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Full Text

Theory of Laser-Assisted Nuclear Fusion

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Abstract

The process of nuclear fusion in the presence of a laser field is analyzed theoretically. The analysis is applicable to most fusion reactions and different kinds of currently available intense lasers, from X-ray free electron lasers to solid-state near-infrared lasers. Laser fields are shown to enhance the fusion yields and the mechanism of the enhancement is explained. Low-frequency lasers are shown to be more efficient in enhancing fusion than high-frequency lasers. Calculation results show enhancements of fusion yields by orders of magnitude with currently available intense low-frequency laser fields. The temperature requirement of controlled nuclear fusion may be reduced with the aid of intense laser fields.

Keywords: Nuclear fusion, Intense lasers, Enhancement of fusion

Introduction

Controlled nuclear fusion is an active research field with the ultimate goal of supplying sustainable and clean energy solutions to mankind [1–6]. Yet it is difficult to achieve ignition in self-sustained nuclear fusion under laboratory conditions, essentially due to the fact that nuclear fusion cross sections are very small. To increase the cross section, the nuclear fuel needs to be heated to very high temperatures, typically on the order of 10^7 K. Achieving and maintaining such high temperatures is very challenging in practice. It is therefore sensible and meaningful to consider methods that may increase the fusion cross section and reduce the temperature requirement.

The possibility of using advanced light sources to influence and enhance nuclear-fusion yields has attracted attention recently [7–19]. These are interesting and timely attempts, observing rapid progress in light-source technologies, especially those with extremities in intensity or frequency (photon energy). Light sources with extreme intensities include, for example, the extreme light infrastructure (ELI) of Europe [20–22] and the superintense ultrafast laser facility (SULF) of Shanghai [23–25]. These lasers are expected to reach peak intensities on the order of 10^{23} W/cm^2 in the coming years. The frequency is in the near-infrared regime with very small single-photon energies (around 1.5 eV). Light sources with extreme frequencies include synchrotron radiations and X-ray free-electron lasers [26, 27]. These sources generate light with photon energies on the order of 1 to 10 keV. The possibility of using these light sources to influence nuclear processes, such as α decay [28–33], nuclear fission [34], quantum optical effects in nuclei [35–38], nonlinear optical effects in nuclei [39, 40], muon production [41], measurements of cross sections and astrophysical S-factor in plasma [42–44], and isomeric excitation [40, 42, 45–60], has been considered or experimentally demonstrated recently.

It is not unreasonable to expect that these light sources can influence the nuclear fusion process. The relevant energy scale of controlled nuclear fusion is on the order of 1 keV ($= 1.16 \times 10^7$ K). For high-frequency light sources, absorption of a single photon will lift the energy of the fusion system by an order of 1 keV.

For low-frequency-high-intensity light sources, simultaneous absorption of 1,000 photons will lift the energy of the fusion system by a similar amount, and it will be shown later that this is actually not difficult with intensities that are readily available nowadays.

Existing works on this topic focus either on light sources with high frequencies [7, 8, 17] or those with low frequencies [9, 18]. This is mainly due to the theoretical techniques used to deal with the problem. For example, Queisser et al. employ a Floquet scattering method [7, 17], and Lv et al. adopt the Kramers-Henneberger approximation [8], both of which are feasible only for high frequencies. Several studies have comparatively discussed different approaches for laser-assisted nuclear fusion to a certain extent [13, 14, 16]. A comprehensive theoretical analysis that is applicable to both high and low laser frequencies is still lacking. Without an analysis that puts different laser frequencies on the same footing, conclusions cannot be drawn on what kind of laser would be most efficient in enhancing fusion. Is an X-ray free-electron laser more efficient in enhancing fusion than a near-infrared laser? One might think the answer would be yes because absorbing a single photon from an X-ray laser is equivalent to absorbing 1,000 photons from a near-infrared laser. However, it will be shown that the answer is actually no.

