The Importance of Nuclear Potential in the ^{16}O- and ^{40}Ca-Accompanied Ternary Fission of ^{256}Fm

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Abstract

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Full Text

Preamble

The importance of nuclear potential in the 16O and 40Ca accompanied ternary fission of 256Fm isotope

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Abstract

This study analyzes the ternary fission of the 256Fm isotope in the fragments collinear geometry with A2 and A3 fragments located in the middle position, incorporating 16O and 40Ca as accompanied particles. The driving potentials were calculated using the Proximity, the Akyüz-Winther, and the Broglia-Winther potentials to assess their influence on the fission process for each available fragment configuration. Among all possible configurations, fragment combinations with positive Q-values were selected and grouped based on their proton numbers (Z) for a comprehensive comparison. The calculated results indicate that the collinear geometry is energetically favorable, with variations in driving potential highlighting the impact of nuclear structure, such as magic and near-magic nuclei, on the selection of combinations. The interaction barrier includes Coulomb, nuclear, and centrifugal potentials. To check the effects of nuclear potential, three types of potential are considered. The comparative analysis between the Proximity, the Akyüz-Winther, and the Broglia-Winther potentials reveals notable differences in values of penetration probabilities and decay constants. These findings provide critical insights into the mechanisms governing the ternary fission and the role of nuclear structure in determining fragment configurations.

Keywords: ternary fission; collinear geometries; driving potential; penetration probability; Woods-Saxon potential; Proximity potential; half-life.

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Introduction

The concept of ternary fission dates back to the pioneering work of San-Tsiang et al. in the 1940s [1–5]. Subsequent experimental studies, such as those by Titterton [6] and Flynn [7], significantly advanced our understanding of this rare phenomenon. Unlike binary fission, which results in two primary fragments, ternary fission involves the simultaneous emission of three fragments. Among these, true ternary fission, where all three fragments have comparable masses, is an exceptionally rare process. It predominantly occurs in heavy nuclei with high fissility parameters ($Z^2/A > 31$), where the Coulomb repulsion surpasses the nuclear binding forces [8].

Rosen and Hudson [9] observed that the frequency of symmetrical true ternary fission is about 6.7 ± 3 per 10^6 binary fission events in the ^{235}U isotope. For simplicity of calculation, often one of the three fragments in ternary fission is considered fixed. Most ternary fission events involve light-charged-particle (LCP)-accompanied ternary fission, where the fixed light particle is emitted perpendicular to the line joining heavier fragments due to Coulomb repulsion, leading to equatorial ternary fission. In contrast, true ternary fission is characterized by the collinear escape of fragments, offering a distinct avenue for understanding nuclear dynamics.

The impressive work of Blocki et al. [10] introduced a theorem for understanding proximity forces between gently curved surfaces, showing that these forces are proportional to the interaction potential per unit area of flat surfaces, modulated by the mean curvature of objects. This work provides a formula for nuclear interaction potentials as a product of a geometric factor and a universal function of nuclear separation, derived using the nuclear Thomas-Fermi approximation. This proximity energy enhanced traditional energy expansions in nuclear systems, offering valuable insights into phenomena such as neck formation during fission and improving the accuracy of nuclear interaction models, even for light isotopes.

Binary fission, a well-studied process due to its relevance to nuclear energy production, occurs spontaneously or is induced by neutrons, charged particles, and energetic γ-rays [11–19]. Despite its rarity, ternary fission provides unique insights into nuclear breakup dynamics and mechanisms. Recent theoretical developments have applied geometrical three-cluster models (TCM), considering prolately deformed nuclei connected linearly and incorporating higher multipole deformations like hexadecapole components. For instance, Hess et al. [20] utilized these models to investigate configurations such as ^{96}Sr + ^{10}Be + ^{146}Ba, illustrating the significance of TCM in studying ternary fission.

Lestone [21] proposed a statistical theory for particle evaporation, highlighting a mechanism where quasi-evaporated charged particles exist transiently near the nuclear surface and occasionally escape during the scission process. Similarly, Florescu et al. [22] explored the preformation amplitudes of light clusters, such as α particles and ^{10}Be, in the cold ternary fission of ^{252}Cf. Their findings indicated that cluster formation is maximized near the scission point, influenced by the spatial configuration of the fragments.

