Online Beam Phase Calibration with Detuning Compensation for Normal-Conducting Cavities under Closed-Loop Operation

Authors: Qiu, Dr. F, Xu, Mr. Chengye, Zeng, Prof. Rihua, Prof. Yuan He (何源), Wei, Shihui, Peng, Mr. Jiayi, Yang, Dr. Lijuan, Yang, Prof. Ziqin, Zhang, Mr. Zhouli, Wang, Dr. Zhi-Jun, Dr. Muyuan Wang, Maiano, Dr. Cecilia, Pierini, Dr. Paolo, Qiu, Dr. F

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Abstract

Accurate calibration of the beam phase (the phase of the beam arrival relative to the cavity accelerating field) is essential for maintaining the stability and efficiency of linear accelerators. Conventional offline phase-scan methods, typically performed during commissioning or maintenance, require considerable manual effort and can disrupt regular operation. Moreover, these methods cannot effectively track gradual drifts caused by ambient conditions. An online beam-phase calibration technique using beam-induced radio-frequency (RF) transients was initially developed at DESY for superconducting cavities operating under open-loop conditions. Extending DESY's method to normal-conducting cavities at the European Spallation Source (ESS) introduces challenges. When the beam pulse length approaches the cavity time constant (= 1/ *{0.5}), where (* {0.5}) is the cavity half-bandwidth, detuning effects distort the trajectory of the beam-induced RF transient and degrade the beam phase measurement accuracy. Furthermore, open-loop operation is generally not advisable for high-current proton linacs because of stability and safety concerns. To address these issues, we revisited the cavity differential equations and proposed a detuning compensation method that corrects the distorted trajectory in the in-phase/quadrature plane. In addition, by analyzing the initial 1.4~( )s transient response before low-level RF (LLRF) feedback becomes active, beam phase calibration can be achieved under closed-loop operation. Experimental results indicate that the proposed method agrees well with beam position monitor (BPM)-based measurements. This approach enables real-time beam phase monitoring without interrupting closed-loop operation and can be adapted to similar accelerator systems.

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Feng Qiu,[1,2,*] Chengye Xu,[1] Rihua Zeng,[3] Yuan He,[1,2,†] Shihui Wei,[1] Jiayi Peng,[1,2] Lijuan Yang,[1] Ziqin Yang,[1] Zhouli Zhang,[1] Zhijun Wang,[1] Muyuan Wang,[4] Cecilia Maiano,[4] and Paolo Pierini[4]

[1]Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

[2]School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China

[3]Advanced Energy Science and Technology Guangdong Laboratory, Huizhou 516000, China

[4]European Spallation Source ERIC, 22100 Lund, Sweden

Abstract. Accurate calibration of the beam phase (the phase of the beam arrival relative to the cavity accelerating field) is essential for maintaining the stability and efficiency of linear accelerators. Conventional offline phase-scan methods, typically performed during commissioning or maintenance, require considerable manual effort and can disrupt regular operation. Moreover, these methods cannot effectively track gradual drifts caused by ambient conditions. An online beam-phase calibration technique using beam-induced radio-frequency (RF) transients was initially developed at DESY for superconducting cavities operating under open-loop conditions.

Extending DESY's method to normal-conducting cavities at the European Spallation Source (ESS) introduces challenges. When the beam pulse length approaches the cavity time constant $\tau = 1/\omega_{0\cdot 5}$, where $\omega_{0\cdot 5}$ is the cavity half-bandwidth, detuning effects distort the trajectory of the beam-induced RF transient and degrade the beam phase measurement accuracy. Furthermore, open-loop operation is generally not advisable for high-current proton linacs because of stability and safety concerns. To address these issues, we revisited the cavity differential equations and proposed a detuning compensation method that corrects the distorted trajectory in the in-phase/quadrature plane. In addition, by analyzing the initial 1.4 µs transient response before low-level RF (LLRF) feedback becomes active, beam phase calibration can be achieved under closed-loop operation.