The goal of the current article is to provide a theoretical analysis that covers both high-frequency lasers, such as X-ray free-electron lasers, and low-frequency lasers, such as near-infrared solid-state lasers. Different lasers are treated on the same footing. Conclusions are drawn on preferable laser parameters to enhance fusion yields. The analysis is physically oriented with the aim of providing physical understanding using the least possible numerical calculations. Fundamentally, the process of laser-assisted nuclear fusion is a complex many-body problem, and an ab initio calculation starting from quantum chromodynamics remains impossible. A feasible theoretical treatment inevitably involves approximations of different levels. A highly precise and numerically intense theoretical technique is not desirable at the current stage.

This article is organized as follows. In Section II the effects of laser fields on each stage (region) of a nuclear fusion process are analyzed. Calculation results are presented in Section III. Discussions on various aspects of our analyses are given in Section IV. A summary and outlook is given in Section V to conclude the article.

II. Analyses of the Laser-Assisted Nuclear Fusion Process

A. Nuclear Fusion Without Laser Fields

We start from nuclear fusion without the presence of laser fields. A nuclear fusion process is usually divided into three regions according to the relative distance between the two nuclei, as illustrated in Fig. 1 [Figure 1: see original paper] (a). Viewing from the rest frame of one nucleus (denoted nucleus 1 for convenience), the other nucleus (nucleus 2) is initially in region III with an asymptotic energy

E, which is usually between 1 and 10 keV depending on the temperature of the fusion environment. As it approaches, nucleus 2 will reach a classical turning point where the Coulomb repulsive energy between the two nuclei equals the energy E. Via a quantum tunneling effect, nucleus 2 nevertheless enters and passes through region II with a small probability. In region I, the two nuclei are very close together and fusion reactions happen. The spatial range of region I is on the order of 1 fm (10^{-15} m), and the spatial range of region II is on the order of 100 fm for typical energies of controlled fusion research.

Indeed, a fusion cross section is usually written in the following form corresponding to the three-region division [61]:

$$\sigma(E) = S(E)\left(\frac{B_G}{E}\right)\exp\left(-\sqrt{\frac{E_G}{E}}\right)$$

where the parameters A_i's and B_i's are determined by fitting to experimental data. For the DT fusion, the values of these parameters are given as follows [63]:

$$S(E) = \frac{A_1 + E(A_2 + E(A_3 + EA_4))}{1 + E(B_1 + E(B_2 + E(B_3 + EB_4)))}$$

with $A_1 = 6.927 \times 10^4$, $A_2 = 7.454 \times 10^8$, $A_3 = 2.050 \times 10^6$, $A_4 = 5.200 \times 10^4$, $B_1 = 6.380 \times 10^1$, $B_2 = -9.95 \times 10^{-1}$, $B_3 = 6.981 \times 10^{-5}$, and $B_4 = 1.728 \times 10^{-4}$. Note that these values are accompanied with the value of E in keV when Eq. (2) is used. The DT fusion cross section $\sigma(E)$ and the function $S(E)$ are shown in Fig. 1 (b) and (c), respectively. $S(E)$ is a slow-varying function, but $\sigma(E)$ is an exponential function due to the tunneling process. $\sigma(E)$ depends on E very sensitively, especially for relatively low E values.

B. Two Nuclei in a Laser Field: The Center-of-Mass Reference Frame

Now let us consider effects of an external laser field on the fusion process. Consider two nuclei with charge and mass $\{q_1, m_1\}$ and $\{q_2, m_2\}$ placed in a plane-wave laser field. There are two commonly used gauges in light-matter interactions. One is the so-called velocity gauge, with which the laser field is characterized by a vector potential $A(t)$. The total Hamiltonian is written as:

$$H = \frac{[p_1 - q_1 A(t)]^2}{2m_1} + \frac{[p_2 - q_2 A(t)]^2}{2m_2} + \frac{q_1 q_2}{r}$$

where p_1 and p_2 are the momenta of the two nuclei, and $r = |r_1 - r_2|$ is the distance between the two nuclei. Atomic units have been used with which $4\pi\epsilon_0 = 1$, hence the form of the Coulomb potential. We have assumed the validity of a long-wavelength approximation by omitting the spatial dependency of the vector potential. This is justified by the fact that the spatial range relevant to nuclear fusion is much smaller than the wavelengths of available intense lasers. Further discussions on this point will be given in Section IV.