Further investigations, such as those by Sowmya and Manjunatha [23], have emphasized comparative studies of binary and ternary fission in uranium isotopes, providing detailed analyses of potential energy surfaces and branching ratios that elucidate the dominant modes of nuclear decay. Expanding on this, Vijayaraghavan et al. [24] discussed dynamics of true ternary fission for heavier fragments (A $>$ 30), where collinear geometries and closed-shell configurations significantly enhance specific decay channels.

The almost sequential mechanism of collinear ternary fission, as studied by Tashkhodjaev et al. [25], explored sequential decay pathways influenced by the Coulomb field of the initially separated fragment. Their analysis demonstrated measurable yields of heavy clusters, such as ^{70}Ni and ^{94}Kr, during the fission of

^{252}Cf, correlating well with experimental observations and highlighting the role of potential-energy landscapes in determining fragment distributions.

Santhosh et al. [26] investigated the cold ternary fission of ^{242}Cm, focusing on LCPs such as ^{4}He, ^{10}Be, and ^{14}C. Using Coulomb and proximity potentials, they highlighted the role of doubly magic and near-doubly magic nuclei in determining favorable fragmentation channels, including configurations like ^{104}Mo + ^{134}Te + ^{4}He and ^{94}Sr + ^{134}Te + ^{14}C.

In the context of superheavy elements, Balasubramaniam et al. [27] emphasized the influence of shell effects and potential energy surfaces. Their research demonstrated that neutron magic numbers significantly impact the preferred ternary fragmentation pathways, with a strong preference for configurations where the lightest fragment resides centrally. The effects of nuclear proximity potential in ternary fission induced by heavy-ion reactions were directly observed by Harrach et al. [28], revealing high probabilities for three-body events originating from two-step reactions, with strong effects of proximity potential and scission-to-scission times on the order of 10^{-21} s.

Ismail et al. [29] analyzed the ternary fission of ^{260}No in the collinear geometry, employing the TCM and considering fragment deformation and orientation. Their findings identified favored fragmentation channels, such as ^{150}Ce + ^{41}Be + ^{100}Zr and ^{106}Mo + ^{48}Ar + ^{106}Mo, based on local minima in driving potentials and high Q-values. They also emphasized the influence of nuclear deformations on the selection of prolate and closed-shell fragments.

In our earlier work on the equatorial geometry of ternary fission in ^{248}Cf [30], we investigated the ^{4}He, ^{10}Be, ^{14}C, ^{14}N, and ^{16}O LCPs accompanied ternary fission in both equatorial and collinear geometries. Additionally, we compared the equatorial and collinear fragment geometries for ^{8}Be, ^{28}Si, ^{58}Ni, and ^{90}Zr accompanied ternary fission of the ^{244}Fm isotope, applying the Proximity Potential [31]. In that work, we showed that the collinear geometry of fragments is more favorable than the equatorial configuration for heavy fixed fragments. Here, we analyze the ^{16}O and ^{40}Ca accompanied ternary fission of the ^{256}Fm isotope using the collinear configuration, mediating A2 and A3 by applying the Proximity, the Broglia-Winther, and the Akyüz-Winther different nuclear potentials.

The choice of the ^{256}Fm isotope for studying ternary fission is strongly supported by its decay characteristics. According to isotopic data [32], the ^{256}Fm isotope undergoes spontaneous fission (SF) with a branching ratio of 91.9%, while only 8.1% of its decay occurs via α-emission. The high SF probability increases the likelihood of observing fission events, including ternary fission. These properties make the ^{256}Fm isotope an ideal candidate for probing ternary fission mechanisms and associated particle yields.

Theoretical Calculations Approach

The ternary fission process closely resembles binary fission, differing primarily in the emission of LCPs during the final stage of the process at the scission point. Study of ternary fission provides insights not only into fission dynamics but also into nuclear structure. Theoretical frameworks have been developed to track LCP trajectories, offering tools to analyze statistical and dynamic characteristics of fission and scission criteria [33, 34].

