Constraining the Nuclear Matter Equation of State Using Massive Neutron Stars within the Relativistic Mean-Field Model Framework

Authors: Wenjie Xie, Xia Chengjun

Date: 2025-04-30T14:56:44+00:00

Abstract

With the increasing volume of astronomical observational data, data-driven approaches for inferring the equation of state of neutron star matter have become viable. This paper employs Bayesian inference and astrophysical observational constraints on relativistic mean-field (RMF) models to investigate the nuclear equation of state (EOS) and neutron star structure. Through analysis of the density-dependent behavior of coupling constants at different transition densities, we find that higher transition densities strengthen the constraints from astrophysical observational data on the equation of state, leading to softening of the intermediate-density behavior of massive neutron stars and increased central energy density. In particular, we find a high probability that the squared speed of sound exceeds the conformal limit (v>1/3), even within the cores of massive neutron stars. The inferred maximum neutron star mass (Mmax \$ \$2.5M) is consistent with interpretations of gravitational wave events such as GW190814, where the low-mass companion likely corresponds to a massive neutron star. Furthermore, the symmetry energy and pressure at extreme densities also exhibit transition density dependence, in accordance with multi-messenger constraints.

Full Text

Constraining Nuclear Equations of State with Massive Neutron Stars in the Framework of Relativistic Mean-Field Models

Wen-Jie Xie^{1,2,3} and **Cheng-Jun Xia**^{4}

^{1}Department of Physics and Electronic Engineering, Yuncheng University, Yuncheng 044000, China
^{2}Shanxi Province Intelligent Optoelectronic Sensing Application Technology

Innovation Center, Yuncheng University, Yuncheng 044000, China
^{3}Guangxi Key Laboratory of Nuclear Physics and Nuclear Technology, Guangxi Normal University, Guilin 541004, China
^{4}Center for Gravitation and Cosmology, Yangzhou University, Yangzhou 225009, China

Funding: This work was supported by the National Program on Key Science and Technology Research Projects (No. 2020SKA0120300), the National Natural Science Foundation of China (No. 12275234), the Shanxi Provincial Foundation for Returned Overseas Scholars (No. 20220037), the Natural Science Foundation of Shanxi Province (No. 20210302123085), the Open Project of Guangxi Key Laboratory of Nuclear Physics and Nuclear Technology (No. NLK2023-03), and the Central Government Guidance Funds for Local Scientific and Technological Development (No. Guike ZY22096024).

First author: Wen-Jie Xie, born 1980, received his Ph.D. from Beijing Normal University in 2014. Research interests: particle physics and nuclear physics.
Corresponding author: Cheng-Jun Xia, E-mail: cjxia@yzu.edu.cn
Received: 2025-04-00; **Revised**: 20XX-00-00

Abstract

Background: With the increasing volume of astronomical observational data, data-driven approaches to inferring the equation of state (EOS) of neutron star matter have become feasible.
Purpose: This study aims to constrain nuclear equations of state using massive neutron stars within the framework of relativistic mean-field (RMF) models, incorporating various astrophysical observations of neutron star properties.
Methods: We investigate the nuclear equation of state and neutron star structures using RMF models constrained by Bayesian inference and astrophysical observations.
Results: By analyzing density-dependent coupling constants across different critical densities, we demonstrate that higher critical densities tighten observational constraints on the EOS, leading to softer intermediate-density behavior and increased central energy densities for massive neutron stars. Notably, we find a high probability that the squared sound speed in massive neutron star cores exceeds the conformal limit ($v_s^2 > 1/3$). The inferred maximum neutron star masses ($M_{\max} \geq 2.5M_\odot$) align with interpretations of gravitational wave events such as GW190814, where the low-mass companion may correspond to a massive neutron star. Additionally, the symmetry energy and pressure at extreme densities exhibit critical-density dependence consistent with multi-messenger constraints.
Conclusions: These findings highlight the interplay between EOS stiffness, phase transitions, and observational constraints, providing critical insights for future studies to refine nuclear matter properties through multi-messenger data

and advanced density functional analyses.

Keywords: Relativistic mean-field model, Bayesian inference, neutron stars, equation of state, gravitational waves.

1. Introduction

While nuclear matter properties near saturation density ($n_0 = 0.16$ fm^{-3}) are relatively well understood [?, ?, ?, ?, ?, ?], the behavior of strongly interacting matter at supranuclear densities ($n \gtrsim 2n_0$) remains uncertain [?, ?, ?, ?]. This uncertainty primarily stems from the non-perturbative nature of quantum chromodynamics (QCD), with reliable perturbative calculations only achievable at extremely high densities ($n \gtrsim 40n_0$) [?, ?]. Fortunately, astrophysical observations of neutron stars have reached unprecedented precision. As the densest objects in the universe, neutron stars serve as natural laboratories for probing the properties of strongly interacting matter at extreme densities.

For instance, reproducing the measured masses of pulsars PSR J1614-2230 ($1.928 \pm 0.017 M_\odot$) [?, ?] and PSR J0348+0432 ($2.01 \pm 0.04 M_\odot$) [?] requires the neutron star matter equation of state to be sufficiently stiff. However, measurements of tidal deformability ($70 \leq \Lambda_{1.4} \leq 580$) and radius ($R = 11.9 \pm 1.4$ km) for $1.4M_\odot$ neutron stars from the binary neutron star merger event GRB 170817A-GW170817-AT 2017gfo [?] suggest the corresponding EOS should be sufficiently soft. Pulse profile modeling using NICER and XMM-Newton data has enabled joint mass-radius measurements for PSR J0030+0451, PSR J0740+6620, and PSR J0437-4715 [?, ?, ?, ?, ?]. These observations support constraints from other measurements, indicating that the EOS is soft at low densities and stiff at high densities. This suggests a unique feature in the neutron star matter sound speed v_s, which may exhibit a peak corresponding to a possible deconfinement phase transition [?, ?, ?, ?, ?, ?, ?, ?].