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Keywords: beam phase calibration, transient beam-loading, detuning effect, closed-loop operation, low-level RF systems, normal-conducting cavity, particle

accelerator

INTRODUCTION

The European Spallation Source (ESS) in Lund, Sweden, is constructing a high-intensity, 2 GeV proton linear accelerator designed for 5 MW operation [1]. The accelerator comprises a normal-conducting (NC) front end operating at 352.21 MHz followed by a superconducting (SC) linac at 704.42 MHz (excluding the spoke cavities). As shown in Fig. 1 [Figure 1: see original paper], the Drift Tube Linac (DTL) and SC sections are subdivided into individual tanks and cryomodules, with the grey-shaded cryomodules indicating those that will remain unpowered during the initial 2 MW operation at 800 MeV [2]. The high-level parameters for ESS are summarized in Table 1 . Designed for a 62.5 mA peak beam current with a 14 Hz repetition rate, ESS demands accurate and precise RF phase control in each cavity to maintain beam quality and minimize beam losses. Accurate measurement of the beam phase (the phase experienced by the beam relative to the RF field) is critical, as deviations cause off-nominal energy gains, phase slip, emittance growth, and potential beam loss, especially significant in high-power machines. By 2023, initial commissioning had accelerated beam through the early NC sections to approximately 74 MeV.

Various methods exist for calibrating beam phase. Direct approaches include phase-scan techniques (e.g., ΔT phase scan [4–7], phase-scan signature matching [7, 8]), which vary the RF phase and observe downstream beam changes. While effective, these methods often need dedicated beam studies, requiring sequential calibration of individual cavities with adjacent ones deactivated or significantly detuned, thus disrupting normal operations. They are typically performed offline during commissioning or maintenance, making them labor-intensive. Furthermore, gradual ambient drifts, such as changes in temperature and humidity, within low-level RF (LLRF) control loops necessitate periodic recalibration during routine operation.

Alternative strategies are designed to minimize disruption to the normal operation of the accelerator. The "drifting beam method," pioneered at SNS [9], uses the LLRF system to measure transient RF signals excited by the beam in an unpowered SC cavity. Analyzing these transients yields the relative beam phase. This avoids extra beam diagnostics but requires temporarily unpowering the cavity. Researchers at DESY proposed another method based on linear fitting of the trajectory of the RF transient induced by a beam pulse in the in-phase/quadrature (I/Q) plane. The beam pulse is typically short, on the order of tens of microseconds, and the measurement is performed within a powered SC cavity [10–14]. DESY's approach allows online measurement but usually requires operating the SC cavity in an open-loop configuration.

Adapting DESY's approach to the ESS NC buncher cavity poses a distinct challenge [15, 16]. The buncher cavity has a measured half-bandwidth ($\omega_{0\cdot5}$) of approximately 18 kHz, corresponding to a time constant ($\tau = 1/\omega_{0\cdot5}$) of about 9 µs. When using short diagnostic beam pulses (e.g., 5 µs) whose duration is comparable to τ, cavity detuning causes a significant non-linear rotation (deflection) of the beam-induced RF transient trajectory. As a result, the trajectory deviates from the expected linear behavior, which substantially reduces the accuracy of beam phase calibration based on linear fitting. This issue is less pronounced in SRF cavities, where time constants are typically on the order of milliseconds due to their much higher loaded quality factors (Q) [17–21].

On the other hand, operating RF cavities in open-loop mode is neither practical nor safe for high-current proton linacs such as ESS. In open-loop operation, the RF field can deviate from its setpoint, and strong beam loading [22] further increases the risk of beam loss. Therefore, it is preferable to maintain closed-loop LLRF operation during measurements. To avoid interference from the feedback system during phase calibration, we deliberately select a calibration window approximately equal to the LLRF loop delay (τ_d 1.4 µs) and restrict the analysis to the beam-induced RF transient before the feedback loop becomes active. This ensures that only the portion of the transient signal unaffected by feedback is considered, allowing beam phase calibration under closed-loop conditions.