For a fusion process, it is the relative motion between the two nuclei that is most relevant. It is therefore more convenient to work in the center-of-mass (CM) reference frame. Define:

$$R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}, \quad P = p_1 + p_2, \quad r = r_1 - r_2, \quad p = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2}$$

Then, after some straightforward algebra, the Hamiltonian in Eq. (3) can be rewritten as:

$$H = \frac{[P - QA(t)]^2}{2M} + \frac{[p - qA(t)]^2}{2\mu} + \frac{q_1 q_2}{r}$$

where $M = m_1 + m_2$, $\mu = \frac{m_1 m_2}{m_1 + m_2}$, $Q = q_1 + q_2$, and $q = \frac{q_1 m_2 - q_2 m_1}{m_1 + m_2}$. One sees from Eq. (8) that the Hamiltonian can be separated into a motion of the CM with charge and mass $\{Q, M\}$, and a motion of a virtual particle with charge and mass $\{q, \mu\}$. The mutual Coulomb potential is not affected. The motion of the CM is not of concern here because it is not relevant to the fusion process.

Alternatively, one may use the so-called length gauge, and the Hamiltonian is given as:

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{q_1 q_2}{r} - (q_1 r_1 + q_2 r_2) \cdot E(t)$$

where $E(t)$ is the laser electric field. Again, the long-wavelength approximation has been assumed by neglecting the spatial dependency of the laser electric field. The above Hamiltonian can also be written in the CM reference frame as:

$$H = \frac{P^2}{2M} - QR \cdot E(t) + \frac{p^2}{2\mu} - qr \cdot E(t) + \frac{q_1 q_2}{r}$$

The motion of the two nuclei is equivalent to a motion of the CM with $\{Q, M\}$ plus a motion of a virtual relative-motion particle with $\{q, \mu\}$.

C. Assumption on Region I

The size of region I is on the order of 1 fm. In this region, the two nuclei fuse and new particles are generated. For the DT fusion reaction, an α particle and a neutron are generated as a result. This is a complex many-body nuclear process, and an ab initio treatment of this process is usually impossible. Adding influences from an external laser field makes the process even more complicated to deal with.

It is, however, reasonable to expect that the effect of a laser field on this region is negligibly small. From the uncertainty principle, the spatial confinement of $\Delta x \sim 1$ fm $= 1.9 \times 10^{-5}$ a.u. indicates an uncertainty in momentum of $\Delta p \sim 5.3 \times 10^4$ a.u., or an uncertainty in energy of $(\Delta p)^2/2\mu \sim 6.4 \times 10^5$ a.u. ≈ 17 MeV. Here $\mu = 2204$ a.u. has been taken as the reduced mass of deuteron and triton. In comparison, the effect of an intense laser field on the fusion process is on the order of 1 keV in energy, which is about 4 orders of magnitude smaller. It seems safe to neglect effects of external laser fields on the nuclear processes happening in region I. This is the basic assumption that our analyses start with.

D. Effects of Laser Fields on Region II

The size of region II is on the order of 100 fm. We shall show in this section that an intense laser field has a finite but small effect on the tunneling process happening in this region. The probability of tunneling, or the "penetrability," through a laser-modified Coulomb potential barrier can be calculated using the Wentzel-Kramers-Brillouin (WKB) method as:

$$P(\theta, t) = \exp\left(-\int_{r_1}^{r_2} \sqrt{2\mu[V_C(r) - E + V_I(r,\theta,t)]}\, dr\right)$$

where $V_C(r) = q_1 q_2 / r$ is the Coulomb potential and $V_I(r,\theta,t) = -qr \cdot E(t) = -qrE(t)\cos\theta$ is the laser potential from Eq. (14). Here θ is the angle between the laser polarization direction and the DT collision (i.e., relative-motion) direction. The integration is performed between two classical turning points r_1 and r_2, which are also the two ends of region II.