Unlike the Chandrasekhar mass limit for white dwarfs, the maximum mass of neutron stars remains unknown. Beyond the confirmed $2M_\odot$ neutron stars [?, ?, ?], there are indications that the maximum mass may be substantially larger. Specifically, the binary compact object merger events GW190814 and GW200210 suggest companion masses of $2.6M_\odot$ and $2.8M_\odot$, respectively [?, ?], exceeding the $2M_\odot$ limit and falling within the hypothesized low-mass gap of $2.5 - 5M_\odot$ [?]. If these companions are neutron stars, the neutron star matter EOS must be significantly stiffer, potentially eliminating the peak in sound speed v_s.

Consequently, our research has two primary objectives: to constrain the nuclear matter EOS based on neutron star observations and to investigate the structure of the most massive neutron stars. To achieve these goals, we employ relativistic mean-field (RMF) models [?, ?, ?, ?, ?, ?, ?, ?, ?]. Specifically, we perform detailed statistical analyses of density-dependent coupling constants in point-coupling RMF models using Bayesian methods, incorporating various constraints from neutron star observations. The constrained coupling constants are then used to determine the nuclear matter EOS and study the mass and composition of the most massive neutron stars.

This paper is organized as follows. Section 2 introduces the theoretical framework of relativistic mean-field models and density-dependent coupling constants. Section 3 applies Bayesian inference to determine parameters from astrophysical observations and presents the resulting neutron star equations of state and structures. Section 4 presents our conclusions.

---

2. Theoretical Framework

2.1 Relativistic Mean-Field Model

The relativistic mean-field model has proven successful in describing both finite nuclei [?, ?, ?, ?, ?, ?] and nuclear matter [?, ?, ?, ?, ?]. The Lagrangian density for the meson-exchange RMF model is given by:

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu}-M)\psi+\frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}\sigma^{2}-\frac{1}{3}g_{2}\sigma^{3}-\frac{1}{4}g_{3}\sigma^{4}+g_{\sigma}\bar{\psi}\psi\sigma-\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu}+\frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu}-g_{\omega}\bar{\psi}\gamma^{\mu}\psi\omega_{\mu}-\frac{1}{4}\rho^{\mu\nu}\rho_{\mu\nu}$$

where M denotes the nucleon mass, τ_3 represents the third component of isospin, and $\omega_{\mu\nu} = \partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu}$ and $\rho_{\mu\nu} = \partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu}$ are field tensors.

For uniform nuclear matter, the Lagrangian density can be simplified to a point-coupling form [?]:

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - M)\psi + \frac{1}{2}\alpha_{S}n_{S}^{2} + \frac{1}{2}\alpha_{V}n_{V}^{2} + \frac{1}{2}\alpha_{TV}n_{TV}^{2}$$

where the coupling constants are $\alpha_S = g_{\sigma}^2/m_{\sigma}^2$, $\alpha_V = g_{\omega}^2/m_{\omega}^2$, and $\alpha_{TV} = g_{\rho}^2/m_{\rho}^2$. The nucleon chemical potentials are determined by:

$$\mu_i = \sqrt{\nu_i^2 + M^{*2}} + \alpha_V n_V + \tau_{3i}\alpha_{TV}n_{TV}$$

where $M^* = M - \alpha_S n_S$ represents the effective nucleon mass, ν_i denotes the Fermi momentum, the particle number density is $n_i = \nu_i^3/3\pi^2$, $n_V = n_n + n_p$, and $n_{TV} = n_n - n_p$ represents the vector density. If the coupling constants are density-dependent, we have:

$$\alpha_i(n_V) = \alpha_i(n_S)$$

Finally, the scalar density, energy density, and pressure of nuclear matter are given by:

$$n_S = \sum_i \frac{M^*}{\pi^2} \left[\nu_i \sqrt{\nu_i^2 + M^{*2}} - M^{*2} \ln \left(\frac{\nu_i + \sqrt{\nu_i^2 + M^{*2}}}{M^*} \right) \right]$$

$$E = \sum_i \frac{1}{\pi^2} \left[\frac{1}{8} \nu_i \sqrt{\nu_i^2 + M^{*2}} (2\nu_i^2 + M^{*2}) - \frac{1}{8} M^{*4} \ln \left(\frac{\nu_i + \sqrt{\nu_i^2 + M^{*2}}}{M^*} \right) \right] + \frac{1}{2}\alpha_S n_S^2 + \frac{1}{2}\alpha_V n_V^2 + \frac{1}{2}\alpha_{TV} n_{TV}^2$$

$$P = \sum_i \frac{1}{3\pi^2} \left[\nu_i \sqrt{\nu_i^2 + M^{*2}} (\nu_i^2 - \frac{3}{2} M^{*2}) + \frac{3}{2} M^{*4} \ln \left(\frac{\nu_i + \sqrt{\nu_i^2 + M^{*2}}}{M^*} \right) \right] + \frac{1}{2}\alpha_S n_S^2 + \frac{1}{2}\alpha_V n_V^2 + \frac{1}{2}\alpha_{TV} n_{TV}^2$$

where $x_i = \nu_i / M^*$.