To mitigate detuning-induced errors, we developed a detuning compensation algorithm by solving the cavity's differential equations to predict the theoretical deflection angle (α{th}) *of the transient trajectory. Subtracting the calculated* α{th} from the fitted angle from the RF transient trajectory enables accurate beam phase calibration.

In the following sections, we first revisit the cavity differential equation and analyze the impact of cavity detuning on the beam-induced RF transient. Based on this analysis, a detuning-aware beam phase calibration method is proposed, together with a practical implementation strategy under closed-loop conditions. Experimental validation is performed using the ESS MEBT buncher cavity, demonstrating the effectiveness of the proposed technique under realistic operating scenarios.

PRINCIPLE OF BEAM PHASE CALIBRATION USING BEAM-INDUCED RF TRANSIENTS

The beam phase calibration method based on beam-induced RF transients in powered RF cavities was originally proposed by DESY [13, 14]. In the following subsection, we present the theoretical basis of DESY's approach, including the derivation of the transient field trajectory and the conditions under which the linear approximation directly applies.

Transient RF Trajectory Calibration: Concept and Example

Figure 2 illustrates the basic principle of beam phase measurement using beam-induced RF transients. As shown in Fig. 2a, the cavity field is initially in a steady state with normalized amplitude and zero phase. When a short beam pulse passes through the cavity in open-loop mode, it induces a transient perturbation, denoted as $V_{cb}(t)$, shifting the cavity voltage from its nominal value to a new complex value $A_{tail} e^{j _{tail}}$. This transient response traces a trajectory in the I/Q plane (Fig. 2b), where the horizontal axis represents the in-phase (I, real) component and the vertical axis the quadrature (Q, imaginary) component. The angle ϕ_b between the steady-state V_c vector and the direction of this trajectory reflects the beam phase [11, 13, 14, 23, 24].

Figure 3 shows the measured beam-induced RF transient in the ESS buncher cavity B2 (see Fig. 1) under open-loop conditions, generated by a 5 µs, 55 mA beam pulse with a beam phase of approximately -44°. The relevant RF parameters are summarized in Table 2 . As seen in Fig. 3a, the cavity remains in a steady state prior to beam arrival. At approximately t = 177 µs, the cavity voltage $V_{c1}(t)$ in the presence of the beam exhibits a pronounced drop in both amplitude and phase relative to the no-beam case $V_{c0}(t)$. A zoomed-in view of the shaded region in Fig. 3a, shown in Fig. 3b, reveals the detailed structure of the beam-induced RF transient. Figure 3c displays the corresponding transient trajectory of $V_{cb}(t)$ in the I/Q plane, reconstructed from sampled RF waveforms acquired by the digital LLRF system.

The time evolution of the cavity field, with and without beam loading, can be described by the following differential equations [23, 25–29]:

$$dV_{c0}(t)/dt + (\omega_{0\cdot5} - j\Delta\omega_0)V_{c0}(t) = (2\beta\omega_{0\cdot5})/(1+\beta)\ V_{f0}(t)$$
$$dV_{c1}(t)/dt + (\omega_{0\cdot5} - j\Delta\omega_1)V_{c1}(t) = (2\beta\omega_{0\cdot5})/(1+\beta)\ V_{f1}(t) + \omega_{0\cdot5}V_b(t)$$

In these equations, t denotes time. The variables V_{f0} and V_{f1} denote the cavity forward voltages without and with beam. The detuning terms $\Delta\omega_0$ and $\Delta\omega_1$ account for cavity detuning in the absence and presence of the beam, respectively. The parameter $\omega_{0\cdot5}$ is the cavity half-bandwidth, assumed to be independent of beam loading, and β is the coupling coefficient of the input coupler. The beam-induced voltage $V_b(t)$ is given by:

$$V_b(t) = R_L I_b(t)$$

where $I_b(t)$ is the RF component of the bunched beam current. The commonly referenced beam current typically refers to the average (DC) value. For short bunches, the RF current amplitude satisfies $|I_b|$ $2I_{b0}$, where I_{b0} is the DC beam current [25]. The loaded resistance R_L is defined as:

$$R_L = 0.5\ (r/Q)\ Q_L$$

where Q_L is the loaded quality factor of the cavity, and r/Q is the normalized shunt impedance.