What is implicit in writing Eq. (15) is a quasi-static approximation. That is, the laser potential can be viewed as static at each time. This approximation is valid when the period of the laser field is much longer than the time scale of the tunneling process. More discussions on this point will be given later in Section IV.

It can be estimated that the magnitude of V_I is much smaller than that of V_C, or that of $V_0 \equiv V_C - E$. The Coulomb potential V_C in region II can be estimated to be on the order of 100 keV, and the relative collision energy E is on the order of 1 keV in typical nuclear fusion experiments. With an intensity of 10^{20} W/cm^2, the magnitude of V_I can be estimated to be on the order of 10 eV. With an intensity of 10^{22} W/cm^2, the magnitude of V_I is on the order of 100 eV. Therefore we may expand Eq. (15) as:

$$\begin{aligned}
P(\theta, t) &= \exp\left(-\int_{r_1}^{r_2} \sqrt{2\mu[V_0(r) + V_I(r,\theta,t)]}\, dr\right) \\
&\approx \exp\left(-\int_{r_1}^{r_2} \sqrt{2\mu V_0(r)}\left[1 + \frac{V_I(r,\theta,t)}{2V_0(r)}\right] dr\right) \\
&= \exp\left(-\int_{r_1}^{r_2} \sqrt{2\mu V_0(r)}\, dr\right) \times \exp\left(-\int_{r_1}^{r_2} \frac{\sqrt{2\mu} V_I(r,\theta,t)}{2\sqrt{V_0(r)}}\, dr\right) \\
&\approx \exp\left(-\int_{r_1}^{r_2} \sqrt{2\mu V_0(r)}\, dr\right)\left[1 + \gamma^{(1)}\right]
\end{aligned}$$

$$= P(E = 0)\left[1 + \gamma^{(1)}\right]$$

where $P(E = 0)$ is the penetrability without external laser fields and $\gamma^{(1)}$ denotes the first-order correction induced by the laser field. Substituting the expression of V_I we get an explicit formula:

$$\gamma^{(1)}(\theta, t) = \frac{qE(t)\cos\theta}{2}\int_{r_1}^{r_2}\frac{r}{\sqrt{V_0(r)}}\,dr$$

The magnitude of $\gamma^{(1)}$ is maximum at the times when $E(t)$ reaches peaks and when $\cos\theta = 1$. The upper limit of integration r_2 is determined by equating the Coulomb potential $V_C(r)$ to the collision energy E. For $E = 5$ keV, we get $r_2 = 288.3$ fm. The lower limit of integration r_1 is determined by a D+T touching condition: $r_1 = r_D + r_T = 1.13(A_1^{1/3} + A_2^{1/3})$ fm $= 3.05$ fm. We find that for intensity 10^{20} W/cm^2, $\gamma^{(1)}$ takes a maximum value of 0.18%. For intensity 10^{22} W/cm^2, the maximum value is 1.8%. For $E = 10$ keV, $r_2 = 144.2$ fm. The corresponding maximum value of $\gamma^{(1)}$ is 0.03% for 10^{20} W/cm^2, and 0.3% for 10^{22} W/cm^2. Therefore one can see that for currently available state-of-the-art laser intensities, the effects of laser fields on region II are finite but quite small.

E. Effects of Laser Fields on Region III

Major effects of a laser field on the fusion process come from region III. The laser field can substantially influence the collision energy E. Without external laser fields, the incoming relative-motion virtual particle is usually described asymptotically as a plane wave state:

$$\psi(r, t) = \exp(ip \cdot r - iEt)$$

where the momentum has magnitude $p = \sqrt{2\mu E}$. This plane-wave state has a well-defined energy E.