2.2 Density Dependence of Coupling Constants

To obtain correct nuclear matter properties, medium effects must be considered. In nonlinear RMF models [**?**, **?**, **?**, **?**, **?**], medium effects are treated through nonlinear self-couplings of mesons:

$$U(\sigma, \omega) = -\frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + c_3 \omega^4$$

which effectively introduces density-dependent coupling constants. Alternatively, density-dependent coupling constants can be obtained directly based on the Typel-Wolter hypothesis [**?**]:

$$\alpha_i(n_V) = \alpha_i(n_S)$$

[Figure 1: see original paper] shows the effective density-dependent coupling constants in the isoscalar-scalar ($\alpha_S = g_\sigma^2$), isoscalar-vector ($\alpha_V = g_\omega^2$), and isovector-vector ($\alpha_{TV} = g_\rho^2$) channels in symmetric nuclear matter predicted by various relativistic density functionals.

[Figure 1: see original paper] The coupling constants as functions of density in symmetric nuclear matter, predicted by various relativistic density functionals including NL3 [**?**], PK1 [**?**], PK1r [**?**], TM1 [**?**], DD-LZ1 [**?**], DDMEX [**?**], PKDD [**?**], DD-ME2 [**?**], DDV [**?**], and TW99 [**?**]. The coupling constants corresponding to nonlinear self-coupling terms are estimated according to Eqs. (9)-(11). Notably, these relativistic density functionals successfully describe ground-state properties of finite nuclei, with predicted total binding energies within 2-4 MeV and charge radii within 0.01-0.03 fm. As shown in [Figure 1: see original paper], the coupling constants generally decrease with density, with the isoscalar-scalar (α_S) and isoscalar-vector (α_V) channels exhibiting similar values near saturation density, primarily reflecting the saturation properties of nuclear matter that require zero pressure in symmetric nuclear matter near saturation.

[Figure 2: see original paper] shows the potential energy per baryon in symmetric nuclear matter predicted by these functionals. By assuming the isoscalar-scalar channel contribution is sufficiently small, we can expand the kinetic energy density as $E_k(M^*) = E_k(M) - \alpha_S n_S^2$. The potential energy density of nuclear matter can then be obtained from Eq. (6):

$$U = E_N M - E_k(M) = \alpha_V n_V^2$$

for symmetric nuclear matter where $n_{TV} = 0$. We can expand the scalar density to second order:

$$n_S = n_V - 0.04\frac{n_V^2}{n_0}$$

where $n_0 = 0.1\ \text{fm}^{-3}$. This expression well describes the relationship between scalar density n_S and vector density n_V in symmetric nuclear matter.

Combining the coupling constants from [Figure 1: see original paper], we can determine the potential energy density of symmetric nuclear matter. [Figure 2: see original paper] shows the potential energy per nucleon as a function of density. Remarkably, despite different density dependencies of α_S and α_V across functionals, the resulting potential energy densities are consistent, particularly near saturation density ($n_V \approx 0.16\ \text{fm}^{-3}$) where U/n_V reaches its minimum value ($U/n_V \approx -30$ MeV).

Furthermore, the potential energy density in the isovector-vector channel is $\alpha_{TV} n_{TV}^2$. Consequently, nuclear symmetry energy increases with α_{TV}. Near $n_0 = 0.1\ \text{fm}^{-3}$, values of $\alpha_{TV}(n_0)$ predicted by various functionals are consistent. Indeed, reproducing finite nuclear properties yields nearly identical symmetry energies near n_0 across different functionals [**?**, **?**], indicating that α_{TV} is well-constrained at $n_V = n_0$. The derivative of α_{TV} at n_0 determines the slope L of symmetry energy, which can be determined from neutron skin thickness of nuclei. However, measurements of neutron skin thickness in ^{208}Pb and ^{48}Ca yield contradictory results for L [**?**, **?**], requiring more detailed theoretical and experimental investigation.

presents the adopted zeroth, first, and second-order derivatives for the coupling constants in Eqs. (17)-(19) at subsaturation density $n_0 \leq n_V \leq n_0$, fixed by the relativistic density functional DD-ME2 [**?**]. Note that DD-ME2 predicts saturation density $n_0 = 0.152\ \text{fm}^{-3}$, energy per nucleon $\varepsilon_0 = -16.13$ MeV, incompressibility $K = 250.8$ MeV, skewness coefficient $J = 477$ MeV, symmetry energy $\varepsilon_{\text{sym}} = 32.3$ MeV, slope $L = 51.2$ MeV, and curvature parameter $K_{\text{sym}} = -87$ MeV. The corresponding nucleon mass is $M = 938.5$ MeV.

In this work, we constrain coupling constants based on astrophysical observations by dividing the density domain into three regions: $n_0 \leq n_V \leq n_0$, $n_0 \leq n_V \leq n_{\text{crit}}$, and $n_V > n_{\text{crit}}$, where $n_0 = 0.1$ fm^{-3}. In each region, we parameterize the coupling constants as:

$$\alpha_i(n_V) = \alpha_i(n_I) + \alpha_i'(n_I)(n_V - n_I) + \frac{1}{2}\alpha_i''(n_I)(n_V - n_I)^2$$

where n_I, n_0, and n_{crit} represent intersection densities where coupling constants match: $\alpha_S(n_I) = \alpha_S(n_I)$, $\alpha_V(n_I) = \alpha_V(n_I)$, and $\alpha_{TV}(n_I) = \alpha_{TV}(n_I)$. The second derivatives $\alpha_S''(n_I)$, $\alpha_V''(n_I)$, and $\alpha_{TV}''(n_I)$ serve as independent parameters controlling the density dependence in each region. At subsaturation density $n_0 \leq n_V \leq n_0$, coefficients are determined from DD-ME2 [**?**] and listed in . Note that the choice of intersection density n_I is not unique. To demonstrate variations arising from different n_I choices, we adopt $n_{\text{crit}} = 2n_0$ and $3n_0$.