To simplify the analysis without loss of generality, we assume that the beam-induced voltage $V_b(t)$ points along the X-axis (real axis) in the complex plane, i.e., its phase angle is set to zero. Under this assumption, $V_b(t)$ $^{+}$. For analytical convenience, we further model the beam input as a unit step function: $V_b(t) = u(t)$, where $u(t) = 0$ for $t < 0$ and $u(t) = 1$ for $t \geq 0$.

This approximation is justified by the fact that, as will be shown later in Fig. 12 [Figure 12: see original paper], the beam current pulse typically resembles a quasi-rectangular waveform with a sub-microsecond rise time. Hence, in the early stage when the beam first enters the cavity, its effect closely resembles that of a step excitation. With this simplification, the solution to Eq. (1) becomes [26]:

$V_{cb},step(t) = \omega_{0\cdot 5} \;\; _0 \;\; e^{-\lambda(t-x)} V_b(x)\,dx$, for $t \geq 0$

where $\lambda = \omega_{0\cdot 5} - j\Delta\omega$.

The general solution of Eq. (4) for zero initial condition (i.e., the zero-state response) is given by:

$V_{cb}(t) = \omega_{0\cdot 5} \;\; _0 \;\; e^{-\lambda(t-x)} V_b(x)\,dx$, for $t \geq 0$

If the cavity operates in open-loop mode, the cavity forward signal remains approximately unchanged during beam injection, such that $V_{f0}(t) \quad V_{f1}(t)$. We also assume that the cavity detuning is approximately constant throughout the beam pulse. Under these assumptions, subtracting the first equation in Eq. (1) from the second yields:

$dV_{cb}(t)/dt + (\omega_{0\cdot 5} - j\Delta\omega)V_{cb}(t) = \omega_{0\cdot 5} \, V_b(t)$

The general solution of Eq. (4) for zero initial condition (i.e., the zero-state response) is given by:

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$V_{cb},step(t) = \omega_{0\cdot 5} \;\; _0 \;\; e^{-\lambda(t-x)} V_b(x)\,dx$, for $t \geq 0$

If the cavity operates exactly on-resonance (i.e., $\Delta\omega = 0$), Eq. (6) simplifies to $V_{cb},step(t) = 1 - e^{-\omega_{0\cdot 5}t}$.

Accordingly, the ratio between the imaginary and real components is:

[V_{cb},step(t)] / [V_{cb},step(t)] (Δω t) / (1 - ½ $\omega_{0}\cdot_{5}$t) Δω t, for t → 0^{+}.

According to Eq. (9), as t → 0^{+}, the ratio between the imaginary and real components approaches zero, indicating that the imaginary part becomes negligible compared to the real part. Consequently, V_{cb},step(t) remains nearly aligned with the direction of V_b, which is assumed to lie along the X-axis.

Figure 4 [Figure 4: see original paper] shows V_{cb},step(t) under various detuning conditions. In Fig. 4a, both real and imaginary components evolve over time, with the response angle determined by the detuning angle $\Delta = \tan^{-1}(\Delta\omega/\omega_{0}\cdot_{5})$. The early-time behavior ($t < 0.5\tau$) is highlighted in Fig. 4b, where the imaginary component remains small. The I/Q trajectories in Fig. 4c illustrate that detuning causes the response to deviate from the direction of V_b, which is assumed to lie along the X-axis. For negative detuning, the real part remains unchanged while the imaginary part reverses sign, resulting in symmetry about the X-axis (not shown). As shown in Fig. 4d, the very initial trajectory ($t < 0.02\tau$) remains nearly aligned with the X-axis even for detuning angles up to 40°, consistent with Eq. (9).

In this section, the beam excitation is approximated as a quasi-rectangular pulse with a sub-microsecond rise time, as typically observed in accelerator systems [22]. Based on this, the beam input is modeled as a unit step function to analyze detuning effects on beam phase measurement for both SC and NC cavities. Although this assumption simplifies the analysis, the qualitative conclusions remain valid for arbitrary beam pulse shapes. A general derivation is provided in Appendix A.