In the presence of a laser field, the asymptotic plane-wave state becomes a Volkov state [64]:

$$\psi_V(r, t) = \exp\left(ip \cdot r - iEt - i\int^t H_I(t')\,dt'\right)$$

where H_I is the interaction Hamiltonian with the external laser field. It is convenient to use the velocity gauge here, and from Eq. (8) $H_I(t) = -\frac{p\cdot A(t)}{\mu} + \frac{q^2A^2(t)}{2\mu}$. Let us assume that the laser field is linearly polarized along the z axis, and the vector potential $A(t) = \hat{z}A_0\sin\omega t$. The case of elliptical or circular polarization will be discussed later in Section IV. Note that because the laser field can be very intense, the A^2 term cannot be simply ignored as in low-intensity situations.

The Volkov state can be expanded in terms of photon number:

$$\psi_V(r,t) = e^{ip\cdot r} \sum_n F_n(u,v) e^{-i(E+U_p+n\omega)t}$$

where the coefficient $F_n(u,v)$ is given by the following integral:

$$F_n(u,v) = \frac{1}{2\pi} \int_0^{2\pi} e^{-iu\cos\xi + iv\sin 2\xi + in\xi}\, d\xi$$

For convenience we have defined $U_p = q^2 A_0^2/4\mu$ (the ponderomotive energy), $u = u(\theta) = qpA_0 \cos\theta/\mu\omega$, and $v = q^2 A_0^2/8\mu\omega$. Here θ is the angle between p and the $+z$ axis, and θ enters into the formalism through u. In a thermal environment, the direction between p and the laser polarization axis is random.

One sees from Eq. (21) that in the presence of a laser field, the collision energy is no longer a well-defined single value. Instead, the energy becomes a distribution, which is centered at $E + U_p$ (ponderomotive shift) and separated by the photon energy. The energy of the particle can be higher or lower than $E + U_p$, corresponding to situations of absorption or emission of photons. The probability of finding the system with energy $E_n = E + U_p + n\omega$ is $P_n(u,v) = |F_n(u,v)|^2$. The total probability summing over the photon number n is equal to unity: $\sum_n P_n = 1$.

III. Numerical Results

A. Energy Distribution with Different Laser Parameters

The energy distribution P_n depends sensitively on laser parameters, especially on the frequency (photon energy). With the same laser intensity, the energy distribution can be very different for lasers with different photon energies. This is illustrated in Fig. 2 [Figure 2: see original paper], which shows the energy distributions for six different photon energies under the same intensity. The bare collision energy without the laser fields is assumed to be 5 keV (corresponding to a temperature of 5.8×10^7 K).

One can see that for a high-frequency laser with photon energy 1 keV (1,000 eV), almost all the population remains with the original collision energy 5 keV. The probability of absorbing (emitting) a photon and changing the energy to 6 keV (4 keV) is very small, with a value of 2.6×10^{-6}. This probability is not visually distinguishable in linear scale as shown in Fig. 2 (a). The probability of absorbing (emitting) two photons and changing the energy to 7 keV (3 keV) is on the order of 10^{-12}. It is very difficult to absorb (emit) energy from (to) a high-frequency laser field, even when the intensity is high. The probability of absorbing or emitting more photons shows a perturbative feature; that is, the probability drops substantially as the number of photons increases.

As the photon energy decreases to 100 eV [Fig. 2 (b)], the probability of absorbing (emitting) a photon increases to about 0.025. This means that 2.5% of the population has an energy of 5.1 keV and another 2.5% of the population has an energy of 4.9 keV. The probability of absorbing (emitting) two photons is on the order of 10^{-4}. As the photon energy decreases to 50 eV [Fig. 2 (c)], the probability of absorbing (emitting) one photon is about 27%, and that of absorbing (emitting) two photons is about 3.3%. The probability of remaining with the original collision energy drops to 39%.

As the photon energy decreases to 30 eV [Fig. 2 (d)], the energy distribution shows clear nonperturbative features. For example, the probability of absorbing (emitting) two photons is higher than that of absorbing (emitting) one photon. The number of photons absorbed or emitted is about 5, and the energy range populated is roughly from 4.85 to 5.15 keV.