This leaves six independent parameters: $\alpha_S''(n_0)$, $\alpha_V''(n_0)$, $\alpha_{TV}''(n_0)$, $\alpha_S''(n_{\text{crit}})$, $\alpha_V''(n_{\text{crit}})$, and $\alpha_{TV}''(n_{\text{crit}})$, which will be determined from neutron star observations. To determine the neutron star matter EOS, lepton contributions must be included, with energy density and pressure given by:

$$E_l = \frac{m_l^4}{8\pi^2}\left[x_l\sqrt{1+x_l^2}(2x_l^2+1) - \ln\left(x_l + \sqrt{1+x_l^2}\right)\right]$$

$$P_l = \frac{m_l^4}{24\pi^2}\left[x_l\sqrt{1+x_l^2}(2x_l^2-3) + 3\ln\left(x_l + \sqrt{1+x_l^2}\right)\right]$$

where $x_l = \nu_l/m_l$, with Fermi momentum fixed at given number density n_l by $\nu_l = (3\pi^2 n_l)^{1/3}$. Lepton masses are $m_e = 0.511$ MeV and $m_\mu = 105.66$ MeV. At a given total baryon density n_V, the number densities of protons, neutrons, electrons, and muons are determined by charge neutrality and beta-equilibrium conditions: $n_p = n_e + n_\mu$ and $\mu_n = \mu_p + \mu_e = \mu_p + \mu_\mu$.

The total energy density and pressure of neutron star matter are calculated as:

$$E = E_N + E_l, \quad P = P_N + P_l$$

For neutron star matter at $n_V < n_0$, we adopt a unified EOS accounting for nonuniform structure formation [**?**]. Neutron star masses and radii are determined by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [**?**, **?**]:

$$\frac{dP}{dr} = -\frac{G(E+P)(M+4\pi r^3 P)}{r(r-2GM)}, \quad \frac{dM}{dr} = 4\pi r^2 E$$

where the gravitational constant $G = 6.707 \times 10^{-45}$ MeV^{-2}. The dimensionless tidal deformability Λ is calculated using [?, ?, ?]:

$$\Lambda = \frac{2}{3} k_2 \left(\frac{R}{M} \right)^5$$

where R and M are the neutron star radius and mass, and k_2 is the second Love number determined from the EOS and TOV equations with appropriate boundary conditions.

2.3 Bayesian Inference Method

To determine the six independent parameters $p_{i=1,2,\ldots,6} = \{\alpha''_S(n_0), \alpha''_V(n_0), \alpha''_{TV}(n_0), \alpha''_S(n_{\rm crit}), \alpha''_V(n_{\rm crit}), \alpha''_{TV}(n$ for $n_{\rm crit} = 2n_0$ and $3n_0$, we employ Bayesian inference. According to Bayes' theorem, the posterior probability $P(M|D)$ given dataset D is:

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

The likelihood function $P(D|M)$ represents the conditional probability that model M correctly predicts data D, while $P(M)$ is the prior probability, which we take as uniform. The denominator serves as a normalization constant.

We randomly sample the six independent parameters $p_{i=1,2,\ldots,6}$ according to the prior probability $P(M)$, then determine the corresponding EOS and neutron star structure using Eqs. (24)-(27). The likelihood of the model reproducing observed radii $R_{{\rm obs},j=1,2,\ldots,7}$ from dataset $D(R_{1,2,\ldots,7})$ shown in is evaluated as:

$$P(D|M) = \prod_{j=1}^{7} \frac{1}{\sqrt{2\pi}\sigma_{{\rm obs},j}} \exp\left[-\frac{(R_{{\rm th},j} - R_{{\rm obs},j})^2}{2\sigma^2_{{\rm obs},j}} \right]$$

where $\sigma_{{\rm obs},j}$ denotes the 1σ error associated with observation j, which can take different values to approximate asymmetric (non-Gaussian) distributions. For different constraints from the same source, we compute the likelihood by randomly selecting each constraint with equal weighting. The total likelihood is determined by multiplying individual likelihood components:

$$P[D|M(p_{1,2,\ldots,6})] = P_{\rm filter} \times P_{\rm mass,max}$$

Here, $P_{\rm filter}$ imposes thermodynamic stability ($dP/dE \geq 0$), causality ($dP/dE \leq 1$), and positivity of effective nucleon mass ($M^* \geq 0$) at all densities. $P_{\rm mass,max}$ ensures the EOS is sufficiently stiff to support a maximum neutron star mass $M_{\rm max} \geq 1.97 M_\odot$.

Finally, we employ Markov Chain Monte Carlo (MCMC) methods with the Metropolis-Hastings algorithm [?, ?, ?, ?] to determine the posterior probability density functions (PDFs) of model parameters, using the astrophysical constraints shown in . We assume mean values within the uncertainty ranges of neutron star masses and adopt corresponding radius constraints. lists the most probable values and credible intervals for model parameters at $n_{\text{crit}} = 2n_0$ and $3n_0$, with stronger parameter constraints observed for larger n_{crit} values.

