted from the post-merger phase of BNS mergers are significantly influenced by the constituents of neutron star cores. The present work aims to investigate the potential quark-hadron transition in neutron stars through BNS mergers.

The dynamical mechanism of hadron-quark transition remains an open question. No definitive evidence currently exists to conclusively determine whether the transition is first-order or crossover along the density axis. When the hadron-quark transition is a first-order phase transition, the energy density of matter exhibits a distinct gap. This gap can either be independent of density or show a soft dependence, creating a soft hadron-quark mixing phase. EoSs with the former characteristic are termed Maxwell-type [?, ?, ?, ?, ?, ?], while the latter are referred to as Gibbs-type [?, ?, ?]. Moreover, lattice QCD simulations have indicated that the transition from hadrons to quark-gluon plasma at finite temperature and zero chemical potential, albeit with nonzero quark masses, is a continuous crossover [?, ?, ?]. Although cold dense matter cannot be directly accessed by lattice QCD simulations due to the sign problem, it is reasonable to contemplate the possibility of a crossover scenario for the transition at finite density within neutron stars [?, ?]. For explorations of various hadron-quark transitions in dense nuclear matter, ranging from first-order phase transitions to continuous crossovers, we refer to, e.g., [?, ?, ?, ?, ?]. The details of these three types of quark-hadron transitions will be discussed later.

In the literature, the effects of hadron-quark transition on GWs from BNS mergers have been studied through numerical relativity simulations and Bayesian analysis [?, ?]. In [?, ?, ?], a mean-field EoS with a strong first-order hadron-quark phase transition, characterized by the Polyakov loop, was utilized to study GW signals. The authors show that the appearance of quarks alters the post-merger signals considerably compared to expected signals from the inspiral phase dominated by hadrons, potentially leading to rapid black hole (BH) formation. It is also found that observations of g-modes with frequencies between 1 and 1.5 kHz could be interpreted as evidence of a sharp hadron-quark phase transition in neutron star cores [?]. Bauswein et al. studied GW signals using an EoS featuring a mixing phase of hadrons and quarks—the Gibbs-type EoS [?, ?]. They found that a phase transition takes place during the merger, resulting in a distinctive increase in the dominant post-merger GW frequency relative to the tidal deformability characterizing the inspiral phase. By combining a low-density pure hadronic EoS with a quark matter EoS modeled using an extended bag model, Prakash et al. developed a finite-temperature composition-dependent EoS with a first-order phase transition and simulated BNS merger processes [?]. They found that the softening of the EoS due to the phase transition led to the formation of more compact remnants and a shorter duration before collapsing to black holes. In addition, they found that the phase transition influenced the post-merger GW signal in terms of duration, amplitude, and peak frequency. In [?, ?], the authors studied GW signals from BNS mergers using an EoS with hadron-quark smooth crossover transition at intermediate density, constrained by perturbative QCD (pQCD) at super-high density, and compared the results with those obtained using an EoS with a first-order phase transition. Their findings suggested that early black hole collapse in the post-merger phase signals a softening of the EoS associated with quark matter onset in the crossover scenario.

In this paper, we conduct a comprehensive study of the potential impacts of hadron-quark transition on GW signals without considering strange quark degrees of freedom. Instead of focusing on a concrete model, we utilize EoSs generally parameterized in terms of Maxwell construction (MC) [?, ?], Gibbs construction (GC) [?, ?], and crossover construction (CC) [?, ?], as all effective models of EoSs can be categorized into one of these frameworks. In the low-density region, without loss of generality, we use a typical EoS derived from a pure hadronic model to investigate the effect of hadron-quark transition. In the super-high-density region, all EoSs are constrained by pQCD. The main progress distinct from existing literature can be summarized as follows:

1. When a hadron-quark phase transition occurs, we explicitly compare GW signals in both time and frequency domains by varying the parameters—the location of the phase transition and the energy gap relating to the energy difference between the ending and beginning of the coexisting phase —in each construction (MC and GC). We explicitly show that distinctions arise from the phase transition by illustrating the evolution of the EoSs covered in the BNS merger. The variation of these parameters can be attributed to thermal effects in compact star matter [?]. If a certain type of phase transition is favored, our results would be helpful for estimating the parameters in the construction, such as the location of the phase transition.