Calculation of Q-values

The energy released in ternary fission (Q-value) is calculated using:

$$Q = M_{\text{parent}} - \sum_{i=1}^{3} m_i$$

where M is the mass excess of the parent nucleus, and m_i (i = 1, 2, 3) are the mass excess values of the three fragments in MeV units [35]. A positive Q-value is required for fission to occur, which is shared as the kinetic energies of the fragments.

Interaction Potentials

The interaction potential between fragments consists of the Coulomb and nuclear potentials:

$$V_{\text{total}} = \sum_{i<j} \left(V_{ij}^{\text{Coulomb}} + V_{ij}^{\text{Nuclear}} \right)$$

1. The Coulomb Potential The Coulomb potential for the interaction of spherical fragments is defined as follows:

$$V_{ij}^{\text{Coulomb}} = \frac{Z_i Z_j e^2}{R_{ij}}$$

where Z_i and Z_j are atomic numbers, and R_{ij} is the center-to-center distance between interacting fragments.

2. The Proximity Potential The nuclear proximity potential $V_p(s)$ is defined as [10]:

$$V_p(s) = 4\pi\gamma R \Phi\left(\frac{s}{b}\right)$$

where $\Phi(\xi)$ as the universal function depends on the separation s/b between fragments and is given by [36]:

$$\Phi(\xi) = \begin{cases} -2(\xi - 2.54)^2 - 0.0852(\xi - 2.54)^3, & \xi < 1.2511 \\ -3.437 \exp(-\xi/0.75), & \xi \geq 1.2511 \end{cases}$$

The effective radius R_x for each fragment is defined as:

$$R_x = 1.28 A_x^{1/3} - 0.76 + 0.8 A_x^{-1/3}$$

with A_x as the mass number of a given fragment.

3. The Broglia-Winther Potential The Broglia-Winther potential is defined by the following equations [37]:

$$V_N = -\frac{V_0}{1 + \exp\left(\frac{r - R_0}{a}\right)}$$

where $a = 0.63$ and $R_0 = R_1 + R_2 + 0.29$. The surface tension parameter in this potential is defined as [37]:

$$\gamma(Z_i, N_i, A_i, Z_j, N_j, A_j) = 0.95 \left[1 - 1.8 \left(\frac{Z_i - N_i}{A_i}\right) \left(\frac{Z_j - N_j}{A_j}\right)\right]$$

and nuclear radius is obtained by:

$$R(A_x) = 1.233 A_x^{1/3} - 0.98 A_x^{-1/3}$$

4. The Akyüz-Winther Potential The Akyüz-Winther potential is defined by [38]:

$$U(r) = \frac{V_0}{1 + \exp\left(\frac{r - R_i - R_j}{a}\right)}$$

where the potential depth is calculated using [38]:

$$V_0 = 16\pi\gamma a R_{ij}, \quad R_{ij} = R_i + R_j$$

$$R_i = 1.20 A_i^{1/3} - 0.09 \text{ fm}$$

and the diffuseness parameter is [38]:

$$a = 0.63 \left[1 + 0.53 A^{-1/3} + A^{-1/3}\right] \text{ fm}$$

while the surface tension parameter is evaluated using [39]:

$$\gamma = 0.95\left[1 - 1.8\left(\frac{N_i - Z_i}{A_i}\right)\left(\frac{N_j - Z_j}{A_j}\right)\right]\ \mathrm{MeV}\cdot\mathrm{fm}^{-2}$$

The Collinear Geometry

In the collinear configuration, the fragments align along a straight line, moving along a straight line connecting their centers. The parameter s describes the distance between near surfaces of interacting fragments: $s = 0$ represents a touching configuration, $s < 0$ denotes overlapping fragments, and $s > 0$ shows separated fragments. With A_3 positioned between the other two fragments, the distance between the surfaces of fragments 1 and 3, or 2 and 3, is denoted as:

$$s = s_{13} = s_{23}$$

The center-to-center distance between fragments 1 and 2 is then given by:

$$s_{12} = 2(R_3 + s)$$

The Penetration Probability

The penetration probability P is calculated using the WKB approximation as [40]:

$$P = \exp\left(-2\int_{S_{\mathrm{in}}}^{S_{\mathrm{out}}} \sqrt{2\mu(V - Q)}\, dS\right)$$