---

3. Results and Discussion

[Figure 3: see original paper] shows the posterior probability distribution functions of the squared speed of sound and central energy densities for neutron stars with $M = 1.4M_\odot$ and $2.0M_\odot$, calculated using transition densities $n_{\text{crit}} = 2n_0$ and $3n_0$.

As shown in , after constraining parameters with various astrophysical observations, we obtain corresponding constraints on nuclear matter properties. In [Figure 3: see original paper], we present the posterior PDFs of central energy densities for $1.4M_\odot$ and $2.0M_\odot$ neutron stars, along with the squared speed of sound at the cores of maximum-mass neutron stars for two critical density thresholds ($n_{\text{crit}} = 2n_0$ and $3n_0$). The results indicate that central energy densities are highly sensitive to the choice of critical density. For $1.4M_\odot$ neutron stars with $n_{\text{crit}} = 3n_0$, the posterior PDF exhibits a prominent shoulder structure, while the distribution for $2.0M_\odot$ neutron stars shows a bimodal feature. Both cases yield higher central energy densities and broader posterior distributions compared to $n_{\text{crit}} = 2n_0$.

Our analysis demonstrates that adopting a higher critical density ($n_{\text{crit}} = 3n_0$) produces tighter observational constraints on the EOS while inducing softening behavior. This explains the systematic increase in central densities: at 68% credibility, the most probable central energy density increases by approximately 48% for $1.4M_\odot$ neutron stars and over 89% for $2.0M_\odot$ neutron stars when n_{crit} changes from $2n_0$ to $3n_0$. Conversely, the squared sound speed decreases slightly with increasing critical density, as higher sound speeds correlate with stiffer EOSs. Notably, our results indicate a high probability that the squared sound speed in massive neutron star cores exceeds the conformal limit of 1/3, consistent with previous findings that this limit is violated when maximum neutron star masses exceed $\sim 2.0M_\odot$ [?, ?, ?, ?]. At extremely high densities ($\sim 40n_0$), the squared sound speed is expected to approach the conformal limit. The case of compact stars composed entirely of quark matter presents an interesting topic for future investigation.

summarizes the most probable values and credible intervals (68% and 90%) for derived central energy densities, tidal deformability, squared sound speed, and maximum mass for neutron stars with masses of $1.4M_\odot$ and $2.0M_\odot$.

[Figure 4: see original paper] displays the nucleon effective masses in neutron star matter within their 90% credible intervals as functions of baryon density. The results reveal two possible trends: the Dirac effective mass either decreases non-monotonically with density before increasing, or shows pure monotonic decrease. Notably, the choice of critical density significantly impacts constraint precision at densities below $2.5n_0$. When $3n_0$ is adopted as the critical density, the 90% credible interval narrows dramatically to $390 - 420$ MeV, whereas the $2n_0$ threshold yields a broader range of $380 - 520$ MeV.

[Figure 5: see original paper] presents the posterior PDFs of tidal deformability for $1.4M_\odot$ and $2.0M_\odot$ neutron stars when switching $n_{\rm crit}$ from $2n_0$ to $3n_0$. The observed reduction in tidal deformability—approximately 29% for $1.4M_\odot$ and 72% for $2.0M_\odot$ neutron stars—directly correlates with EOS softening at higher critical densities. This mass-dependent sensitivity arises because stiffer EOSs (associated with lower critical densities) enhance tidal deformation through stronger gravitational multipole response. The 68% and 90% credible intervals summarized in demonstrate that the $2.0M_\odot$ case shows significantly broader constraints for $n_{\rm crit} = 2n_0$ compared to $n_{\rm crit} = 3n_0$. The enhanced sensitivity of massive neutron stars to critical density variations implies stronger coupling between their internal EOS and tidal deformation mechanisms, consistent with recent studies indicating that massive neutron stars primarily probe the quark-hadron transition region where EOS stiffness undergoes significant density-dependent changes.

[Figure 6: see original paper] shows the posterior PDFs of maximum neutron star masses. We explore the possibility that the secondary objects observed in GW190814 and GW200210 could be neutron stars [?, ?]. If true, the maximum neutron star mass must exceed $2.5M_\odot$, i.e., $M_{\rm max} > 2.5M_\odot$. [Figure 7: see original paper] presents the 68% credible intervals for sound speed, pressure, symmetry energy, and energy per nucleon in symmetric matter as functions of reduced density n_V/n_0, conditioned on neutron stars supporting masses exceeding $2.5M_\odot$. The posterior PDF of maximum neutron star mass shows that while $n_{\rm crit} = 2n_0$ yields a stiffer EOS supporting larger mass configurations, even the $n_{\rm crit} = 3n_0$ scenario predicts maximum masses above $2.5M_\odot$, consistent with interpreting GW190814's secondary as a neutron star. Notably, the $3n_0$ critical density provides tighter EOS constraints, primarily driven by modifications to the symmetry energy sector: symmetry energy at $2n_0$ shows minimal sensitivity to critical density choice (50.7^{+7}_{-6} MeV for $2n_0$ versus 53.9 ± 4.6 MeV for $3n_0$), consistent with heavy-ion collision analyses and multi-messenger neutron star observations (51 ± 13 MeV). In contrast, pressure exhibits clear critical density dependence (14.8 ± 4.8 MeV/fm^3 for $n_{\rm crit} = 2n_0$ versus 16.5 ± 2.7 MeV/fm^3 for $n_{\rm crit} = 3n_0$), falling within bounds from LIGO/Virgo collaborations (green bands) [?]. The sound speed curves show characteristic peaks near critical density, confirming our recent findings [?] of possible hadron-quark phase transitions.