2. GW signals for the CC are also calculated by choosing three sets of parameters to investigate the impact of the transition region. Instead of choosing parameters randomly, we select their values in each set such that the EoSs from all three parameter sets coincide at very low and high densities. Therefore, the effects of the central value of the crossover density region, the intensity of the transition density, the location of the transition, and the energy gap can be explicitly illustrated.

3. We also compare GW signals stemming from BNS systems with different component masses, considering that the size of the quark core in a neutron star depends on the NS mass for a specific EoS. We find that the amplitude of GWs in the inspiral phase is very sensitive to the component mass of the BNS system.

4. By suitably choosing parameters in MC, GC, and CC-type transitions such that the EoSs converge to coincidence at low and high densities but differ in intermediate densities, we compare GW signals from these three types

of EoSs to investigate the effect of the transition type on GW signals. We find that the peak of the frequency spectrum can be used to distinguish the type of transition. This is one of our main qualitative conclusions in this work.

To our knowledge, these factors have not been systematically simulated and compared in a single work. Although some qualitative conclusions can be naively expected, we hope our detailed analysis and comparison in this work can provide a comprehensive understanding of hadron-quark transition and its GW diagnosis. The rest of this paper is arranged as follows. In Section 2 we discuss the EoSs constructed using MC, GC, and CC. We present the details of the techniques used in this work in Section 3. In Section 4 we illustrate the GWs stemming from BNS merger by varying the parameters in each model to explore the parameter dependence of GW signals. The effect of the size of the quark core in the parent stars on GW signals is probed in Section 5 by varying the component mass in the binary system. Finally, in Section 6 we investigate the impact of the type of hadron-quark transition on GW signals by unifying the EoSs at low and high densities. Our conclusion and discussion are given in the last section.

2. Equation of State with Hadron-quark Transition

By assuming that the hadron-quark transition is first-order, one can phenomenologically construct the hadron-quark mixed phase using a polytropic EoS of the form [**?**]

$$p = \kappa n^{\gamma} \tag{1}$$

where p is pressure, γ is the polytropic index, and κ is a parameter. By applying the first law of thermodynamics, which should be fulfilled in compact star matter, the energy density ε can be obtained as a function of density based on the polytropic form (1).

When the polytropic index is set to $\gamma = 0$, the system characterizes a sharp hadron-quark phase transition where pressure is independent of matter density. This transition, referred to as the MC, is encapsulated through the following EoS [**?**, **?**]

$$\varepsilon_{\mathrm{MC}}(p) = \begin{cases} \varepsilon_h(p) & p < p_{\mathrm{tr}} \\ \varepsilon_h(p_{\mathrm{tr}}) + \Delta\varepsilon & p = p_{\mathrm{tr}} \\ \varepsilon_h(p_{\mathrm{tr}}) + \Delta\varepsilon + c_s^{-2}(p - p_{\mathrm{tr}}) & p > p_{\mathrm{tr}} \end{cases} \tag{2}$$

Here $\varepsilon_{\mathrm{MC}}$ stands for the energy density of the MC, ε_h represents the energy density derived from pure hadronic models that will be taken, for example, from the APR EoS [**?**] in this work, p_{tr} denotes the pressure at which the phase transition occurs, and $\Delta\varepsilon$ signifies the energy gap between the two phases. The sound speed c_s adheres to the causality constraint $c_s \leq 1$ (in natural units) and is treated as constant in the constant speed of sound (CSS) approximation. Therefore, in the MC there are three free parameters: p_{tr}, $\Delta\varepsilon$, and c_s. In previous studies [**?**, **?**, **?**, **?**], NS properties were studied using this MC-type EoS by taking p_{tr}, $\Delta\varepsilon$, and c_s as free parameters or by estimating them using NJL-type models. It should be noted that when $\Delta\varepsilon \to 0$, the EoS approaches the model used in [**?**, **?**]. In this work, we treat these parameters as free variables to investigate their impact on GW signals.

On the other hand, if the polytropic index $\gamma \neq 0$, the EoS describes a soft phase transition termed GC [**?**, **?**]

$$\varepsilon_{\mathrm{GC}}(p) = \begin{cases} \varepsilon_h(p) & p < p_{\mathrm{tr}} \\ \varepsilon_h(p_{\mathrm{tr}}) + \Lambda(p - p_{\mathrm{tr}})^{\gamma_m} & p_{\mathrm{tr}} \leq p \leq p_{\mathrm{css}} \\ \varepsilon_h(p_{\mathrm{tr}}) + \Lambda(p_{\mathrm{css}} - p_{\mathrm{tr}})^{\gamma_m} + c_s^{-2}(p - p_{\mathrm{css}}) & p > p_{\mathrm{css}} \end{cases} \tag{3}$$