where S_{in} and S_{out} are the inner and outer turning points, and μ is the fragments' reduced mass obtained using:

$$\mu = m\frac{A_1A_2A_3}{A_1A_2 + A_1A_3 + A_2A_3}$$

The Half-Life of Ternary Fission

Half-life of the ternary fission is evaluated by the following equation:

$$\lambda = \nu P, \quad T_{1/2} = \frac{\ln 2}{\lambda}$$

where ν is the assault frequency that is simply defined as follows:

$$\nu = \frac{v}{R_0}$$

The vibrational energy E_ν is then calculated using the following phenomenological relation [41]:

$$E_\nu = Q\left[0.056 + 0.039\exp\left(\frac{4 - A_{\text{LCP}}}{A_{\text{LCP}}}\right)\right]$$

Finally, the relative yield $Y(A_i, Z_i)$ for each fragmentation in a given geometry is determined as follows:

$$Y(A_i, Z_i) = \frac{P(A_i, Z_i)}{\sum P(A_i, Z_i)}$$

Results and Discussion

This study covers the ternary fission of the ^{256}Fm isotope based on collinear configurations with fixed fragments ^{16}O and ^{40}Ca. The main objective is to explore the influence of nuclear potential on driving potentials, penetration probabilities, and fragmentation yields. Fragment combinations were selected based on their Q-values and penetration probabilities. Three different nuclear potentials are considered: the Proximity (V_P), the Broglia-Winther ($V_{W.B}$), and the Akyüz-Winther ($V_{A.W}$) potentials. The results highlight the significant impact of geometric configuration, structural properties (such as magic and near-magic proton and neutron numbers), and the choice of potential model on ternary fission outcomes.

The Q-values of selected combinations for both fixed fragments are illustrated in Fig. 1 [Figure 1: see original paper], with the highest Q-value for each fixed fragment explicitly identified. The calculated driving potentials for collinear configurations are illustrated in Fig. 2 [Figure 2: see original paper]. Comparisons are made between the Proximity, Akyüz-Winther, and Broglia-Winther potentials for mid-placement of different fragments. The corresponding penetration probabilities for these configurations are shown in Fig. 3 [Figure 3: see original paper], highlighting the role of nuclear potential in influencing fission dynamics. The most favorable combination, indicated by a vertical line in both figures, has the structure of the combination explicitly labeled.

Configurations with ^{16}O as the Fixed Fragment

For configurations with ^{16}O as the fixed fragment, over 660 combinations leading to positive Q-values were analyzed, with the most favorable combination selected based on its penetration probability. The standout combination, ^{108}Mo + ^{132}Sn + ^{16}O, exhibited the highest Q-value (237.51 MeV), the lowest potential barrier, and the highest penetrating probability across all three nuclear potentials. For the A_2 fragment occupying the middle position, the driving potentials were $V_P = 38.67$ MeV, $V_{W.B} = 38.53$ MeV, and $V_{A.W} = 36.26$ MeV for the Proximity, Akyüz-Winther, and Broglia-Winther potentials, respectively.

For A_3 as the middle fragment, the driving potentials decreased significantly to $V_P = 0.06$ MeV, $V_{W.B} = 16.14$ MeV, and $V_{A.W} = 8.79$ MeV, reflecting reduction of potential barriers and enhancement of penetration probabilities.

The penetration probabilities and yields for A_2 as the middle fragment were $P_{P-A2} = 6.75 \times 10^{-23}$ and $Y_{P-A2} = 96.7\%$, respectively. For A_3 positioned in the middle, the Proximity potential yielded the highest penetration probability ($P_{P-A3} = 1.64 \times 10^{-4}$) and yield (85.77%). The nuclear structure of these fragments includes the even-even ^{108}Mo (Z = 42, N = 66) fragment and the doubly magic ^{132}Sn (Z = 50, N = 82) ones, whose neutron and proton shell closures stabilize the configuration. The lighter A_3 fragment in the geometry when occupied in the middle further reduces the driving potential.

The calculated data for configurations with ^{16}O as the fixed fragment are tabulated in Table I (A_2 as the middle fragment) and Table II (A_3 as the middle fragment). Driving potentials are illustrated in Figures 2a and 2b and penetration probabilities are visualized in Figures 3a and 3b for A_2 and A_3 occupied in the middle position, respectively.