BEAM PHASE CALIBRATION ALGORITHM WITH DETUNING COMPENSATION

To enable accurate phase calibration under detuning, this section presents a correction framework based on theoretical deflection estimation. We first derive the deflection angle induced by detuning and then discuss its application under closed-loop operation.

Theoretical Estimation of Deflection Angle Caused by Detuning

According to Eq. (6) in Section II, the theoretical phase deflection angle *α{th} induced by detuning Δω at time T_w (T_w represents the calibration window for beam-phase, T_w < T*{ps}) can be expressed as:

α_{th} = $\tan^{-1}$([V_{cb}(t)|_{t=T_w}] / [V_{cb}(t)|_{t=T_w}])

As shown in Fig. 6 [Figure 6: see original paper], the step response V_{cb}(t) in Eq. (6) is discretized as t = nT_s, where T_s is the sampling period of the digital LLRF data acquisition system (T_s 0.119 µs). Assuming that V_b points along the X-axis, the resulting discrete samples are plotted as blue dots in the I/Q plane. A linear regression of these points yields the green dashed line, with its angle relative to the X-axis denoted by α{fit}. *The theoretical deflection angle* α{th}, calculated from Eq. (12), corresponds to the angle between the X-axis and the line connecting the first and last sampled points. Since α{th} *reflects only the trajectory defined by the start and end points, while* α{fit} accounts for the overall sample distribution, the residual phase error $\Delta\alpha$ = α{fit} - α{th} is generally nonzero. In practical application, α{fit} *is first extracted from the measured data and then corrected by subtracting* α{th} (see Fig. 9 [Figure 9: see original paper]), making it necessary to evaluate whether the resulting $\Delta\alpha$ can be neglected.

Figure 7 [Figure 7: see original paper] presents 2D contour plots of the theoretical deflection angle α{th} *(a) and the residual phase error* $\Delta\alpha$ *(b) as functions of the detuning angle* Δ *and the observation time T_w, where T_w [0, 0.5*τ*] and* Δ *[-45°, 45°]. Although* α{th} and α{fit} *are derived differently, Fig. 7b shows that* $\Delta\alpha$ *remains negligibly small across the entire scan range. This confirms that in practical applications, when the fitting interval is limited to 0.5*τ*, the phasor V_b (or I_b) can be accurately recovered by rotating the fitted trajectory by* -α{th}. Since α_{th} can be directly calculated from Eq. (12), this method enables effective compensation of detuning-induced phase errors in beam phase measurements.

Feasibility of Closed-Loop Implementation

Having established the theoretical framework for detuning compensation, we now turn to its practical application under closed-loop conditions, which are essential for safe and stable beam delivery in high-current proton linacs such as ESS.

Accordingly, we exploit the LLRF loop delay window τ_d, during which the cavity field responds only to the beam and not to feedback corrections. As shown in Fig. 8a [Figure 8: see original paper] and Fig. 8b, the beam enters the cavity at approximately 151.6 µs, and the feedback system begins adjusting V_f at about 153 µs. Within this short interval (τ_d 1.4 µs), the system behaves equivalently to open-loop operation, enabling accurate measurement of the beam-induced transient. The close agreement between the open-loop and closed-loop trajectories in Fig. 8c demonstrates the feasibility of beam phase calibration in this mode.

Figure 9 [Figure 9: see original paper] illustrates the calibration process under closed-loop and detuned conditions. The beam phase ϕ_b is calculated as follows:

1. Normalize the steady-state cavity voltage V_c prior to beam arrival to 1 + 0j, which serves as the reference point in the I/Q plane.
2. Extract the transient response V_{cb}(t) within the loop delay window τ*d, and perform linear fitting to obtain the fitted deflection angle* α{fit}.