where $\varepsilon_{\mathrm{GC}}$ stands for the energy density of the GC, p_{css} denotes the pressure saturating the CSS where the quark phase commences, and unlike MC, there are no discontinuities in energy density. The constant Λ and the polytropic index γ_m are constrained by ensuring continuity of the EoS at transition points p_{tr} and p_{css}. They can be expressed equally through the energy gap between the beginning and ending of the coexisting phase. Hence, once the CSS c_s is chosen as will be done in the following, three free parameters can be identified in GC: p_{tr}, p_{css}, and $\Delta\varepsilon$. The differences in NS properties obtained from MC-type and GC-type EoSs were investigated in [**?**] with quark matter estimated using a bag model. Moreover, GW signals were simulated using the GC-type EoS [**?**, **?**] with some specific choices of parameters.

In addition to the MC and GC transitions discussed above, which are based on the description of a first-order phase transition between hadron and quark phases, a third conceivable hadron-quark transition exists—the smooth crossover transition termed CC. This transition can be introduced phenomenologically through the pressure function as [**?**, **?**]

$$p(n) = p_h(n) + p_q(n) \cdot f_{\mathrm{cross}}(n) \tag{4}$$

where p_h and p_q are the pressure functions described in terms of hadrons and quarks, respectively. The hadron part pressure function $p_h(n)$ can be obtained from the energy density $\varepsilon_h(n)$ in the MC (2) and GC (3) by using the thermal relation derived from (1). Similarly, the quark part of the CC EoS $p_q(n)$ is constructed by using the CSS parameterization of $\varepsilon_{\mathrm{MC}}(n)$ for $p > p_{\mathrm{tr}}$ in (2) through the thermal relation. Extrapolating p_h and p_q to the whole density region and joining them using a smooth function, one obtains the EoS of crossover transition. The smooth function is typically taken as

$$f_{\text{cross}}(n) = \frac{1}{2}\left[1 + \tanh\left(\frac{n - \bar{n}}{\Gamma}\right)\right] \tag{5}$$

where $\bar{n}$ denotes the central value of the crossover density region, and Γ accounts for the intensity of the transition. In this context, the EoS comprising hadronic matter and quark matter forms a globally continuous function of density, blurring the distinction between hadron and quark phases. The EoS constructed in this way has parameters $\bar{n}$ and Γ. These parameters can be estimated by requiring that the CC EoS continuously and monotonically converges to that of MC at low and high densities. A detailed discussion of the differences among these three types of hadron-quark transitions with possible parameter choices and their effects on GW signals will be given later. Star properties were explored with or without strangeness by varying the intensity of the transition in [**?**, **?**, **?**] with quark matter calculated using the Nambu-Jona-Lasinio (NJL) model. Here, the EoS of quark matter is parameterized in terms of the CSS approximation.

3. Methodology

In this section, for completeness, we summarize the main points of the numerical simulation used in this work. The details were presented in the referenced literature. For the purpose of illustrating the impacts of varying constructions of the hadron-quark transition and associated parameters, in the low-density region of compact star matter we choose pure hadronic $\varepsilon_h(p)$ from the typical pure-nucleon APR EoS [**?**]. Furthermore, in the case of MC and GC, we take the CSS parameterization of the EoS which can be viewed as the lowest-order terms of a Taylor expansion of the high-density EoS around the transition pressure [**?**, **?**]. At lower density after p_{tr} in MC and after p_{css} in GC, we take $c_s = 1$ to offer a stiff EoS. Conversely, at higher density, the value is taken to be $c_s = 1/\sqrt{3}$ to approach the conformal limit in perturbative QCD. A comparison of the EoSs for these three constructions will be depicted in the right panel of Figure 7 [Figure 7: see original paper].

In the numerical simulation of GW signals, we apply the following nine-segment isentropic polytropic approximation of the aforementioned EoS for the entire density region—both the low-density region of hadronic matter and the high-density region of quark matter—of neutron star matter up to $\sim 8.0n_0$:

$$\varepsilon(p) = \varepsilon_{i-1} + K_i(n - n_{i-1})^{\Gamma_i} \tag{6}$$

where ε_{i-1} and n_{i-1} ($i = 1$-9) are the energy density and baryon number density at the $i - 1$ segment endpoint, and K_i and Γ_i are the polytropic constant and index of the i-th segment, respectively. Explicitly, the segmentation is obtained by optimizing the density location for each segment with K_i and Γ_i fitted such that minimal deviation between the fitted EoS and original EoS is achieved.