Configurations with ^{40}Ca as the Fixed Fragment

For ^{40}Ca as the fixed fragment, 304 combinations with positive Q-values were analyzed. The most favorable configurations were ^{82}Zn + ^{134}Sn + ^{40}Ca (Q = 229.08 MeV) and ^{86}Ge + ^{130}Cd + ^{40}Ca (Q = 230.85 MeV). The former exhibited lower driving potentials for A_3 as the middle fragment: $V_P = 32.06$ MeV, $V_{W.B} = 38.74$ MeV, and $V_{A.W} = 35.13$ MeV. For A_2, potentials increased to $V_P = 56.50$ MeV, $V_{W.B} = 58.88$ MeV, and $V_{A.W} = 55.46$ MeV for the first combination. Similarly, ^{86}Ge + ^{130}Cd + ^{40}Ca showed $V_P = 33.29$ MeV (A_3) and $V_P = 56.75$ MeV (A_2).

Nuclear shell effects played a critical role in the calculation of ternary fission quantities. For example, combinations including the ^{134}Sn fragment (Z = 50, N = 84) and the ^{130}Cd fragment (Z = 48, N = 82) had lower driving potentials and higher penetration probabilities due to their magic and near-magic proton/neutron numbers, which contributed to shell effects and closure. Meanwhile, the ^{82}Zn (Z = 30, N = 52) and ^{86}Ge (Z = 32, N = 54) fragments enhanced stability due to their near-magic and even-even nature.

Penetration probabilities for ^{82}Zn + ^{134}Sn + ^{40}Ca were $P_{P-A3} = 2.55 \times 10^{-32}$ for A_3 occupied in the middle position and $P_{P-A2} = 2.97 \times 10^{-47}$ for A_2 occupied in the middle position, with yields of 68.7% and 50.2%, respectively. For ^{86}Ge + ^{130}Cd + ^{40}Ca, penetration probabilities decreased slightly to $P_{P-A3} = 1.00 \times 10^{-32}$ and $P_{P-A2} = 2.78 \times 10^{-47}$, yielding 26.9% and 46.9% for A_3 and A_2 occupied in the middle position, respectively.

Final results for combinations with ^{40}Ca as the fixed fragment are summarized in Tables III and IV. Driving Potentials for A_2 and A_3 in the middle position are indicated in Figs. 2c and 2d and penetration probabilities illustrated in Figs.

3c and 3d. The dominance of combinations with A_3 positioned in the middle for lowering barriers mirrors trends observed for ^{16}O, underscoring the universal role of fragment geometry and shell closures in ternary fission.

[Figure 1: see original paper] The calculated Q-values as a function of fragment mass number A_1 for the ^{16}O and ^{40}Ca-accompanied ternary fission of ^{256}Fm isotope in the collinear geometry.

[Figure 2: see original paper] The calculated driving potentials as a function of fragment atomic numbers A_1 for the ternary fission of ^{256}Fm isotope in the collinear geometry. Each plot compares the Proximity, the Akyüz-Winther, and the Broglia-Winther potentials for the configurations where ^{16}O or ^{40}Ca are the accompanied fragments, with A_2 or A_3 occupied in the middle position.

[Figure 3: see original paper] Penetration probabilities vs fragment mass number A for the ternary fission of ^{256}Fm isotope in the collinear geometry using the Proximity, the Akyüz-Winther, and the Broglia-Winther potentials for configurations where ^{16}O or ^{40}Ca are the accompanied fragments, with A_2 or A_3 occupied in the middle position.

Conclusions

This study investigates ^{16}O and ^{40}Ca-accompanied ternary fission of ^{256}Fm isotope in the collinear geometry, focusing on nuclear shell effects and the choice of potential model. The Proximity, Akyüz-Winther, and Broglia-Winther potentials are considered to study the influence of nuclear potential on ternary fission quantities. The half-lives for the ternary fission of the ^{256}Fm isotope, calculated using different potentials, are summarized in Table V. The Akyüz-Winther (A.W) potential consistently predicts the shortest half-lives across configurations, particularly for the ^{40}Ca fixed fragment, where its minimized driving potentials ($V_{A.W} = 35.13$ MeV for $^{82}Zn + {}^{134}Sn + {}^{40}Ca$) lead to penetration probabilities orders of magnitude higher than other potentials. For ^{40}Ca, A.W yields half-lives as low as 8.55×10^{52} seconds for A_3 occupied in the middle position, outperforming the Proximity potential (1.05×10^{56} seconds) and Broglia-Winther potential (1.70×10^{55} seconds).