3. Compute the cavity detuning angle Δ and corresponding frequency detuning $\Delta\omega$ at the beam arrival time (assuming detuning remains constant during the beam pulse): Δ = V_c - V_f,
   $\Delta\omega$ = tan(Δ) · $\omega_{0\cdot5}$ Then calculate the theoretical deflection angle at T_w = τ_d using Eq. (12).
4. Finally, compute the beam phase using: $\phi b = \pi$ - (α{fit} - α_{th})

EXPERIMENTAL VALIDATION

Having established the theoretical foundation of the detuning-compensated beam phase calibration algorithm and analyzed its feasibility under closed-loop operation, we now proceed to experimental validation using the normal-conducting buncher cavity (B2) in the MEBT section of the ESS accelerator (see Fig. 1). Throughout the experiment, the beam pulse width was fixed at T_{ps} = 5 µs, while the beam current and phase were adjusted as needed. The beam current and phase were independently measured using beam current transformers (BCTs) and a BPM (in quadrupole magnet Q5). The RF pulse waveforms under both open-loop and closed-loop conditions are shown in Fig. 3 and Fig. 8, respectively, and key RF parameters are summarized in Table 2.

Figure 10 [Figure 10: see original paper] compares the measured beam-induced RF transients V_{cb} with simulated results under various detuning angles Δ in open-loop mode. The measurements, shown as filled circles, align well with the simulated waveforms computed from Eq. (11), confirming the validity of the transient response model. Having validated the theoretical predictions, we next evaluate the performance of the detuning correction algorithm.

Figure 11 [Figure 11: see original paper] presents beam phase measurements before and after applying the detuning correction algorithm under open-loop conditions with a beam current of I_{b0} = 55 mA. Subfigures (a) and (b) correspond to calibration windows T_w = 1.4 µs (equal to the loop delay τ_d) and T_w = 3 µs, respectively. Small markers represent individual measurements, while larger filled circles and squares indicate the averaged values. After applying the correction method proposed in Section III.B, the measured beam phase becomes insensitive to detuning, confirming the effectiveness of the proposed approach. Without correction, detuning angles of $±45°(*correspondingto*Δ $ = $\pm\omega_{0\cdot5}$) result in phase errors of approximately 9° at T_w = 1.4 µs and 18° at T_w = 3 µs (0.34τ). These results are consistent with the theoretical prediction shown in Fig. 7a.

To validate the applicability of the proposed calibration method under realistic accelerator operating conditions, we conducted a series of closed-loop beam phase measurements. Figure 12 shows the measured beam current (top) and beam phase (bottom) obtained using BCT and BPM. Figures 12a and b correspond to low-current (I_{b0} = 5 mA) and high-current (I_{b0} = 60 mA) cases, respectively. Notably, under high-current conditions, the beam phase exhibits fluctuations of approximately 2° across the pulse duration, impacting measurement precision as discussed later.

Figure 13 [Figure 13: see original paper] compares the trajectories of the beam-induced transient signal V_{cb} in the I/Q plane under open-loop and closed-loop conditions. For the 5 mA case (Fig. 13a), the signal is relatively weak and more susceptible to noise, leading to reduced accuracy in phase determination. For the 60 mA case (Fig. 13b), although the feedback introduces slight curvature to the trajectory, the open-loop and closed-loop traces remain nearly identical within the loop delay window (approximately 13 samples, including the initial 1 + 0j sampling point), confirming the feasibility of closed-loop beam phase measurement.

The impact of the calibration window length T_w was further evaluated under both open- and closed-loop modes for beam currents of 5 mA and 60 mA. Figure 14 [Figure 14: see original paper] summarizes the results. Figures 14(a) and (b) show probability density histograms of the measured beam phase for various T_w, where the upper plots correspond to open-loop conditions and the lower plots to closed-loop conditions. Different colors represent different T_w. At 5 mA (Fig. 14(a)), under open-loop operation, increasing the calibration window T_w leads to a narrower beam phase distribution, which corresponds to a lower root-mean-square (RMS) error. The histogram peak stabilizes near -85.5°, consistent with the theoretical prediction shown in Fig. 12a. After switching to closed-loop operation, the measurements remain in good agreement with the open-loop results when $T_w \leq \tau_d$. However, deviations appear for $T_w > \tau d$ *as the feedback response begins to affect* $V\{cb\}$ (see Fig. 8c).