For different constructions, the transition points p_{tr} and p_{css} are taken to be the same when parameterizing the Maxwell and Gibbs constructed EoSs to ensure that the only difference between EoSs arises from the types of constructions. Note that we artificially increased the sound speed slightly during the phase transition of the MC to avoid possible numerical errors that lead to unphysical results.

Furthermore, we also take thermal corrections into account through an ideal, nonrelativistic fermion gas approximation [**?**]. In this approximation, the pressure used in the GW simulation is decomposed as

$$p = p_{\mathrm{cold}} + p_{\mathrm{th}} \tag{7}$$

where p_{cold} is the EoS at zero temperature discussed above, and p_{th} is the thermal correction. In the ideal, nonrelativistic fermion gas approximation, the pressure is

$$p_{\mathrm{th}} = (\Gamma_{\mathrm{th}} - 1)\varepsilon_{\mathrm{th}} \tag{8}$$

where $\varepsilon_{\mathrm{th}}$ is the thermal part of specific internal energy, and the thermal adiabatic index Γ_{th} can be taken as $\Gamma_{\mathrm{th}} \approx 1.75$ [**?**].

The GWs emitted from BNS mergers were investigated by performing numerical relativity simulations using the Einstein Toolkit [**?**, **?**], which is a community-driven software platform of core tools [**?**, **?**, **?**, **?**]. Generally, the simulation can be decomposed into two steps. The first step is to generate the initial data [**?**], such as the initial frequency and angular momentum of the equilibrium state of the BNS system before merger, by using the initial conditions of irrotational binary systems [**?**]—i.e., the separation distance, component masses, and the EoS (7). This step is fulfilled using the public codes of the LORENE library (https://lorene.obspm.fr/). The second step is the relativistic hydrodynamic simulation, which is performed utilizing the Einstein Toolkit [**?**, **?**], a numerical relativity simulation platform based on the Cactus Computational Toolkit [**?**, **?**, **?**, **?**] (https://www.cct.lsu.edu/~eschnett/McLachlan/; http://cactuscode.org/)—a software framework for high-performance computing to advance and support research in relativistic astrophysics and gravitational physics. In this step, the initial data generated in the first step, the grid resolution, and the parameterized EoS including the thermal contribution (8) are input. The main tools we used are Carpet (adaptive mesh refinement methods), GRhydro (hydrodynamic evolution), and McLachlan (spacetime metric evolution). We used a fourth-order Runge-Kutta method for solving differential equations and fourth-order finite difference stencils for spacetime evolution, HLLE for the Riemann solver, WENO for reconstruction, and the system grid uses 2D symmetry across the x-y plane. The grid resolution in our simulation is taken as ~ 260 m.

Using the Einstein Toolkit module WeylScal4, the GW signal is extracted from each simulation with the standard method of calculating the Newman-Penrose scalar ψ_4 during numerical evolution [?]. Its asymptotic limit (for $r \to \infty$) describes gravitational radiation where h is the complex GW strain $h = h_+ + i h_\times$ with h_+ and $h_\times$ being the polarizations in the transverse-traceless (TT) gauge. The Newman-Penrose scalar ψ_4 is computed on a spherical surface far away from the source and then decomposed in spin-weighted spherical harmonics of weight-2, which is done by the thorn Multipole. Since ψ_4 falls off as $1/r$ in asymptotically flat space, results for $r\psi_4$ (and equivalently rh) are often reported. To obtain the GW strain h_{lm}, double time integration must be performed numerically using a simple trapezoidal rule [?] formalized in a first-order gauge-invariant representation of the variables [?, ?, ?] as

$$h_{lm}(t) = \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \psi_{4,lm}(t'') + Q_0 + Q_1 t \tag{9}$$

where Q_0 and Q_1 are integration constants fitted to the resulting strain with a first-order polynomial and subtracted from the strain itself. To eliminate low-frequency unphysical oscillations in the strain amplitude caused by unresolved high-frequency noise aliased into the low-frequency signal during integration, we further subtract from the strain a second-order polynomial fit [?]

$$Q_0 + Q_1 t + Q_2 t^2 \tag{10}$$

which is found sufficient to eliminate the unphysical drift from the dominant (2,2) mode considered in this work. In the following, instead of $\bar{h}_{lm}$, we will present the results of rh_{lm}, which are constant in r when $r \to \infty$.