This study constrains the nuclear matter equation of state and explores neutron star structure within the RMF framework, employing Bayesian inference to reconcile theoretical predictions with astrophysical observations. By analyzing density-dependent coupling constants and their impact on EOS stiffness, we reveal that the choice of critical density (e.g., $n_{\mathrm{crit}} = 2n_0$ versus $3n_0$) significantly affects central energy densities and tidal deformability, with higher critical densities producing softer EOS behavior at intermediate densities. Our results demonstrate a high probability of exceeding the conformal limit ($v_s^2 > 1/3$) in massive neutron star cores. Furthermore, imposing a maximum neutron star mass constraint ($M_{\mathrm{max}} \geq 2.5M_{\odot}$) reveals substantial parameter space satisfying all astrophysical constraints, consistent with interpretations of gravitational wave events like GW190814 where massive companions may be neutron stars. These findings highlight the interplay between EOS stiffness, phase transitions, and observational constraints, providing critical insights for future studies to refine nuclear matter properties through multi-messenger data and advanced density functional analyses.

Acknowledgments

We thank the reviewers and individuals who contributed to this work.

Author Contributions

Wen-Jie Xie performed the calculations and wrote the manuscript. Cheng-Jun Xia provided research guidance and contributed to manuscript writing.

References

[1] S. Shlomo, V. M. Kolomietz, and G. Colò, Eur. Phys. J. A 30, 23 (2006).
[2] B.-A. Li and X. Han, Phys. Lett. B 727, 276 (2013).
[3] M. Oertel, M. Hempel, T. Klähn, and S. Typel, Rev. Mod. Phys. 89, 015007 (2017).
[4] Y. Zhang, M. Liu, C.-J. Xia, Z. Li, and S. K. Biswal, Phys. Rev. C 101, 034303 (2020).
[5] R. Essick, I. Tews, P. Landry, and A. Schwenk, Phys. Rev. Lett. 127, 192701 (2021).
[6] S. Huth et al., Nature 606, (2022), arXiv:2107.06229 [nucl-th].
[7] M. Dutra, O. Lourenço, J. S. Sá Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C 85, 035201 (2012).
[8] M. Dutra, Lourenço, Avancini, Carlson, A. Delfino, D. P. Menezes, C. Providência, S. Typel, and J. R. Stone, Phys. Rev. C 90, 055203 (2014).
[9] C.-J. Xia, T. Maruyama, N. Yasutake, T. Tatsumi, H. Shen, and H. Togashi, Phys. Rev. D 102, 023031 (2020).
[10] K. Hebeler, Phys. Rep. 890, 1 (2021).
[11] E. Fraga and P. Romatschke, Phys. Rev. D 71, 105014 (2005).
[12] E. S. Fraga, A. Kurkela, and A. Vuorinen, Astrophys. J. 781, L25 (2014).

[13] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, Nature 467, 1081 (2010).
[14] E. Fonseca, T. T. Pennucci, J. A. Ellis, I. H. Stairs, D. J. Nice, S. M. Ransom, P. B. Demorest, Z. Arzoumanian, K. Crowter, T. Dolch, R. D. Ferdman, M. E. Gonzalez, G. Jones, M. L. Jones, M. T. Lam, L. Levin, M. A. McLaughlin, K. Stovall, J. K. Swiggum, and W. Zhu, Astrophys. J. 832, 167 (2016).
[15] J. Antoniadis, P. C. C. Freire, N. Wex, T. M. Tauris, R. S. Lynch, M. H. van Kerkwijk, M. Kramer, C. Bassa, S. Dhillon, T. Driebe, J. W. T. Hessels, V. M. Kaspi, V. I. Kondratiev, N. Langer, T. R. Marsh, M. A. McLaughlin, T. T. Pennucci, S. M. Ransom, I. H. Stairs, J. van Leeuwen, J. P. W. Verbiest, and D. G. Whelan, Science 340, 1233232 (2013).
[16] LIGO Scientific and Virgo Collaborations, Phys. Rev. Lett. 121, 161101 (2018).
[17] T. E. Riley, A. L. Watts, S. Bogdanov, P. S. Ray, R. M. Ludlam, S. Guillot, Z. Arzoumanian, C. L. Baker, A. V. Bilous, D. Chakrabarty, K. C. Gendreau, A. K. Harding, W. C. G. Ho, J. M. Lattimer, S. M. Morsink, and T. E. Strohmayer, Astrophys. J. 887, L21 (2019).
[18] T. E. Riley, A. L. Watts, P. S. Ray, S. Bogdanov, S. Guillot, S. M. Morsink, A. V. Bilous, Z. Arzoumanian, D. Choudhury, J. S. Deneva, K. C. Gendreau, A. K. Harding, W. C. G. Ho, J. M. Lattimer, M. Loewenstein, R. M. Ludlam, C. B. Markwardt, T. Okajima, C. Prescod-Weinstein, R. A. Remillard, M. T. Wolff, E. Fonseca, H. T. Cromartie, M. Kerr, T. T. Pennucci, A. Parthasarathy, S. Ransom, I. Stairs, L. Guillemot, and I. Cognard, Astrophys. J. 918, L27 (2021).
[19] M. C. Miller, F. K. Lamb, A. J. Dittmann, S. Bogdanov, Z. Arzoumanian, K. C. Gendreau, S. Guillot, A. K. Harding, W. C. G. Ho, J. M. Lattimer, R. M. Ludlam, S. Mahmoodifar, S. M. Morsink, P. S. Ray, T. E. Strohmayer, K. S. Wood, T. Enoto, R. Foster, T. Okajima, G. Prigozhin, and Y. Soong, Astrophys. J. 887, L24 (2019).
[20] M. C. Miller, F. K. Lamb, A. J. Dittmann, S. Bogdanov, Z. Arzoumanian, K. C. Gendreau, S. Guillot, W. C. G. Ho, J. M. Lattimer, M. Loewenstein, S. M. Morsink, P. S. Ray, M. T. Wolff, C. L. Baker, T. Cazeau, S. Manthripragada, C. B. Markwardt, T. Okajima, S. Pollard, I. Cognard, H. T. Cromartie, E. Fonseca, L. Guillemot, M. Kerr, A. Parthasarathy, T. T. Pennucci, S. Ransom, and I. Stairs, Astrophys. J. 918, L28 (2021).
[21] D. Choudhury, T. Salmi, S. Vinciguerra, T. E. Riley, Y. Kini, A. L. Watts, B. Dorsman, S. Bogdanov, S. Guillot, S. Ray, D. J. Reardon, R. A. Remillard, A. V. Bilous, D. Huppenkothen, J. M. Lattimer, N. Rutherford, Z. Arzoumanian, K. C. Gendreau, S. M. Morsink, and W. C. G. Ho, Astrophys. J. Lett. 971, L20 (2024).
[22] P. Bedaque and A. W. Steiner, Phys. Rev. Lett. 114, 031103 (2015).
[23] Y.-L. Ma and M. Rho, Phys. Rev. D 100, 114003 (2019).
[24] C.-J. Xia, Z. Zhu, X. Zhou, and A. Li, Chin. Phys. C 45, 055104 (2021).
[25] M. Hippert, E. S. Fraga, and J. Noronha, Phys. Rev. D 104, 034011 (2021).
[26] H. Tan, V. Dexheimer, J. Noronha-Hostler, and N. Yunes, Phys. Rev. Lett.