For ^{16}O as the fixed fragment, while the Proximity potential occasionally achieves higher penetration probabilities in specific cases (e.g., $P_{P-A3} = 1.64 \times 10^{-4}$ for $^{108}Mo + {}^{132}Sn + {}^{16}O$ and $P_{P-A3} = 8.43 \times 10^{-6}$ for $^{106}Zr + {}^{134}Te + {}^{16}O$), the A.W potential still dominates in overall half-life reduction. The A_3 middle-positioned configurations with ^{16}O exhibit a half-life of 2.88×10^{28} seconds under A.W, significantly shorter than the Proximity's 3.75×10^{29} seconds, confirming A.W's superiority in balancing nuclear and Coulomb interactions.

Heavier middle fragments (A_2) universally suppress fission, as seen in prolonged half-lives (1.76×10^{46} seconds for ^{16}O and 1.49×10^{73} seconds for ^{40}Ca under the Proximity potential). This trend aligns with the enhanced stability of magic/near-magic fragments like ^{132}Sn (Z = 50, N = 82), where shell closures reduce driving potentials. Even-even fragments such as ^{108}Mo (Z = 42, N = 66) further stabilize configurations, though their lack of magic numbers underscores the nuanced role of pairing effects.

Critically, the A.W potential's accuracy in modeling fission barriers, evidenced by trends for magic number systems, positions it as the most reliable choice for ternary fission studies. While the Proximity potential excels in isolated ^{16}O cases, its overestimation of repulsive forces limits broader applicability. The Broglia-Winther potential, though intermediate, lacks the precision required for systems dominated by shell effects.

These findings underscore the importance of conducting potential-specific analyses in fission modeling, particularly for complex cases involving actinides like ^{256}Fm. By integrating fragment geometry, shell closures, and interaction potentials, this work contributes to the advancement of predictive frameworks for ternary fission, offering valuable insights for the study of nuclear decay.

Tables

Table I

Q-value, penetration probabilities, driving potentials, and yields for ^{16}O as the fixed fragment with A_2 in the middle, using different potentials (the Proximity, the Broglia-Winther, and the Akyüz-Winther) for the ternary fission of ^{256}Fm isotope. Yields below 10^{-4} are represented as 0.

[TABLE:I]

Table II

Q-value, penetration probabilities, driving potentials, and yields for ^{16}O as the fixed fragment with A_3 in the middle, using different potentials (the Proximity, the Broglia-Winther, and the Akyüz-Winther) for the ternary fission of ^{256}Fm isotope. Yields below 10^{-4} are represented as 0.

[TABLE:II]

Table III

Q-value, penetration probabilities, driving potentials, and yields for ^{40}Ca as the fixed fragment with A_2 in the middle, using different potentials (the Proximity, the Broglia-Winther, and the Akyüz-Winther) for the ternary fission of ^{256}Fm isotope. Yields below 10^{-4} are represented as 0.

[TABLE:III]

Table IV

Q-value, penetration probabilities, driving potentials, and yields for ^{40}Ca as the fixed fragment with A_3 in the middle, using different nuclear potentials (the Proximity, the Broglia-Winther, and the Akyüz-Winther) for the ternary fission of ^{256}Fm isotope. Yields below 10^{-4} are represented as 0.

[TABLE:IV]

Table V

Half-lives (in seconds) for the ternary fission of ^{256}Fm isotope with ^{16}O and ^{40}Ca as the fixed fragments. Calculations performed for the middle-positioned fragments (A_2 and A_3) considering the Proximity, the Akyüz-Winther, and the Broglia-Winther potentials separately.

[TABLE:V]

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Note: Figure translations are in progress. See original paper for figures.