At 60 mA (Fig. 14b), under open-loop operation, the phase distributions are more concentrated due to improved signal-to-noise ratio. Nevertheless, intra-pulse beam phase fluctuations, as observed in Fig. 12b, cause the histogram peak to shift with increasing T_w, for example, from approximately -42.5° at T_w = 1 µs to -45° at T_w = 4 µs. Under closed-loop operation, a similar trend is observed: the results match the open-loop reference when $T_w \leq \tau_d$, but deviate for $T_w > \tau d$ *as the feedback response begins to affect* $V\{cb\}$.

Figures 14c and 14d present the average (top) and RMS error (bottom) of the measured beam phase as functions of T_w for the 5 mA and 60 mA cases, respectively. For 5 mA (Fig. 14c), the average phase remains stable across different T_w under open-loop conditions. Under closed-loop operation, it matches the open-loop reference when $T_w \leq \tau_d$, but deviates for larger T_w. The RMS error decreases monotonically with increasing T_w, indicating that a larger calibration window improves phase measurement robustness. For 60 mA (Fig. 14d), although the measurement precision is overall higher, intra-pulse phase fluctuations still introduce deviations in both open- and closed-loop results, as previously discussed.

Figure 15 [Figure 15: see original paper] further compares the calibrated beam phase (blue) under closed-loop operation, obtained from fitting V_{cb}, with the BPM5 measurements (green) across different beam currents. The BPM data are obtained by averaging the phase signal over the 2.5–5 µs interval (see Fig. 12). From the BPM measurements, it can be observed that the beam phase differs at various beam currents. This may be attributed to the defocusing effects of the buncher cavity, where beams of different intensities pass through the cavity along slightly different orbits, causing variations in the BPM readings. As shown in Fig. 15, at low beam currents, the BPM5 measurements match well with the calibrated beam phase from the V_{cb} fit under closed-loop operation. However, at higher currents, increasing deviations are observed due to beam phase fluctuations during the pulse duration (see Fig. 12b).

Figure 16 [Figure 16: see original paper] presents the beam phase measurement results under closed-loop conditions with a calibration window of T_w = τd*, performed at a beam current of* I{b0} = 60 mA. Subfigures (a), (b), and (c) correspond to different beam phase settings. Small dots represent individual measurements, while larger solid markers (circles and squares) denote the averaged results before and after correction, respectively. Before calibration (blue), the measured beam phase exhibits a clear linear dependence on cavity detuning. After applying the correction method described in Section III.B (red), this dependence is effectively eliminated, and the beam phase remains stable across the full detuning range. These results provide strong experimental validation of the proposed detuning correction algorithm. It is also worth noting that, due to intrinsic phase fluctuations within the high-current pulse, the calibrated phase results show an offset of approximately 1.5–2° compared to BPM-based measurements. Overall, the experimental data demonstrate the effectiveness of the proposed method in compensating for detuning effects and achieving beam phase measurement under more realistic accelerator conditions.

CONCLUSIONS AND FUTURE WORK

In this study, we systematically investigated a beam phase measurement method based on transient beam loading and quantitatively analyzed the influence of cavity detuning on the measurement results for both superconducting and normal-conducting RF cavities. A theoretical expression for the detuning-induced phase deflection angle was derived, and a compensation algorithm was proposed. The feasibility of implementing this method was validated experimentally using the buncher cavity (B2) in the MEBT section of the ESS accelerator. The proposed algorithm's effectiveness was demonstrated by comparing open-loop and closed-loop measurements under various detuning scenarios.

The results confirm that within the LLRF loop delay window (τd)*, the transient vector* V{cb} evolves similarly under both open- and closed-loop modes, enabling reliable beam phase determination. Nonetheless, the limited length of τ_d restricts the number of samples available for fitting, and low-current conditions reduce phase resolution due to lower signal-to-noise ratios. Under higher beam currents, intra-pulse phase fluctuations introduce systematic offsets compared with BPM-based measurements. Despite these limitations, closed-loop calibration offers enhanced operational stability and is better suited for continuous beam phase monitoring in linacs.