As the photon energy decreases to 10 eV [Fig. 2 (e)], the number of photons absorbed or emitted is about 40. The populated energy range is between 4.6 and 5.4 keV. Besides, the distribution shows an overall structure with peaks near two ends and a valley in the middle. This means that the collision has substantial probabilities with energies away from the original bare energy. As the photon energy decreases to 1.55 eV [Fig. 2 (f)], which corresponds to a wavelength of 800 nm from Ti:sapphire intense lasers, the number of photons absorbed or emitted is over 1,300, and the energy range between 3.1 and 7.3 keV is substantially populated. In this case the ponderomotive shift $U_p = 109$ eV, and notice that the distribution is not exactly symmetric.

The main message from the above results is that it is easier for low-frequency lasers to deliver energy to the fusion system. Although the energy of a single photon is small, the number of participated photons can be very large so that the populated energy range is wide.

Note that the energy distribution P_n depends on the parameter u, which depends on the angle θ between the laser polarization direction and the collision (relative-motion) direction. We have set $\theta = 0^\circ$ for all the cases shown in Fig. 2. As θ increases from 0° to 90°, the energy distribution becomes narrower, as shown in Fig. 3 [Figure 3: see original paper] for the case of 1.55 eV (800 nm). P_n for $\theta > 90^\circ$ is the same as that for $(180^\circ - \theta)$. It is easier for the laser to deliver energy to the fusion system if the axis of collision and the axis of laser polarization align.

B. Enhancement of Fusion

We see that in the presence of a laser field, the collision energy E changes from a single value to a distribution, the character of which depends on the laser parameters. The fusion system can either absorb energy from the laser field, leading to collision energies higher than E, or lose energy to the laser field, leading to energies lower than E (though the center of the energy distribution is ponderomotively shifted to $E + U_p$). Energies higher than E lead to higher fusion yields, and energies lower than E lead to lower fusion yields.

The net effect, however, is an enhancement of the fusion yield. This is because the cross section function of Eq. (1) depends on the collision energy exponentially (concaving upward) as shown in Fig. 4 [Figure 4: see original paper] in linear scale. The fusion yields gained with higher energies are greater than the fusion yields lost with lower energies. This is the mechanism of fusion-yield enhancement in the presence of a laser field.

C. Effective Fusion Cross Section

It seems sensible to define an effective fusion cross section in the presence of a laser field. We denote this laser-assisted cross section as $\sigma_L(E)$, which can be calculated by averaging over all θ angles between the collision direction and the laser polarization direction:

$$\sigma_L(E) = \int_0^\pi \sigma_L(E,\theta) \sin\theta \, d\theta$$

where $\sigma_L(E,\theta)$ is defined as:

$$\sigma_L(E,\theta) = \sum_n P_n[u(\theta), v]\sigma(E + U_p + n\omega)$$

The effective cross section $\sigma_L(E)$, in comparison to the laser-free cross section $\sigma(E)$, gives a quantitative measure of the effect of a laser field on the nuclear fusion process. It is to be emphasized that the laser field mainly affects the fusion process via modifying the collision energy in region III before the tunneling process. It has very small effects on the processes happening in region I and region II, as explained above.

Fig. 5 [Figure 5: see original paper] shows the laser-assisted effective DT fusion cross section for three bare energies ($E = 1$, 5, and 10 keV) and different laser intensities and photon energies. The three energies represent typical temperatures in thermonuclear fusion experiments. One sees from Fig. 5 that to have noticeable effects on nuclear fusion, the laser intensity needs to be higher than 10^{18} or 10^{19} W/cm^2.

One can also see that σ_L is larger for lower laser frequencies under the same intensity. For all three cases and with the intensity range shown, the fusion enhancement for the higher frequency cases (100 eV and 1,000 eV) is very small. This is the direct consequence of the point explained above: it is low-frequency lasers that are more efficient in delivering energy to the fusion system. It is difficult to absorb energy from a high-frequency laser field.

Substantial enhancements can be seen with the low-frequency 1.55 eV lasers. For $E = 1$ keV, the enhancement is 3 orders of magnitude at intensity 10^{20} W/cm^2 and 9 orders of magnitude at intensity 5×10^{21} W/cm^2. The ratio of enhancement decreases as E increases. This is because the cross section function $\sigma(E)$ of Eq. (1) depends more sensitively on E for smaller E values. The effect of the laser field is more pronounced for smaller E values.