The GW signal in the time domain, as illustrated in the following figures, can be extracted. Consequently, the time-frequency-amplitude relation can be obtained by using the short-time Fourier transform (STFT) [?, ?, ?]

$$\tilde{h}(f,t) = \int_{-\infty}^{\infty} h(t') w(t'-t) e^{-2\pi i f t'} dt' \tag{11}$$

where $w(t)$ is the window function.

In addition, after obtaining $h_{lm}(t)$ with the above procedure, the radiated energy can be obtained via

$$E_{\mathrm{GW}} = \frac{1}{32\pi} \sum_{l,m} \int_{-\infty}^{\infty} |\dot{h}_{lm}(f)|^2 df \tag{12}$$

4. Impact of the Parameters in the EoS on GW Signals

We first discuss the impact of parameters in different constructions on GWs from BNS mergers. For this purpose, we choose equal-mass systems with component mass $1.1M_{\odot}$ and an initial separation of 40 km.

We depict the global properties of neutron stars calculated using MC-type transition in Figure 1 [Figure 1: see original paper]. The left panel indicates that the smaller the latent heat $\Delta\varepsilon$, the stiffer the EoS after phase transition. When nuclear matter at low density is described by the APR EoS, the latent heat is estimated to be $\lesssim$ 200 MeV fm^{-3} with respect to constraints imposed by NICER. The right panel shows that the tidal deformability calculated from our parameter choice saturates the constraints extracted from GW170817.

From Figure 1 [Figure 1: see original paper] one can see that a quark core appears in neutron stars with mass $\gtrsim 1.25M_{\odot}$ when the transition pressure is $p_{\mathrm{tr}} \approx 20$ MeV/fm^3 ($\sim 2.8n_0$) and the stars become hybrid ones. When the latent heat is large, e.g., $\Delta\varepsilon \gtrsim 350$ MeV/fm^3, the twin star scenario arises due to the hadron-quark phase transition [**?**, **?**, **?**]. Since the initial configurations we consider are either BNS systems with mass $1.1M_{\odot}$ or a heavier star with $\Delta\varepsilon = 150$ MeV/fm^3, the twin star configuration does not affect our initial choices.

The GW signals resulting from BNS mergers using MC-type transition are plotted in Figure 2 [Figure 2: see original paper]. In the upper row we fix $p_{\mathrm{tr}} \approx 20$ MeV/fm^3 ($\sim 2.8n_0$) and vary latent energy $\Delta\varepsilon$, while in the lower row we choose $\Delta\varepsilon = 150$ MeV/fm^3 and vary p_{tr} to investigate the model-parameter dependence of GW signals. The upper-left panel depicts waveforms obtained with varying latent energy $\Delta\varepsilon$. We can see that differences between waveforms start to appear only after the merger of the binary stars. This is because, for stars with mass $1.1M_{\odot}$, the hadron-quark transition does not affect the star properties, so GWs in the inspiral phase are not affected by the transition.

However, in the post-merger phase, since the central density of the remnant exceeds the density where the transition occurs, the waveforms are notably affected by the hadron-quark transition. The upper-middle panel illustrates the evolution of GW frequencies. One can see that frequencies remain nearly constant during the inspiral phase since the two stars are far apart and the inspiral frequency is nearly steady. However, when the two stars are close in the inspiral phase and during merger, the frequency steadily rises due to violent matter exchange. A noticeable trend emerges where higher latent heat, or softer EoS, leads to a swifter frequency increase. The frequency spectrum is shown in the upper-right panel. The result shows that after chirp—the coalescence of the two stars—the frequency spectrum keeps decreasing. Given that the phase transition density exceeds the core density of the parent stars, the chirp frequencies and spectrum are not significantly different.

The green line in Figure 3 [Figure 3: see original paper] shows the evolution of the EoSs covered in the BNS merger. One can clearly see that up to post-merger time $t - t_{\mathrm{merger}} = 2$ ms, the phase transition does not enter the evolution, so the GWs are identical. However, at $t - t_{\mathrm{merger}} = 5$ ms, the central densities are in the quark phase and the larger the latent heat, the larger the quark cores involved, and therefore the GWs differ in the post-merger phase.