128, 161101 (2022).
[27] E. Annala, T. Gorda, J. Hirvonen, O. Komoltsev, A. Kurkela, J. Nättilä, and A. Vuorinen, Nat. Commun. 14, 8451 (2023).
[28] M.-Z. Han, Y.-J. Huang, S.-P. Tang, and Y.-Z. Fan, Science Bulletin 68, 913 (2023).
[29] C.-J. Xia, W.-J. Xie, and M. Bakhiet, Phys. Rev. D 110, 114009 (2024).
[30] LIGO Scientific and Virgo Collaborations, Astrophys. J. 896, L44 (2020).
[31] J.-P. Zhu, S. Wu, Y. Qin, B. Zhang, H. Gao, and Z. Cao, Astrophys. J. 928, 167 (2022).
[32] Y. Yang, V. Gayathri, I. Bartos, Z. Haiman, M. Safarzadeh, and H. Tagawa, Astrophys. J. 901, L34 (2020).
[33] R. Brockmann and W. Weise, Phys. Lett. B 69, 167 (1977).
[34] J. Boguta and S. Bohrmann, Phys. Lett. B 102, 93 (1981).
[35] J. Mareš and J. Žofka, Z. Phys. A 333, 209 (1989).
[36] J. Mareš and B. K. Jennings, Phys. Rev. C 49, 2472 (1994).
[37] Y. Sugahara and H. Toki, Prog. Theor. Phys. 92, 803 (1994).
[38] C. Y. Song, J. M. Yao, H. F. LV, and J. Meng, Int. J. Mod. Phys. E 19, 2538 (2010).
[39] Y. Tanimura and K. Hagino, Phys. Rev. C 85, 014306 (2012).
[40] X.-S. WANG, H.-Y. SANG, J.-H. WANG, and H.-F. LV, Commun. Theor. Phys. 60, 479 (2013).
[41] J. Meng, ed., Relativistic Density Functional for Nuclear Structure, International Review of Nuclear Physics, Vol. 10 (World Scientific Pub Co Pte Lt, 2016).
[42] P.-G. Reinhard, Rep. Prog. Phys. 52, 439 (1989).
[43] P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996).
[44] J. Meng, H. Toki, S. Zhou, S. Zhang, W. Long, and L. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).
[45] N. Paar, D. Vretenar, E. Khan, and G. Colò, Rep. Prog. Phys. 70, 691 (2007).
[46] J. Meng and S. G. Zhou, J. Phys. G: Nucl. Part. Phys. 42, 093101 (2015).
[47] C. Chen, Q.-K. Sun, Y.-X. Li, and T.-T. Sun, Sci. China Phys. Mech. Astron. 64, 282011 (2021).
[48] S. Typel and H. Wolter, Nucl. Phys. A 656, 331 (1999).
[49] D. Vretenar, W. Pöschl, G. A. Lalazissis, and P. Ring, Phys. Rev. C 57, R1060 (1998).
[50] B.-N. Lu, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C 84, 014328 (2011).
[51] B. Wei, Q. Zhao, Z.-H. Wang, J. Geng, B.-Y. Sun, Y.-F. Niu, and W.-H. Long, Chin. Phys. C 44, 074107 (2020).
[52] A. Taninah, S. Agbemava, A. Afanasjev, and P. Ring, Phys. Lett. B 800, 135065 (2020).
[53] N. Glendenning, Compact Stars. Nuclear Physics, Particle Physics, and General Relativity, 2nd ed., 978-0-387-98977-8 (Springer-Verlag, Berlin, 2000).
[54] S. F. Ban, J. Li, S. Q. Zhang, H. Y. Jia, J. P. Sang, and J. Meng, Phys. Rev. C 69, 045805 (2004).
[55] F. Weber, R. Negreiros, P. Rosenfield, and M. Stejner, Prog. Part. Nucl.