The possibility of using lasers to reduce the temperature requirement of fusion reactions can be seen. For example, without laser fields the DT fusion cross section at $E = 1$ keV is 1.37×10^{-11} barn. With an 800-nm (1.55-eV) laser field of intensity 10^{20} W/cm^2, the effective fusion cross section becomes 1.02×10^{-8} barn, which is equal to the cross section value at $E = 1.6$ keV without laser fields. If the laser intensity is 5×10^{21} W/cm^2, the effective cross section becomes 0.027 barn, which is equal to the cross section value at $E = 10$ keV without laser fields. Putting it another way, the enormous gap in the DT fusion cross section between 1 keV (1.16×10^7 K) and 10 keV (1.16×10^8 K) is filled, or compensated, completely by the intense laser field.

IV. Discussions

A. Applicability to Other Fusion Reactions

Although the results presented above are for the DT fusion reaction, our analyses apply to other fusion reactions as well, for example, the neutronless proton-boron fusion reaction [65–69].

However, laser fields have no effect, at least within the approximations adopted in the current article, on fusion reactions with the two nuclei having the same charge-to-mass ratio. An example of this kind is the deuteron-deuteron fusion reaction. If so, one can find from Eq. (12) that the charge of the relative-motion virtual particle is zero, and the laser ceases to have an effect on the relative-motion degree of freedom. This is easy to understand because the motion of a charged particle in a laser field is determined by its charge-to-mass ratio. If two nuclei have the same charge-to-mass ratio, then their motion in the laser field will be the same, and the laser field has no tendency to separate them apart or press them closer; i.e., the laser field has no effect on the relative motion between the two nuclei.

B. The Long-Wavelength Approximation

We have assumed the validity of the long-wavelength approximation by neglecting the spatial dependency of the laser vector potential or the electric field. This is justified if the spatial range relevant to fusion is much smaller than the laser wavelength. The former range can be estimated by a quiver motion amplitude $z_0 = qA_0/\mu\omega$, which is the amplitude of spatial oscillations of a free charged particle in a laser field. The required validity condition of the long-wavelength approximation is $z_0 \ll \lambda = 2\pi c/\omega$, or $A_0 \ll 2\pi c\mu/q$. Putting in the values of μ and q for the DT fusion, we get $A_0 \ll 9.5 \times 10^6$ a.u., or the amplitude of the laser electric field $E_0 = A_0\omega \ll 9.5 \times 10^6\omega$ in a.u. Let us give two examples. For photon energy 1,000 eV, $\omega = 36.8$ a.u., so $E_0 \ll 3.5 \times 10^8$ a.u., or equivalently the intensity $I \ll 4.3 \times 10^{33}$ W/cm^2. For photon energy 1.55 eV, $\omega = 0.0569$ a.u., so $E_0 \ll 5.4 \times 10^5$ a.u., or equivalently the intensity $I \ll 1.0 \times 10^{28}$ W/cm^2. The validity of the long-wavelength approximation can be guaranteed for both examples.

C. The Quasi-Static Approximation

We mentioned in Section II.D that in writing Eq. (15) we have implicitly used the quasi-static approximation. This approximation is valid when the laser period is much longer than the time scale of the tunneling process, which can be estimated using a classical picture. For example, for $E = 5$ keV, the velocity of relative motion is $v = \sqrt{2E/\mu} = 0.41$ a.u. The tunneling length (from the tunneling entrance point to the tunneling exit point) is 5.38×10^{-3} a.u. (285 fm). The estimated time for the tunneling process is therefore 0.013 a.u. or 0.32 as (1 as $= 10^{-18}$ s). As long as the laser period is much longer than 0.32 as, or the photon energy much lower than 13 keV, the quasi-static approximation is valid.

We mention here that the quasi-static approximation is also an important concept in strong-field atomic ionization [70, 71]. The ratio between the classically estimated electron tunneling time and the laser period is called the Keldysh parameter [70]. The quasi-static approximation is valid when the Keldysh parameter $\gamma \ll 1$, that is, when the time scale of tunneling is much shorter than the laser period.