After merger, the BNS remnant can form a differentially rotating hyper-massive neutron star (HMNS) exceeding the maximum mass derived from TOV equations. With decreasing differential rotation, the HMNS would collapse to a black hole, indicated by the null amplitude of GWs shown in the figure. The duration of the ring-down period depends on the EoS. As seen from the upper row of Figure 2 [Figure 2: see original paper], the larger the latent heat $\Delta\varepsilon$, the shorter the HMNS duration before black hole formation—that is, the shorter the ring-down period and the higher the frequency after merger. These phenomena can be elucidated by the softness of the EoS attributed to the phase transition.

The lower row of Figure 2 [Figure 2: see original paper] showcases GW signals from BNS mergers with $\Delta\varepsilon = 150$ MeV/fm^3 but different values of p_{tr} to elucidate the effect of transition location. Unlike the upper-left panel, GW waveforms in this scenario exhibit substantial discrepancies even in the inspiral phase. With a lower p_{tr} (i.e., a larger quark core exists in the NS), the binary system merges quicker compared to those with higher p_{tr}. Therefore, after merger, the GW frequency calculated with $p_{\mathrm{tr}} \approx 15$ MeV/fm^3 ($\sim 2.2n_0$) is smaller than that calculated with $p_{\mathrm{tr}} \approx 20$ MeV/fm^3 (lower-middle panel), and the chirp frequency spectrum is smaller for the former (lower-right panel).

The variation of GW signals in the inspiral phase can be attributed to the tidal deformability Λ of the neutron stars involved, which describes the deformability or the level of difficulty in matter exchange in the BNS system. As illustrated in the right panel of Figure 1 [Figure 1: see original paper], for the selected parameters from the lower row of Figure 2 [Figure 2: see original paper], $\Lambda \approx 1540$ for $p_{\mathrm{tr}} \approx 15$ MeV/fm^3 and $\Lambda \approx 1160$ for $p_{\mathrm{tr}} \approx 20$ MeV/fm^3. Consequently, the duration of the inspiral phase is shorter for the latter scenario.

We next show GW signals from BNS mergers simulated with GC transition in Figure 4 [Figure 4: see original paper]. In the upper row, GWs from BNS mergers with the same transition pressure range (p_{tr} to p_{css}) but varying $\Delta\varepsilon$ are displayed, while the lower row shows simulations with the same $\Delta\varepsilon$ but diverse pressure ranges. In both left panels, GWs exhibit differences in both inspiral and post-merger phases because the hadron-quark transition enters the core of the star with mass $1.1M_{\odot}$ and the latent heat effects differ. The upper-left panel demonstrates that with smaller $\Delta\varepsilon$, the frequency is smaller (albeit marginally), while the amplitudes in the post-merger phase are larger. Conversely, the lower-left panel indicates that a narrower range of transition pressure leads to slightly higher inspiral frequency and larger GW amplitudes in the post-merger phase. From the middle column, one can see that the frequency remains roughly constant in the post-merger phase, decreasing slightly with latent heat (upper panel) while decreasing with location of the phase transition pressure (lower panel). The right column reveals that a higher latent heat value results in the frequency spectrum peak shifting toward higher values while its magnitude is diminished. Conversely, when the phase transition location is reduced, the frequency spectrum peak shifts toward higher values.

Finally, in Figure 5 [Figure 5: see original paper] we present GW signals resulting from CC-type transition. To investigate the impact of the transition region, we select three parameter sets as shown in the figure, ensuring coincidence of the EoSs at low and high densities. We find that NS properties from all three parameter sets are consistent with NICER [**?**, **?**]. The results clearly show that lower transition density leads to a shorter inspiral phase, briefer post-merger phase, and reduced wave amplitudes in the post-merger phase. The frequency evolution (middle panel) indicates that in both inspiral and post-merger phases it does not change significantly. In contrast, the magnitude of the frequency spectrum peak is sensitive to the parameters involved and therefore suggests a possible means to distinguish the CC EoS.

5. Effect of Component Mass on GW Signals

We next discuss the variation of GWs arising from different component masses of the BNS system. We first set $p_{\mathrm{tr}} \approx 20$ MeV/fm^3 and $\Delta\varepsilon = 150$ MeV/fm^3 and consider equal-mass systems with component masses $1.1M_\odot$, $1.3M_\odot$, and $1.5M_\odot$. From the left panel of Figure 1 [Figure 1: see original paper] one concludes that the cores of stars with masses $1.3M_\odot$ and $1.5M_\odot$ are in the quark phase, although their sizes differ. The upper row of Figure 6 [Figure 6: see original paper] clearly demonstrates that component mass significantly impacts the maximum GW amplitude and merger time of BNS. As the mass of individual stars increases, the inspiral period shortens, the maximum amplitude grows, and the ring-down period decreases. These distinctions align with our previous discussions on EoS stiffness.