When both detuning and feedback are present, the trajectory of V_{cb} in the I/Q plane becomes more complex but still follows identifiable patterns. Future research may leverage artificial intelligence (AI) techniques to overcome the current T_w $\leq$ τ_d limitation, enabling accurate phase recovery even beyond the initial transient regime. Preliminary investigations in this direction have shown promising results [16].

APPENDIX A: EARLY-TIME EXPANSION OF THE CAVITY RESPONSE

To analyze the early-time behavior of the cavity response under beam-induced excitation, consider the first-order differential equation:

dV_{cb}(t)/dt + λV_{cb}(t) = $\omega_{0\cdot5}$ V_b(t), V_{cb}(0) = 0, (A.1)

where λ = $\omega_{0\cdot5}$ - j$\Delta\omega$, and V_b(t) is an arbitrary causal input with V_b(t < 0) = 0.

We focus on the limit t $\rightarrow 0^+$, immediately after the beam pulse begins. Figure 17 [Figure 17: see original paper] illustrates the typical shapes of V_b(t) as t $\rightarrow 0^+$, distinguishing between the cases V_b(0) $\neq$ 0 and V_b(0) = 0. To simplify the analysis without loss of generality, we assume the beam phase remains constant within the pulse duration and set the initial beam phase to zero. Thus, V_b(t) $^+$.

The system's zero-state solution is:

V_{cb}(t) = $\omega_{0\cdot5}$ $_0$ e^{-λ(t-x)} V_b(x) dx. (A.2)

Expand V_b(x) near x = 0:

V_b(x) = V_b(0) + V'_b(0) x + O(x^2). (A.3)

Substitute Eq. (A.3) into Eq. (A.2), and change variables via s = t - x:

V_{cb}(t) $\omega_{0\cdot5}$ $_0$ e^{-λs} [V_b(0) + V'_b(0)(t - s)] ds. (A.4)

Using e^{-λs} 1 - λs as s $\rightarrow$ 0, the integral becomes:

$_0$ (1 - λs) ds = t - ½λt^2 (A.6)
$_0$ (t - s)(1 - λs) ds = t^3 (A.7)

Substituting Eqs. (A.6)–(A.7) into Eq. (A.5), the response becomes:

V_{cb}(t) $\omega_{0\cdot5}$ [V_b(0) (t - ½λt^2) + V'_b(0) (t^3)]. (A.8)

Separating into real and imaginary parts:

[V_{cb}(t)] $\omega_{0\cdot5}$ [V_b(0) t - ½ $\omega_{0\cdot5}$ V_b(0) t^2 + O(t^3)] (A.9)
[V_{cb}(t)] $\omega_{0\cdot5}$ [½ $\Delta\omega$ V_b(0) t^2 + O(t^3)] (A.10)

The ratio between imaginary and real components is:

[V_{cb}(t)] / [V_{cb}(t)] { ½ $\Delta\omega$ t, if V_b(0) $\neq$ 0;
$\Delta\omega$ t, if V_b(0) = 0 } + O(t^2). (A.11)

For a unit step input V_b(t) = u(t), we have V_b(0) = 1, V'_b(0) = 0. Substituting into Eq. (A.13) yields:

[V_{cb}(t)] / [V_{cb}(t)] ½ $\Delta\omega$ t [1 - ½ $\omega_{0\cdot5}$t] + O(t^2) ½ $\Delta\omega$ t, for t $\to$ 0^+. (A.12)

This result can be interpreted physically as follows. At early times, the cavity response vector aligns with the initial phase of the input signal. The detuning term $\Delta\omega$ contributes only to higher-order terms and does not cause immediate phase rotation. While the derivation assumes a constant beam phase, the result still holds if the phase varies smoothly near t = 0, as such variations affect only higher-order corrections. A detailed treatment of time-varying phase is beyond the scope of this appendix.

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