D. The Coulomb-Volkov State

For simplicity, we have been using the (plane-wave) Volkov state to describe the relative-motion virtual particle. The Volkov state is the solution of the time-dependent Schrödinger equation for a free charged particle in the presence of a laser field. The Coulomb potential between the two nuclei is neglected.

The quantum state of the full Coulomb-plus-laser system does not have general analytical solutions. An approximate solution is the so-called Coulomb-Volkov state [72, 73], which has the same form as the plane-wave Volkov state except to replace the $e^{ip\cdot r}$ in Eqs. (19) or (21) with a Coulomb wave function $\phi_p(r)$. The temporal part, hence the energy distribution, remains the same. Therefore the results and discussions presented above are not affected if Coulomb-Volkov states are used.

E. Elliptical or Circular Polarization

Extension of the above formalism to elliptically or circularly polarized laser fields is straightforward, so we only outline a few steps here. Assume the vector potential is in the z-y plane: $A(t) = \hat{z}A_\varepsilon \sin\omega t + \hat{y}\varepsilon A_\varepsilon \cos\omega t$, with ellipticity ε and amplitude $A_\varepsilon = A_0/\sqrt{1+\varepsilon^2}$. Then the Volkov wave function can be written in the same form as Eq. (21) except that $F_n = F_n(u, w, v)$ has an additional argument w due to the additional polarization direction. Here $u = u(\theta) = qpA_\varepsilon \cos\theta/\mu\omega$, $w = w(\theta, \phi) = qpA_\varepsilon \sin\theta \sin\phi/\mu\omega$, and $v = q^2A_\varepsilon^2(1+\varepsilon^2)/8\mu\omega$. (θ, ϕ) are the direction angles of the momentum p. The coefficient F_n is obtained via the integral:

$$F_n(u,w,v) = \frac{1}{2\pi}\int_0^{2\pi} e^{-iu\cos\xi + iw\sin\xi + iv\sin 2\xi + in\xi}\, d\xi$$

Because elliptical polarization breaks the cylindrical symmetry of linear polarization, the effective fusion cross section σ_L needs to be averaged over both θ and ϕ. Fig. 6 [Figure 6: see original paper] shows how the angle-averaged σ_L changes with ε for intensity 1×10^{20} W/cm^2. Elliptical or circular polarization does not lead to further enhancements in the fusion yield. The efficiency is (slightly) lower than linear polarization, mainly due to the reduction of the field amplitude from A_0 to A_ε.

V. Summary and Outlook

In summary, we have considered the process of nuclear fusion in the presence of a laser field. Without laser fields, a nuclear fusion process is usually treated as a three-region process, and we have analyzed the effects of laser fields on each of the three regions. Our analysis is physically oriented, aiming to provide a clear physical understanding of the laser-assisted nuclear fusion process. We show that major effects of the laser field on the nuclear fusion process come from influencing the collision energy before tunneling. We explain why this influence in the collision energy leads to enhanced fusion yields. By treating lasers with different frequencies on the same footing, we are able to draw conclusions on optimal laser parameters to enhance fusion. We show that intense low-frequency lasers are most efficient in delivering energy to the fusion system and enhancing the fusion yield.

The possibility is pointed out that lasers may be used to reduce the temperature requirement of controlled fusion research. The vast difference between fusion cross sections at different temperatures shrinks in the presence of laser fields. Controlled fusion experiments might be performed at lower temperatures with the aid of intense laser fields.

In this article, we only consider the (pure) system of two nuclei plus a laser field. We have not considered a more complicated plasma environment with nuclei, electrons, laser-plasma interactions, etc. These complications are important but outside the scope of the current article. As a first step, we need to understand the laser-assisted nuclear fusion process, and this is the goal of the current article. Adding the above-mentioned complications into the picture and evaluating the effect of laser fields is the next step to go.

Note: Figure translations are in progress. See original paper for figures.