The frequency spectrum shown in the upper-middle panel indicates that when quark matter enters the cores of the parent stars, the frequency becomes larger compared to stars without quark cores. One can conclude from the upper-right panel that at low frequencies, the amplitude of GWs from BNS with heavier component masses is easier to detect, but at high frequencies the amplitude is not sensitive to component mass.

To investigate whether the above conclusion on component mass effects depends on phase transition location, we illustrate results with $p_{\mathrm{tr}} \approx 15$ MeV/fm^3 but fix other parameters in the lower row of Figure 6 [Figure 6: see original paper]. It can be easily seen that the inspiral period is further shortened due to the larger quark core (lower-left panel), the frequency domain is not sensitive to component mass since all three combinations considered include quark cores (lower-middle panel), and the tendency of frequency dependence of amplitude remains intact (lower-right panel). The same conclusions and reasoning apply to results from GC- and CC-type transitions, so we will not repeat them here.

6. Effect of the Type of the Hadron-quark Transition

Finally, we discuss how the nature of the hadron-quark transition impacts GWs from BNS mergers. For this purpose, we choose typical parameter values of $(p_{\mathrm{tr}}, \Delta\varepsilon) = (20, 150)$ MeV/fm^3 for MC and $(p_{\mathrm{tr}}, p_{\mathrm{css}}, \Delta\varepsilon) = (15, 50, 223)$ MeV/fm^3 for GC. With these parameters, we ensure precise alignment of the EoSs of MC and GC before and after phase transitions, as shown in the right panel of Figure 7 [Figure 7: see original paper].

Similarly, as shown in Figure 7 [Figure 7: see original paper], the CC is constructed to ensure that the EoS is monotonically and continuously convergent to that of MC at low and high densities. With this requirement, the parameters are taken as $(p_{\mathrm{tr}}, \Delta\varepsilon, \bar{n}, \Gamma) = (35\ \mathrm{MeV\ fm}^{-3}, 150\ \mathrm{MeV\ fm}^{-3}, 3.1, 0.6)$. In this way, differences in GWs arise only from the constructions of the transition.

In the left panel of Figure 7 [Figure 7: see original paper], we present simulations for a binary neutron star with equal masses of $1.1M_{\odot}$. One can see that in the inspiral phase, the amplitudes of GWs from MC and CC are similar but smaller than those from GC. Among the three constructions, MC results in the shortest inspiral period, while GC yields the longest inspiral period. With increasing maximum density and pressure during the merger process, the transition starts to play a considerable role in the post-merger phase. This is reflected in waveform differences after $t - t_{\mathrm{merger}} > 0$ ms, as shown in the figure, where CC gives much smaller amplitude than MC and GC.

The impact of transition becomes more evident in the time-frequency diagram in Figure 8 [Figure 8: see original paper], derived from Figure 7 [Figure 7: see original paper] using the short-time Fourier transform technique. Across all three constructions, GW frequencies show a swift rise during the inspiral phase at $t - t_{\mathrm{merge}} < 0$, accompanied by peak GW amplitudes indicated by brighter colors in the graphs. Notably, a compelling characteristic is that GW frequency from the Maxwell construction increases even after merger, while those from Gibbs and crossover constructions decrease.

We compare the radiated energy (power) carried by gravitational waves with different transition constructions in Figure 9 [Figure 9: see original paper]. The EoSs used are the same as those utilized in Figure 7 [Figure 7: see original paper]. One can see that the widths of the power spectra vary depending on the specific EoSs: the larger the latent heat, the broader the power for MC and GC. Since hadronic and quark components coexist across all density ranges within neutron stars, the power spectrum is narrower for the CC scenario. Regarding the radiated energy, one can estimate the remnant masses as $\Delta m \approx (2.18, 2.178, 2.186)M_{\odot}$ for MC, GC, and CC, respectively. Concerning the maximum NS masses available within these three constructions, one can easily understand the waveforms of Figures 2 [Figure 2: see original paper], 4 [Figure 4: see original paper], and 5 [Figure 5: see original paper].