Phys. 59, 94 (2007).
[56] W. H. Long, B. Y. Sun, K. Hagino, and H. Sagawa, Phys. Rev. C 85, 025806 (2012).
[57] T. T. Sun, B. Y. Sun, and J. Meng, Phys. Rev. C 86, 014305 (2012).
[58] S. Wang, H. F. Zhang, and J. M. Dong, Phys. Rev. C 90, 055801 (2014).
[59] A. Fedoseew and H. Lenske, Phys. Rev. C 91, 034307 (2015).
[60] Z.-F. Gao, N. Wang, H. Shan, X.-D. Li, and W. Wang, Astrophys. J. 849, 19 (2017).
[61] B. A. Nikolaus, T. Hoch, and D. G. Madland, Phys. Rev. C 46, 1757 (1992).
[62] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[63] W.-H. Long, J. Meng, N. V. Giai, and S.-G. Zhou, Phys. Rev. C 69, 034319 (2004).
[64] Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994).
[65] N. K. Glendenning and S. A. Moszkowski, Phys. Rev. Lett. 67, 2414 (1991).
[66] T. Maruyama, T. Tatsumi, D. N. Voskresensky, T. Tanigawa, and S. Chiba, Phys. Rev. C 72, 015802 (2005).
[67] G. A. Lalazissis, T. Nikšić, D. Vretenar, and P. Ring, Phys. Rev. C 71, 024312 (2005).
[68] S. Typel and D. Alvear Terrero, Eur. Phys. J. A 56, 160 (2020), arXiv:2003.02085 [nucl-th].
[69] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009).
[70] B. A. Brown, Phys. Rev. Lett. 111, 232502 (2013).
[71] PREX Collaboration, Phys. Rev. Lett. 126, 172502 (2021).
[72] CREX Collaboration, Phys. Rev. Lett. 129, 042501 (2022).
[73] C.-J. Xia, T. Maruyama, A. Li, B. Y. Sun, W.-H. Long, and Y.-X. Zhang, Commun. Theor. Phys. 74, 095303 (2022).
[74] R. C. Tolman, Proc. Nat. Acad. Sci. 20, 169 (1934).
[75] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. 55, 374 (1939).
[76] F. J. Fattoyev, J. Carvajal, W. G. Newton, and B.-A. Li, Phys. Rev. C 87, 015806 (2013).
[77] T. Malik, N. Alam, M. Fortin, C. Providência, B. K. Agrawal, T. K. Jha, B. Kumar, and S. K. Patra, Phys. Rev. C 98, 035804 (2018).
[78] T. Hinderer, B. D. Lackey, R. N. Lang, and J. S. Read, Phys. Rev. D 81, 123016 (2010).
[79] S. De, D. Finstad, J. M. Lattimer, D. A. Brown, E. Berger, and C. M. Biwer, Phys. Rev. Lett. 121, 091102 (2018).
[80] J. Lattimer and A. Steiner, Eur. Phys. J. A 50, 40 (2014).
[81] E. Fonseca, H. T. Cromartie, T. T. Pennucci, P. S. Ray, A. Y. Kirichenko, S. M. Ransom, P. B. Demorest, I. H. Stairs, Z. Arzoumanian, L. Guillemot, A. Parthasarathy, M. Kerr, I. Cognard, T. Baker, H. Blumer, P. R. Brook, M. DeCesar, T. Dolch, F. A. Dong, E. C. Ferrara, W. Fiore, N. Garver-Daniels, D. C. Good, R. Jennings, M. L. Jones, V. M. Kaspi, M. T. Lam, D. R. Lorimer, J. Luo, A. McEwen, J. W. McKee, M. A. McLaughlin, N. McMann, B. W.

Meyers, A. Naidu, C. Ng, D. J. Nice, N. Pol, H. A. Radovan, B. Shapiro-Albert, C. M. Tan, S. P. Tendulkar, J. K. Swiggum, H. M. Wahl, and W. W. Zhu, Astrophys. J. 915, L12 (2021).
[82] V. Doroshenko, V. Suleimanov, G. Pühlhofer, and A. Santangelo, Nat. Astron. 6, 1444 (2022).
[83] W.-J. Xie and B.-A. Li, Astrophys. J. 883, 174 (2019).
[84] W.-J. Xie and B.-A. Li, Astrophys. J. 899, 4 (2020).
[85] W.-J. Xie, Z.-W. Ma, and J.-H. Guo, Nucl. Sci. Tech. 34, 91 (2023).
[86] A. Kurkela, E. S. Fraga, J. Schaffner-Bielich, and A. Vuorinen, Astrophys. J. 789, 127 (2014).
[87] J. Alsing, H. O. Silva, and E. Berti, Mon. Not. Roy. Astron. Soc. 478, 1377 (2018), arXiv:1709.07889 [astro-ph.HE].
[88] B.-J. Cai, B.-A. Li, and Z. Zhang, Phys. Rev. D 108, 103041 (2023).
[89] B. P. Abbott, R. Abbott, T. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al., Physical Review Letters 121, 161101 (2018).

Note: Figure translations are in progress. See original paper for figures.