Figure 10 [Figure 10: see original paper] illustrates the spatial density evolution resulting from BNS mergers employing different transition constructions. The top, middle, and bottom rows correspond to simulations with MC, GC, and CC-type transitions, respectively. From the density contours in the top row we can see that the CSS phase persists within the compact star cores throughout the inspiral phase. In contrast, in the GC scenario depicted in the middle row, the CSS phase emerges in the post-merger phase once the density reaches a critical value. In the bottom row (crossover scenario), no distinct contours are present, as it is impossible to distinguish the exact density where the transition begins and ends.

7. Conclusion and Discussion

We comprehensively analyzed the impacts of various types of hadron-quark transition on neutron star properties and GWs emitted during BNS mergers. Explicitly, we compared differences in GW signals in both time and frequency domains arising from parameter choices in the EoS, scenarios of hadron-quark transition, and masses of binary components. The main results can be summarized as follows:

1. In MC, GW signals are sensitive to the location of the phase transition and the latent heat. The smaller the transition density, the larger the GW frequency in the post-merger phase, the shorter the inspiral period, and the smaller the frequency of the amplitude peak. The larger the latent heat, the higher the GW frequency in the post-merger phase and the shorter the post-merger period. The latent heat does not significantly affect the frequency amplitude. The same conclusions are found for GC-type transition. For CC-type transition, the inspiral period is sensitive to the hadron-quark transition parameter p_{tr} in the CSS parameterization of quark matter.
2. GW signals are sensitive to the size of the quark core in neutron stars. The larger the quark core, the shorter the inspiral period and the larger the GW frequency in the post-merger phase.
3. A comparison of GW signals emitted using the three types of constructions shows that in the post-merger phase, the GW frequency increases with evolution in MC-type transition but is not sensitive in the other two types. The radiated energy is largest in GC-type transition and smallest in CC-type transition.

These findings indicate that features of time-frequency evolution, inspiral duration, peak frequency, and post-merger phases serve as distinguishing factors to identify the transition type, in addition to precise GWs. By integrating present GW signal results, electromagnetic signals from other observations, and future GW detections, there is potential to differentiate between various types of hadron-quark transitions.

In the above numerical simulation, we used a typical resolution of 260 m and fixed mesh grid. To check simulation convergence and uncertainties arising from resolution, we increased the resolution to 120 m. The results illustrated in Figure 11 [Figure 11: see original paper] show that although there are tiny differences in GW signals, the conclusions discussed above remain intact. This simple check indicates that our results are not affected by numerical resolution.

We only presented the (2,2) mode, which is the dominant and quasi-symmetric quadrupole radiation mode of the multipole expansion of gravitational radiation, since we focused on equal-mass binary systems in the above discussion. In some merger processes such as unequal-mass binary systems, higher modes like the (2,1) mode may also be interesting since the symmetry of equal-mass systems is lost [**?**]. We leave detailed discussion of these aspects to future publication.

Although it is difficult to measure GWs after merger in BNS systems at present, it is promising to establish the Neutron Star Extreme Matter Observatory (NEMO) to measure kilohertz GWs in the upcoming decade [**?**, **?**], as well as potential detection in O5 or 3G [**?**]. Therefore, it is essential to conduct thorough analysis of available EoSs and establish a database for future matching between observational data and theoretical simulations. This process will enable refinement and constraint of compact star models based on matched results.

In this work, we only considered zero-temperature EoS with finite temperature effects implemented through the ideal, nonrelativistic fermion gas approximation [**?**]. Although this approximation works well for hadronic matter, the deviation is significant for quark matter. The temperature dependence of the EoS and phase transition boundaries is highly relevant for BNS mergers and GW signals from the post-merger phase, which might be measurable in next-generation observatories [**?**, **?**]. This dependence should be properly included in explicit models if one wants to pin down physics by matching model calculations with observational data. Since we varied the type of hadron-quark transitions and model parameters to investigate their effects on GW signals, thermal effects are beyond the scope of this work and our conclusions remain intact.

Finally, we note that as one can see from the present calculation and existing literature, the lower the transition pressure/density, the softer the EoS and thereby the easier the compact quark matter object collapses to a black hole. In the early universe before hadronization, there may have been compact objects composed solely of quarks. As temperature decreases, these compact objects may either collapse to or annihilate into black holes, potentially providing the origin of certain black holes [**?**, **?**]. We will investigate this aspect in future work.

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ORCID iDs

Ling-Jun Guo https://orcid.org/0009-0007-9449-9576
Yong-Liang Ma https://orcid.org/0000-0001-9154-529X

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